Macroeconomics

Lecture 8

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The Modigliani-Miller Theorem

The dividend stream of a tree is governed by a 2-state Markov process. Dividend takes either $\sigma_1 > 0$ or $\sigma_2 = 0$. Furthermore, once dividends have assumed the value zero, they remain zero forever. (The tree has died and the firm has been bankrupted)

The Modigliani-Miller Theorem

- **Hence, the dividend stream $\{d_t\}$ follows a two-state Markov chain with transition matrix**

$$P = \begin{bmatrix} p_{11} & (1-p_{11}) \\ 0 & 1 \end{bmatrix}, \qquad 0 < p_{11} < 1,$$

$$where \;\; p_{ij} = prob\left\{d_{t+1} = \sigma_j \mid d_t = \sigma_i\right\}$$

$$Thus, \;\; p_{11} \; is \; the \; probability \; that \; a \; living \; tree \; will \; be \; alive \; next \; period.$$

- **Suppose that a tree represents firm (high risk firm). This firm issues bonds (i.e. asset with fixed but low return) and equities (i.e. asset of which returns depended on the performance of the firm).**

The Modigliani-Miller Theorem

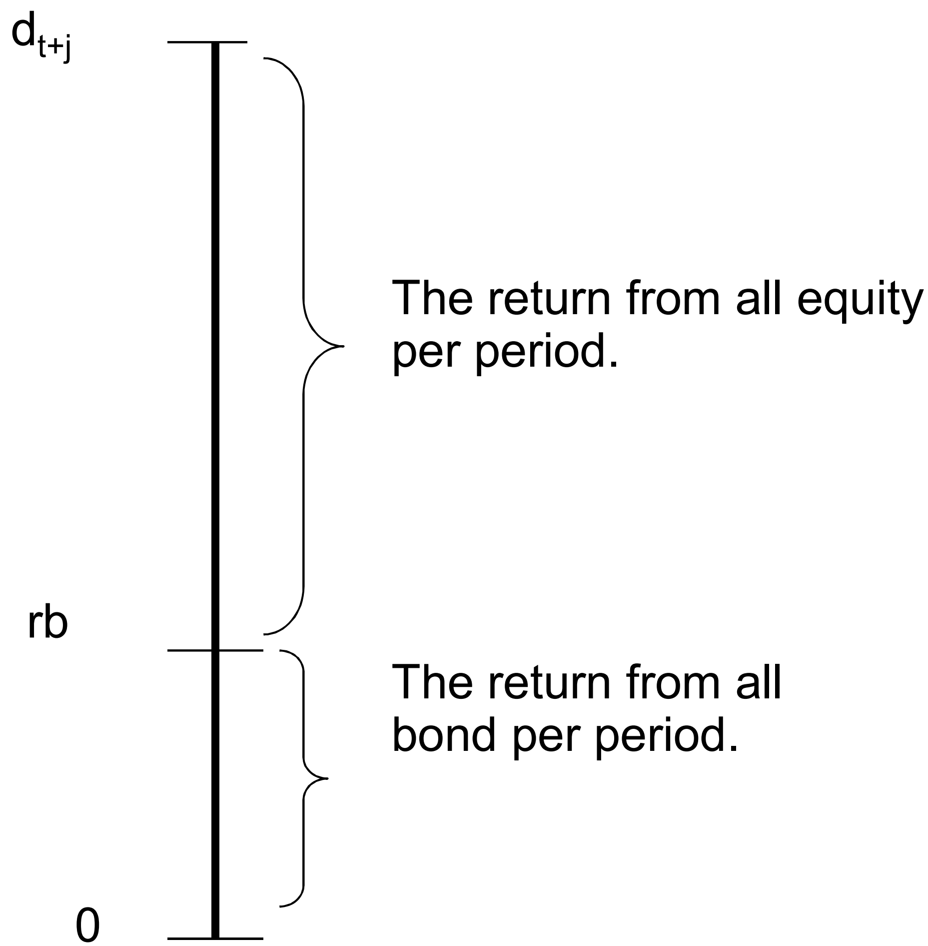
- A firm issues b units of bonds.
- Each bond pays r per period if (i.e., r is market risk free interest rate)

  $rb \le d_{t+j}$,

  and pays $r = d_{t+j}/b$ per period if

  $rb > d_{t+j}$. (since this firm can pay no more than d_{t+j})
- Equity pays $(d_{t+j} - rb)$ per period if $rb < d_{t+j}$,

  and pays 0 if $rb \ge d_{t+j}$.


d_{t+j}
The return from all equity per period.
rb
The return from all bond per period.
0

The Modigliani-Miller Theorem

The total value of bonds is

$$B_t = \sum_{j=1}^{\infty} \int_{rb \le d_{t+j} = x'} rbq^j \left(x_{t+j}, x_t\right) dx_{t+j} + \sum_{j=1}^{\infty} \int_{rb > d_{t+j} = x'} d_{t+j} q^j \left(x_{t+j}, x_t\right) dx_{t+j} \qquad (6)$$

The total value of equities is

$$S_t = \sum_{j=1}^{\infty} \int_{rb < d_{t+j} = x'} \left(d_{t+j} - rb\right) q^j \left(x_{t+j}, x_t\right) dx_{t+j} \qquad (7)$$

The total value of the firm is

$$B_t + S_t = \sum_{j=1}^{\infty} \int_{0}^{d_{t+j} = x'} d_{t+j} q^j \left(x_{t+j}, x_t\right) dx_{t+j} \qquad (8)$$

- **The value of the firm is independent of the number of bonds b outstanding and the coupon rate r.**

The Modigliani-Miller Theorem

- The value of the firm depends on d_{t+j} and $q^j(x_{t+j},x_t)$ because;
- (a) for each j; assumes that

$$prob\left(d_{t+j} \le d' \mid d_t = d\right) = \int_0^{d'} f(s,d)\,ds = F(d',d)$$

- where the state of the economy is $(d_t)=x_t$, so that

$$\int_0^{d'} f(s,d)\,ds = \int_0^{x'} q^j\left(x_{t+j},x_t\right)dx_{t+j}$$

hence, the conditional expectation in (8), the objective function of the firm, is well defined with respect to the above transition probability.

- (b) we know the law of motion of an the price of firm, p_t. (since $p_t=h(d_t)$ and we know the above transition probability)

Government Debt and the Ricardian Proposition

- g_t is per capita government expenditures at t.
- Let $\{g_t\}$, where $g_t<d_t$ for all t, be a nonnegative stochastic process.
- τ_t is a lump-sum per capita taxes.
- $\{\tau_t\}$ is a stochastic process expressible as a function of x_t at time t.
- The government issues one-period debt that is state contingent.

$$The\ government's\ budget\ constraint\ is$$

$$g_t = \tau_t + \int q(x_{t+1}, x_t) b(x_{t+1}) dx_{t+1} - b(x_t) \qquad (9)$$

where $b(x_{t+1})$ *is the amount of* $(t+1)$ *goods that the government promises at* t *to deliver at* $(t+1)$ *when the economy is in state* x_{t+1}.

If the debt is risk free, then $b(x_{t+1}) = b_{t+1}$ *for all* x_{t+1}

$$\int q(x_{t+1}, x_t) b(x_{t+1}) dx_{t+1} = b_{t+1} \int q(x_{t+1}, x_t) dx_{t+1} = \frac{b_{t+1}}{R_{1t}}$$

Hence, Eq.(9) *then becomes*

$$g_t = \tau_t + \frac{b_{t+1}}{R_{1t}} - b_t \qquad (10)$$

Assume that d_t is given by a Maarkov process with transition density $f\left(d_{t+1}, d_t\right)$.

Assume that g_t, d_t are jointly described by a Markov process with transition density $h\left(g_{t+1}, d_{t+1} \mid g_t, d_t\right)$, where

$$prob\left\{d_{t+1} \le d', g_{t+1} \le g' \mid g_t = g,\ d_t = d\right\} = \int_0^{d'}\int_0^{g'} h\left(z, s; g, d\right) ds dz$$

The state of the economy is now $\left(g_t, d_t\right) \equiv x_t$.

If we write (9) in the form

$$b\left(x_t\right) = \tau_t\left(x_t\right) - g_t\left(x_t\right) + \int_0^{x'} q\left(x_{t+1}, x_t\right) b\left(x_{t+1}\right) dx_{t+1},$$

and iterate to eliminate $b\left(x_{t+1}\right)$,

$$b\left(x_t\right) = \tau_t - g_t + \sum_{j=1}^{\infty}\int_0^{x'}\left[\tau_{t+j}\left(x_{t+j}\right) - g_{t+j}\left(x_{t+j}\right)\right] q^j\left(x_{t+j}, x_t\right) dx_{t+j} \quad (11)$$

If all government debt is perfectly safe, (11) simplifies to

$$b_t = \tau_t - g_t + \sum_{j=1}^{\infty}\left(\tau_{t+j} - g_{t+j}\right) R_{jt}^{-1}$$

where $R_{jt}^{-1} = \int_0^{x'} q^j\left(x_{t+j}, x_t\right) dx_{t+j}$

The social planning problem is to

$$\max E_0\{\sum_{t=0}^{\infty} \beta^t U(c_t)\}$$

$$s.t. \quad c_t \le d_t - g_t$$

The solution is $c_t = d_t - g_t$.

The equilibrium state-contingent prices are

$$q(x_{t+1}, x_t) = \beta \frac{\partial U(d_{t+1} - g_{t+1}) / d(d_{t+1} - g_{t+1})}{\partial U(d_t - g_t) / d(d_t - g_t)} . h(d_{t+1}, g_{t+1} \mid d_t, g_t)$$

Ricardian Proposition

- The equilibrium consumption stream and state-contingent prices depend only on the stochastic process for output d_t and government expenditure g_t. They are both independent of the stochastic process τ_t for taxes.
- The choices of the time pattern of taxes and government bond issues have no effect on any relevant equilibrium price or quantity.

Homework

$$Let \quad R_{jt}^{-1} = \beta^j \int_{x_{t+j}\in\Omega} q^j(x_{t+j}, x_t)dx_{t+j}\,,\ \beta = 1\,,\ j=1,2,3,\ldots$$

$$\beta = discount\ rate$$

Hence $\quad R_{1t}^{-1} = \int_{x_{t+1}\in\Omega} q^1(x_{t+1}, x_t)dx_{t+1}\,.$

Then, please show that,

$$\int_{x_{t+2}\in\Omega} q^2(x_{t+2}, x_t)dx_{t+2}$$

$$= \int_{x_{t+2}\in\Omega}\int_{x_{t+1}\in\Omega} q^1(x_{t+2}, x_{t+1})q^1(x_{t+1}, x_t)dx_{t+1}dx_{t+2}$$