Question 1

If $(s \vee \sim r) \rightarrow (\sim q \vee t)$, $\sim p \vee s$ and $\sim q \rightarrow r$, then $[(p \wedge q) \vee \sim r] \rightarrow t$

Proof by contradiction assume $\sim([(p \wedge q) \vee \sim r] \rightarrow t)$ is true.

1. $\sim([(p \wedge q) \vee \sim r] \rightarrow t)$	Assumption
2. $[(p \wedge q) \vee \sim r] \wedge \sim t$	1, De Morgan's Law, Law of implication, Law of Substitution
3. $(p \vee \sim r) \wedge (q \vee \sim r) \wedge \sim t$	2, Distributive Law
4. $r \rightarrow p$, $r \rightarrow q$ and $\sim t$	3, Definition of Conjunction, Law of implication, Law of Substitution
5. $\sim q \rightarrow r$	Assumption
6. $\sim q \rightarrow p$	4,5, Law of Hypothetical Syllogism
7. $q \vee p$	6, Law of implication, Law of substitution
8. $\sim q \rightarrow q$	4,5, Law of Hypothetical Syllogism
9. $q \vee q$	8, Law of Implication, Law of Substitution
10. q	9, Idempotent law
11. $q \wedge \sim t$	4,10, Definition of Conjunction
12. $(s \vee \sim r) \rightarrow (\sim q \vee t)$	Assumption
13. $(q \wedge \sim t) \rightarrow (\sim s \wedge r)$	12, De Morgan's Law, Law of Contraposition, Rule of substitution
14. $\sim s \wedge r$	11,13, Rule of Modus Ponens
15. $\sim s$ and r	14, Definition of Conjunction
16. p	4, 15, Rule of Modus Ponens
17. $\sim p \vee s$	Assumption
18. s	16, 17, Disjunctive Syllogism
19. $s \wedge \sim s$	18, 15, Definition of Conjunction

Since $s \wedge \sim s$ is contradiction so $\sim([(p \wedge q) \vee \sim r] \rightarrow t)$ is false.
Therefore, $[(p \wedge q) \vee \sim r] \rightarrow t$ is true.

Question 2

Find the solution of $\dfrac{|1-x|-2}{x+|x|-3} > 1$

We separate it into 3 possible cases

Case 1:

When $x \geq 1$, we will get

$$\frac{|1-x|-2}{x+|x|-3} > 1$$

$$\frac{x-1-2}{x+x-3} > 1$$

$$\frac{x-1-2}{2x-3} > 1$$

$$\frac{x-3}{2x-3} - \frac{2x-3}{2x-3} > 0$$

$$\frac{-x}{2x-3} > 0 \text{ ; the critical values are 0 and } \frac{3}{2}$$

Thus, Solution set for Case 1 is $[1, \frac{3}{2})$

Case 2:

When $0 \leq x < 1$, we will get

$$\frac{|1-x|-2}{x+|x|-3} > 1$$

$$\frac{1-x-2}{x+x-3} > 1$$

$$\frac{-x-1}{2x-3} > 1$$

$$\frac{-x-1}{2x-3} - \frac{2x-3}{2x-3} > 0$$

$$\frac{-3x+2}{2x-3} > 0 \text{ ; the critical values are } \frac{2}{3} \text{ and } \frac{3}{2}$$

Thus, the solution set for Case 2 is $(\frac{2}{3}, 1)$

Case 3:

When $x < 0$, we get

$$\frac{|1-x|-2}{x+|x|-3} > 1$$

$$\frac{1-x-2}{x-x-3} > 1$$

$$\frac{-1-x}{-3} > 1$$

$$-x-1 < -3$$

$$x > 2$$

Thus, the solution set of Case 3 is an empty set.

Hence, the solution set of $\frac{|1-x|-2}{x+|x|-3} > 1$ is $[1, \frac{3}{2}) \cup (\frac{2}{3}, 1)$ which is $(\frac{2}{3}, 1)$.

1. Induction

Proof that for every positive integer n,

$n^2 + n$ is divisible by 2

Solution

- Let S_n be the statement $n^2 + n$ which is divisible by 2 for every positive integer
- Show S_1 "$1^2 + 1 = 2$" which is divisible by 2 which is true
- Assume S_k

  $k^2 + k$ is divisible by 2 for every positive integer

  is equivalent to

  $k^2 + k = 2M$, where M is a positive integer

  we want to show that S_{k+1} is true

  $(k+1)^2 + k+1$

  expand

  $$\begin{aligned}(k+1)^2 + k+1 &= k^2 + 2k + 1^2 + k + 1\\ &= (k^2 + k) + (2k + 2)\\ &= 2M + 2(k + 1) \text{ , by } k^2 + k = 2M\\ &= 2(M + k + 1)\end{aligned}$$

  Therefore, S_{k+1} is true

- **Hence, n^2 + n is divisible by 2 for every positive integer by mathematical induction**

2. Solving the inequality

Find the solution set of the inequality $\frac{\boldsymbol{|x-1|-1}}{\boldsymbol{|x-1|}} \leq \frac{\mathbf{2}}{\mathbf{3}}$

<u>Solution</u>

- From $\frac{|x-1|-1}{|x-1|} \leq \frac{2}{3}$ we can take $\frac{2}{3}$ to minus to the another side

$$\frac{|x-1|-1}{|x-1|} - \frac{2}{3} \leq 0$$

$$= \frac{(3|x-1|-3)-(2|x-1|)}{3|x-1|} \leq 0$$

$$= \frac{|x-1|-3}{3|x-1|} \leq 0$$

$= |x-1|-3 \leq 0$, we can put $3|x-1|$ to multiply 0 because $3|x-1|$ is always positive then it would not affect the $\leq$ sign

$= |x-1| \leq 3$

$= -3 \leq x-1 \leq 3$, by the definition of absolute value

$= -2 \leq x \leq 4$

- **Then the solution set of** $\frac{\boldsymbol{|x-1|-1}}{\boldsymbol{|x-1|}} \leq \frac{\mathbf{2}}{\mathbf{3}}$ **will be [-2,4]**

Prove

1.Prove that $\sim u \lor (r \longrightarrow p), u \land p$ then $\sim r$

1. $u \land p$ assumption
2. u and p 1,definition of conjunction
3. $\sim u \lor (r \longrightarrow p)$ assumption
4. $r \longrightarrow p$ 2,3 Rule of Modus Ponens
5. $\sim r$ 2,4 Rule of Modus Ponens

Since, $\sim u \lor (r \longrightarrow p), u \land p$ then $\sim r$ is true. So we prove that $\sim u \lor (r \longrightarrow p), u \land p$ then $\sim r$

Inequlity

2.Slove that $|5x-2| < 20$

Since $|5x-2| < 20$ then -20 $< 5x - 2 < 20$

Hence, $-\frac{18}{5} < x < \frac{22}{5}$

$$\therefore The\ sol^n\ set\ is\ (-\frac{18}{5}, \frac{22}{5})$$

1. *Proof*

If $(\sim p \vee q) \leftrightarrow (p \to s)$, $p \to (q \to \sim r)$, $\sim s \vee r$,

$r \vee \sim (\sim r \to \sim p)$ and $(\sim r \to \sim s) \to q$,
$Then\ (\sim p \vee r\ ) \to \sim(\sim s \wedge p)$

Answer We assume $\sim p \vee r$ and then prove $\sim(\sim s \wedge p)$.
Since $\sim(\sim s \wedge p) \equiv s \vee \sim p \equiv \sim p \vee s \equiv p \to s$ Hence, we prove $p \to s$ instead.

1. $\sim p \vee r$ Assumption
2. $(\sim r \to \sim s) \to q$ ***Assumption***
3. $(s \to r) \to q$ ***2, Law of Contraposition, substitution***
4. $(\sim s \vee r) \to q$ ***3, Law of Implication, substitution***
5. $\sim s \vee r$ ***Assumption***
6. q ***4, 5, Rule of Modus Ponens***
7. $r \vee \sim (\sim r \to \sim p)$ ***Assumption***
8. $r \vee \sim (p \to r)$ ***7, Law of Contraposition, substitution***
9. $r \vee \sim (\sim p \vee r)$ ***8, Law of Implication, substitution***
10. r ***1, 9, Disjunctive Syllogism, Double Negation***
11. $p \to (q \to \sim r)$ ***Assumption***
12. $p \to (\sim q \vee \sim r)$ ***11, Law of Implication, substitution***
13. $p \to \sim (q \wedge r)$ ***12, De Morgan's Laws, substitution***
14. $q \wedge r$ ***6, 10, Definition of Conjunction***
15. $\sim p$ 13, 14, ***Rule of Modus Tollens***
16. $(\sim p \vee q) \leftrightarrow (p \to s)$ ***Assumption***
17. $[(\sim p \vee q) \to (p \to s)] \wedge [(p \to s) \to (\sim p \vee q)]$ ***16, Law of Equivalence, substitution***
18. $[(\sim p \vee q) \to (p \to s)]$ and $[(p \to s) \to (\sim p \vee q)]$ ***17, Definition of Conjunction***
19. $\sim p \to (\sim p \vee q)$ 15, Law of Addition, ***Substitution***
20. $\sim p \vee q$ 15, 19, ***Rule of Modus Ponens***
21. $p \to s$ ***18, 20, Rule of Modus Ponens***

2. ***Solving the inequality***

Find the solution set of the inequality $\frac{x^2+6x+9}{x^2-8x+16} - \left|\frac{x+3}{x-4}\right| - 30 < 0.$

Answer $\frac{(x+3)^2}{(x-4)^2} - \left|\frac{x+3}{x-4}\right| - 30 < 0$

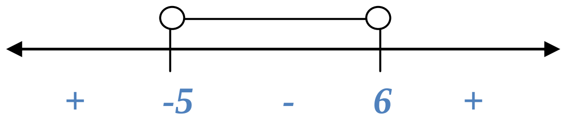
$$\left|\frac{x+3}{x-4}\right|^2 - \left|\frac{x+3}{x-4}\right| - 30 < 0$$

Let $a = \left|\frac{x+3}{x-4}\right|$ ***, we have***

$$a^2 - a - 30 < 0$$

$$(a+5)(a-6) < 0$$

The critical values are -5 and 6



The solution set of $a^2 - a - 30 < 0$ ***is (-5, 6) or*** $-5 < a < 6$

We substitute $a = \left|\frac{x+3}{x-4}\right|$ back into the inequality, we get

$$-5 < \left|\frac{x+3}{x-4}\right| < \mathbf{6}$$

$$-5 < \left|\frac{x+3}{x-4}\right| \quad \textit{and} \quad \left|\frac{x+3}{x-4}\right| < \mathbf{6}$$

From $-5 < \left|\frac{x+3}{x-4}\right|$ ***, we know that this case is always true because absolute is always greater than or equal to 0. Then x can be any real number.***

From $\left|\frac{x+3}{x-4}\right| < \mathbf{6}$ ***, we get*** $|x+3| < \mathbf{6}|x+4|$

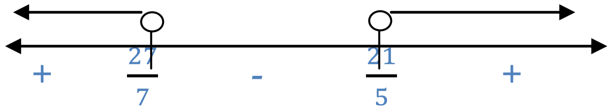
$$|x+3| < |6x+24|$$

$$|x+3|^2 < |6x+24|^2$$

$$(6x+24)^2 - (x+3)^2 > \mathbf{0}$$

$$(7x-27)(5x-21) > 0$$

The critical values are $\frac{27}{7}$ ***and*** $\frac{21}{5}$



The solution set is $(-\infty, \frac{27}{7}) \cup (\frac{21}{5}, \infty)$

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1.Using the given premises, ~e →(b→a) and (e v d)→(c v a) , prove that the conclusion a v ~b v c is true

<u>Proof</u>

1.	~e →(b→a)	assumption
2.	(e v d)→(c v a)	assumption
3.	~(e v d) v (c v a)	2 , Law of implication, sub.
4.	(~e^~) v (c v a)	3 , De Morgan's laws, sub.

5. (~e v c v a) ^ (~d v c v a) — 4 , Distributive laws, sub.
6. (~e v c v a) and (~d v c v a) — 5 , Def. of conjunction
7. d→(c v a) — 6 , Law of implication, sub.
8. ~(c v a)→~t — 7 ,Law of contraposition, sub.
9. ~(c v a)→(b→a) — 8,1 ,Transitive laws, sub.
10. ~(b→a)→(c v a) — 9 ,Law of contraposition,sub.
11. (b^~a) → (c v a) — 10 ,Law of implication, sub.
12. ~(b ^ ~a) v (c v a) — 11 ,Law of implication, sub.
13. (~b v a) v (c v a) — 12 ,De Morgan's laws, sub.
14. (a v a) v ~b v c — 13 ,Associative laws, sub.
15. a v ~b v c — 14 ,Idenpotency laws, sub.

2. $|2x + 3| \geq 8$
This solution will involve setting up two separate inequalities and solving each.

$2x + 3 \leq -8$ or $2x + 3 \geq 8$

$2x \leq -11$ or $2x \geq 5$

$X \leq \frac{11}{2}$ $\quad x \geq \frac{5}{2}$

In interval notation, the answer is $(-\infty, -\frac{11}{2}]$ U $[\frac{5}{2}, \infty)$

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TU152

Method of Proof

Prove that if $(I \vee J) \rightarrow (F \vee H)$, $(G \rightarrow I) \wedge (H \rightarrow J)$, $F \vee (G \vee H)$, and $\sim F$, then H

1.$F \vee (G \vee H)$	Assumption
2.$\sim F$	Assumption
3. $G \vee H$	1, 2, Disjunctive Syllogism
4.$(G \rightarrow I) \wedge (H \rightarrow J)$	Assumption
5. $I \vee J$	3, 4, Constructive Dilemma
6. $(I \vee J) \rightarrow (F \vee H)$	Assumption
7.$F \vee H$	5, 6, Rule of Modus Ponens
8. $\sim F$	Assumption

9.H 7, 8, Disjunctive Syllogism

<u>Solving the inequalities</u>

$$\left|\frac{7-3y}{3}\right| + 2 \geq 4$$

Note: $\left|\frac{7-3y}{3}\right| = \begin{cases} \frac{7-3y}{3} \geq 0 \\ \frac{7-3y}{3} < 0 \end{cases}$

1. We isolate the absolute value expression.

$$\left|\frac{7-3y}{3}\right| \geq 2$$

2. We use the definition of absolute value to set up the inequality without absolute values.
3. Solve the linear inequalities set up in step 2.

We can determine the inequality into 2 cases

<u>Case1</u>

$$\frac{7-3y}{3} \leq -2$$
$$7-3y \leq -6$$
$$-3y \leq -13$$
$$y \geq \frac{13}{3}$$

Hence, the solution set for case1 is $\left[\frac{13}{3}, \infty\right)$

<u>Case2</u>

$$\frac{7-3y}{3} \geq 2$$
$$7-3y \geq 6$$
$$-3y \geq -1$$
$$y \leq \frac{1}{3}$$

Hence, the solution set for case2 is $\left(-\infty, \frac{1}{3}\right]$

Therefore, the solution set of this inequality $\left|\frac{7-3y}{3}\right| + 2 \geq 4$ is $\left(-\infty, \frac{1}{3}\right] \cup \left[\frac{13}{3}, \infty\right)$

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<u>1.Proof</u>

Using the given promises, $(p \land q) \to r$, $\sim (r \lor s)$, p , prove that the conclusion ~q is true.

1. $\sim (r \lor s)$	Assumption
2. $\sim r \land \sim s$	1, De Morgan's Law
3. ~r and ~s	2, Definition of Conjunction
4. $(p \land q) \to r$	Assumption

5.~ (p ∧ q)	3, 4, Rule of Modus Tollens
6.~p ∨ ~q	5, De Morgan’s Law
7. p	Assumption
8. ~q	6, 7, Disjunctive Syllogism, double negation, substitution

Hence, ~q is true according to the proof above.

2. Solving the inequality

$\frac{x^2-1}{|x+1|}$ -1 > 0

$\frac{x^2-1}{|x+1|}$ > 1

Since, $|x+1|$ is always be positive and cannot equal to 0. Therefore, we can move $|x+1|$ to multiplied with another side without changing the inequality symbol , > . Also, when $|x+1|$ cannot equal to 0, this inequality must be marked up that x will not be -1.

$x^2 - 1 > |x+1|$

$(x^2 - 1)^{\ 2} > (x+1)^2$

$(x^2 - 1)^{\ 2} - (x+1)^2 > 0$

$(x^2 - x - 2)\ (x^2 - x) > 0$

x(x – 2) (x + 1)(x+1) > 0

the critical values in this case are 0, -1 ,2

Calculating;

-Assume that x= -2 the solution will be 8 which is true because it is more than 0. So, when x < -1 the inequality is true.

-Assume that x=-0.5 , the solution will be 0.3125 which is true because it is more than 0. So, when -1<x < 0 the inequality is true.

-Assume that x= 1, the solution will be -4 which will make the equality false because it is not more than 0. So, when 0<x<2 the set will not exist on the solution set

-Assume that x=3 , the solution will be 48 which is true because it is more than 0. So, when x>2 the inequality is true.

Due to what we marked up before that X cannot be -1.

In conclusion, the solution set is (-∞, -1) U (-1,0) U (2, ∞) .

1.proof that $\sim(\backsim pVq) \Rightarrow p$ is tautology

1. $[\sim (\backsim p) \wedge \backsim q) \Rightarrow p]$ DeMorgan's Law
2. $[(p \wedge \backsim q) \Rightarrow p]$ Law of Double Negative
3. $[\backsim (p \wedge \backsim q) V p]$ Law of Implication
4. $[\backsim p V \backsim (\backsim q) V p]$ Demorgan's Law
5. $[(\backsim pVq) V p]$ Law of Double Negative
6. $[(\backsim pVp) V q]$ Commutative Laws
7. $[TVq]$ Inverse Law

8.T Domination Laws

$\therefore \sim (\backsim p \vee q) \Rightarrow p$ is tautology

2.Find the solution set of the inequality $\frac{2}{x-2} - \frac{3}{x-4} \leq 1$

$$\frac{2}{x-2} - \frac{3}{x-4} \leq 1$$

$$\frac{2x(x-4)-3x(x-2)-(x-2)(x-4)}{(x-2)(x-4)} \leq 0$$

$$\frac{-2x^2+4x-8}{(x-2)(x-4)} \leq 0$$

$$\frac{2x^2-4x+8}{(x-2)(x-4)} \geq 0$$

Consider $2x^2 - 4x + 8$, we have $b^2 - 4ac = -48 < 0$ hence we cannot factor $2x^2 - 4x + 8$

Since a = 2 > 0, then $2x^2 - 4x + 8 > 0$ for all x so we can only find the values of x that satisfy (x-2)(x-4) > 0

∴ The critical values are 2,4

Hence, the solution set is (-⋈ ,2)∪(4,⋈)

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1.Proof

Using the given promises, (p ∧ q) → r , ~ (r ∨ s) , p , prove that the conclusion ~q is true.

1. ~ (r ∨ s) Assumption
2. ~r ∧ ~s 1, De Morgan's Law
3. ~r and ~s 2, Definition of Conjunction
4. (p ∧ q) → r Assumption
5. ~ (p ∧ q) 3, 4, Rule of Modus Tollens
6. ~p ∨ ~q 5, De Morgan's Law
7. p Assumption
8. ~q 6, 7, Disjunctive Syllogism, double negation, substitution

Hence, ~q is true according to the proof above.

2. Solving the inequality

$\frac{x^2-1}{|x+1|}$ -1 $>$ 0

$\frac{x^2-1}{|x+1|} > 1$

Since, $|x+1|$ is always be positive and cannot equal to 0. Therefore, we can move $|x+1|$ to multiplied with another side without changing the inequality symbol , > . Also, when $|x+1|$ cannot equal to 0, this inequality must be marked up that x will not be -1.

$x^2 - 1 > |x+1|$

$(x^2-1)^2 > (x+1)^2$

$(x^2-1)^2 - (x+1)^2 > 0$

$(x^2 - x - 2)(x^2 - x) > 0$

x(x – 2) (x + 1)(x+1) > 0

the critical values in this case are 0, -1 ,2

Calculating;

-Assume that x= -2 the solution will be 2 which is true because it is more than 0. So, when x < -1 the inequality is true.

-Assume that x=-0.5 , the solution will be -3.5 which is false because it is more than 0. So, when -1<x < 0 the inequality is false.

-Assume that x=3 , the solution will be 1 which is true because it is more than 0. So, when x>2 the inequality is true.

Due to what we marked up before that X cannot be -1.

In conclusion, the solution set is (-∞, -1) U (2, ∞) .

TU152

Method of Proving

If $a \to (b \wedge c)$, $(d \wedge c) \to (e \wedge \sim a)$, $a \vee (d \to \sim e)$ and $d \wedge (b \to e)$, then $\sim e \leftrightarrow \sim a$

Solution Since $[\ \sim e \leftrightarrow \sim a] \leftrightarrow [\ (\sim e \to \sim a)\ \wedge\ (\sim a \to \sim e)]$, by law of Equivalence, we can then prove $(\sim e \to \sim a)\ \wedge\ (\sim a \to \sim e)$ instead of $\sim e \leftrightarrow \sim a$. To prove $(\sim e \to \sim a)\ \wedge\ (\sim a \to \sim e)$,

firstly, we have to prove $\sim e \to \sim a$ by assuming $\sim e$ and show $\sim a$

1.	$\sim e$	assumption
2.	$d \wedge (b \to e)$	assumption
3.	d *and* $(b \to e)$	2, Definition of Conjunction
4.	$\sim b$	1, 3 and Rule of Modus Tollens
5.	$\sim b \to (\sim b \vee \sim c)$	Law of Addition
6.	$\sim b \vee \sim c$	4, 5 and Rule of Modus Ponens
7.	$\sim (b \wedge c) \leftrightarrow (\sim b \vee \sim c)$	De Morgan's Laws
8.	$\sim (b \wedge c)$	6, 7 and Rule of Substitution
9.	$a \to (b \wedge c)$	assumption
10.	$\sim a$	8, 9 and Rule of Modus Tollens

Secondly, we have to prove , $\sim a \to \sim e$ by assuming $\sim a$ and show $\sim e$

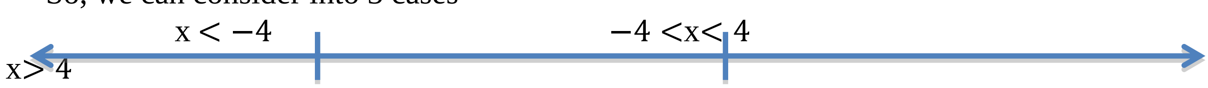
1.	$\sim a$	assumption
2.	$a \vee (d \to \sim e)$	assumption
3.	$d \to \sim e$	1, 2 , and Disjunctive Syllogism
4.	$d \wedge (b \to e)$	assumption
5.	d and $b \to e$	4, Definition of Conjunction
6.	$\sim e$	4, 5 and Rule of Modus Ponens

Since $(\sim e \to \sim a)\ \wedge\ (\sim a \to \sim e)$ is true , $\sim e \leftrightarrow \sim a$ is true.

Solving the inequalities

$$|x-4| + |x+4| < 12$$

Solution Since x-4 = 0 when x = 4 and x+4 = 0 when x = -4 ,
we can see that the critical values are X = -4 and x = 4.
So, we can consider into 3 cases



Case -4 Case 4 Case

when $x < -4$
$|x-4|$ = -x + 4
and $|x+4|$ = - x – 4
$|x-4| + |x+4| < 12$
-x + 4 + (-x – 4) < 12
$-2x < 12$
x > -6
so the solution set of case 1 is (-6 , ∞) $\cap(-\infty, -4)$
= (-6,-4)

Therefore, the solution set of the inequalities is (-6,-4)$\cup(-4,4)\cup(4,6)$ = (-6,6)

When -4 <x< 4
$|x-4|$ = -x + 4
and $|x+4|$ = x + 4
$|x-4| + |x+4| < 12$
-x + 4 + x +4 < 12
8 < 12
Hence, x = R but because -4 <x< 4 ,
The solution set of case 2 is
[-4,4]

When x> 4
$|x-4|$ = x – 4
and $|x+4|$ = x + 4
$|x-4| + |x+4| < 12$
2x < 12
x < 6
Hence, the solution set of case 3 is (-∞,6) $\cap(4,\infty)$ = (4,6)

1) Prove that if x Λ ~ t , t V s V ~ r , then ~ s → ~ r

PROVE we assume ~ s prove ~ r

1) ~ s	assumption
2) x Λ ~ t	assumption
3) x and ~ t	2, Definition of Conjunction
4) t V s V ~ r	assumption
5) t V (s V ~ r)	4, Associative Laws
6) s V ~ r	3, 5, Disjunctive Syllogism, Double Negation, Substitution
7) ~ r	1, 6, Disjunctive Syllogism, Double Negation, Substitution

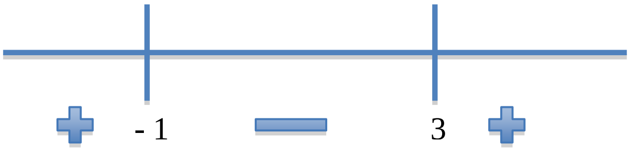
2) Solve the inequality $2x^2 - 4x \geq 6$

SOLUTION

1) $2x^2 - 4x - 6 \geq 0$

2) $(2x + 2)(x - 3) \geq 0$

$\therefore$ The critical values are -1 and 3



$\therefore$ Hence the solution set is $(-\infty,-1] \cup [3,\infty)$

1. Prove that if $(a \vee b) \rightarrow c$, $(a \rightarrow c) \rightarrow \sim(e \wedge f)$, $(e \rightarrow \sim f) \rightarrow (f \vee \sim g)$ and $f \rightarrow \sim(d \vee \sim d)$ then, $\sim g$

1. $(a \vee b) \rightarrow c$	Assumption
2. $\sim(a \vee b) \vee c$	1, Law of implication
3. $(\sim a \wedge \sim b) \vee c$	2, De Morgan's Law
4. $(\sim a \vee c) \wedge (\sim b \vee c)$	3, Distributive Laws
5. $(a \rightarrow c) \wedge (b \rightarrow c)$	4, Law of implication
6. $a \rightarrow c$ and $b \rightarrow c$	5, Definition of conjunction
7. $(a \rightarrow c) \rightarrow \sim(e \wedge f)$	Assumption
8. $\sim(e \wedge f)$	6,7, Rule of Modus Ponens
9. $\sim e \vee \sim f$	8,De Morgan's Law
10. $e \rightarrow \sim f$	9, Law of implication
11. $(e \rightarrow \sim f) \rightarrow (f \vee \sim g)$	Assumption
12. $f \vee \sim g$	10,11 Rule of Modus ponens
13. $d \vee \sim d$	Law of excluded middle
14. $f \rightarrow \sim(d \vee \sim d)$	Assumption
15. $\sim f$	13,14 Rule of Modus Tollens
16. $\sim g$	12,15, Disjunctive Syllogism

2. Find the solution set of the equality $\left|\frac{-x^3+3x^2-18x}{x+2}\right| \geq 4x$

$$\frac{-x^3+3x^2-18x}{x+2} \geq 4x$$
$$\frac{-x^3+3x^2-18x}{x+2} - 4x \geq 0$$
$$\frac{-x^3+3x^2-18x-4x(x+2)}{x+2} \geq 0$$
$$\frac{-x^3+3x^2-18x-4x^2-8x}{x+2} \geq 0$$
$$\frac{-x^3-x^2-26x}{x+2} \geq 0$$
$$\frac{-x(x^2+x+26)}{x+2} \geq 0$$

Since x^2+x+26 cannot be factored because
b^2-4ac =1-4(1)(26)<0 and a=1, then x^2+x+26>0
So, we can only find the values of x that satisfy $\frac{-x}{x+2} \geq 0$

-2 0

\- + -

Noted that x cannot equal to -2 otherwise, the inequity is not defined.
Hence, the solution set for
$\frac{-x^3+3x^2-18x}{x+2} \geq 4x$ is (-2, 0]

or $\frac{-x^3+3x^2-18x}{x+2} \leq -4x$

$$\frac{-x^3+3x^2-18x}{x+2} + 4x \leq 0$$
$$\frac{-x^3+3x^2-18x+4x(x+2)}{x+2} \leq 0$$
$$\frac{-x^3+3x^2-18x+4x^2+8x}{x+2} \leq 0$$
$$\frac{-x^3+7x^2-10x}{x+2} \leq 0$$
$$\frac{-x(x^2-7x+10)}{x+2} \leq 0$$
$$\frac{-x(x-5)(x-2)}{x+2} \leq 0$$

Therefore, the critical values are -2, 0, 2, 5

-2 0 2 5

\- + - + -

Hence, the solution set for

$\frac{-x^3+3x^2-18x}{x+2} \leq -4x$ is

(-∞,-2) U [0,2] U [5,∞)

Therefore, the solution set is

(-2, 0] U (-∞,-2) U [0, 2] U [5, ∞) =

(-∞,-2) U (-2, 2] U [5, ∞) Ans

Prove that if ~(a ∨ ~b) , (b ∨ c) → d , and e→(~d ∨ a) then ~[(~e ∨ f)→ ~d]

1. ~(a ∨ ~b)	Assumption
2. ~a ∧ b	1, De Morgan's Laws
3. ~a and b	2, Definition of Conjunction
4. b→ (b ∨ c)	3, Law of Addition
5. b ∨ c	3, 4, Rule of Modus Ponens
6. (b ∨ c) → d	Assumption
7. d	5, 6, Rule of Modus Ponens
8. d ∧ ~a	3, 8, Definition of Conjunction
9. e→(~d ∨ a)	Assumption
10. e→ ~ (d ∧ ~a)	9, De Morgan's Laws
11. ~e	8, 11, Rule of Modus Tollens
12. ~e →(~e ∨ f)	11, Law of Addition
13. ~e ∨ f	11, 12, Rule of Modus Ponens
14. (~e ∨ f) ∧ d	7, 13, Definition of Conjunction
15. ~[~(~e ∨ f) ∨ ~d]	14, Law of Double Negation, De Morgan's Law
16. ~[(~e ∨ f)→ ~d]	15, Law of Implication

Find the solution set of the inequality $\frac{(2x^2+7x-15)(x^2+4x+9)}{(x+1)} \geq 0$

Solution

$$\frac{(2x^2+7x-15)(x^2+4x+9)}{(x+1)} \geq 0$$

$$\frac{(2x-3)(x+5)(x^2+4x+9)}{(x+1)} \geq 0$$

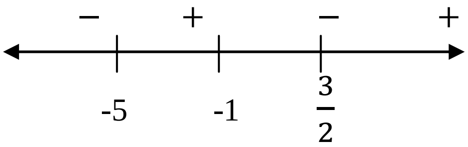
Consider x^2+4x+9 , we have $b^2-4ac<0$ hence, we cannot factor x^2+4x+9

Since a = 1 which is more than 0, then $x^2+4x+9>0$ for all x

So we can only find the value of x that satisfy

$$\frac{(2x-3)(x+5)}{(x+1)} \geq 0$$

The critical value are -5, -1, $\frac{3}{2}$



Hence, the solution set is [-5,1) ∪ [$\frac{3}{2}$, ∞)

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Mathematical Induction

1) Use mathematical induction to prove that

$$S_n : 1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1}n^2 = (-1)^{n-1} \cdot \frac{n}{2}(n+1)$$

is true for every positive integer n.

Proof: Let S_n be the statement

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1}n^2 = (-1)^{n-1} \cdot \frac{n}{2}(n+1)$$

where n is every positive integer.

1) S_1 is "$1^2 = (-1)^{1-1} \cdot \frac{1}{2}(1+1)$", that is "1=1"

which is true.

2) Assume that S_K is true and show that S_{k+1} is true.

That is, we assume

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{k-1}k^2 = (-1)^{k-1} \cdot \frac{k}{2}(k+1) \quad \text{(Assumption)}$$

where k is any positive integer.

We need to show

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{k}(k+1)^2 = (-1)^{k} \cdot \frac{k+1}{2}((k+1)+1)$$

Since $1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{k-1}k^2 = (-1)^{k-1} \cdot \frac{k}{2}(k+1)$ is true

So we add $(-1)^k \times (k+1)^2$ on both sides

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + [(-1)^{k-1}k^2] + [(-1)^k(k+1)^2] = \left[(-1)^{k-1} \cdot \frac{k}{2}(k+1)\right] + [(-1)^k \cdot (k+1)^2\ ]$$

$$= \left[\frac{(-1)^k}{(-1)} \cdot \frac{k}{2}(k+1)\right] + [(-1)^k \cdot (k+1)^2\ ]$$

$$= (-1)^k \cdot (k+1) \cdot \left[\left(\frac{1}{-1}\right) \cdot \frac{k}{2} + (k+1)\right]$$

$$= (-1)^k \cdot (k+1) \cdot \left[-\frac{k}{2} + (k+1)\right]$$

$$= (-1)^k \cdot (k+1) \cdot \left[\frac{k+2}{2}\right]$$

$$= (-1)^k \cdot (k+1) \cdot \left[\frac{(k+1)+1}{2}\right]$$

Therefore, S_{k+1} is true

∴By mathematical induction, we conclude that

$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1}n^2 = (-1)^{n-1} \cdot \frac{n}{2}(n+1)$ for any positive integer n.

Inequalities

2)

$$\frac{x-1}{\sqrt{x^2+4x+4}} > 0$$

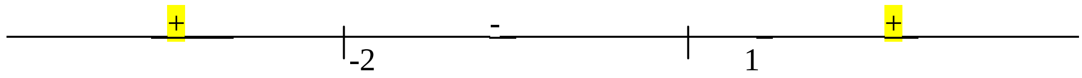
$$\frac{x-1}{\sqrt{(x+2)^2}} > 0$$

$$\frac{x-1}{x+2} > 0$$

Multiply both sides by $(x+2)^2$

So we get; $(x-1)(x+2) > 0 \quad \cap$ denominator $\neq 0$

$(x-1)(x+2) > 0 \quad \cap \ x-2 \neq 0$



Hence, the solution set is $(-\infty, -2) \cup (1, \infty)$

Inequalities

2)

$$\frac{x-1}{\sqrt{x^2+4x+4}} > 0$$

$$\frac{x-1}{\sqrt{(x+2)^2}} > 0$$

$$\frac{x-1}{x+2} > 0$$

Multiply both sides by $(x+2)^2$

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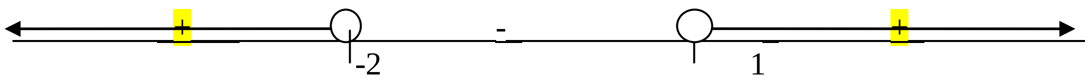
$(x-1)(x+2) > 0 \;\cap\; x-2 \neq 0$



Hence, the solution set is $(-\infty, -2) \cup (1, \infty)$

1) Prove that if p, p → q, and (q v r) → s, then s

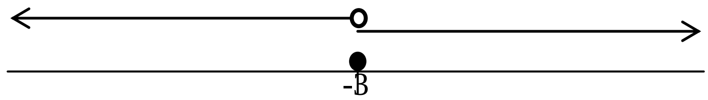
1.	P	Assumption
2.	P → q	Assumption
3.	q	1, 2, Rule of Modus Ponens

4. q → (q v r) 3, Law of Addition
5. q v r 3, 4, Rule of Modus Ponens
6. (q v r) → s Assumption
7. s 5, 6, Rule of Modus Ponens

2) $|x + 3| \leq 5$

<u>Note</u>:

$$|x + 3| = \begin{cases} x + 3; \; x \geq -3 \\ -(x + 3); \; x < -3 \end{cases}$$



<u>Case</u> 1: When $x \geq -3$, we substitute $|x + 3|$ with $x + 3$. So we have,

$$|x + 3| \leq 5$$
$$x + 3 \leq 5$$
$$x \leq 2$$

Hence, the solution set for case 1 is $[-3, \infty) \cap (-\infty, -2) = [-3, -2]$

<u>Case</u> 2: When $x < -3$, we substitute $|x + 3|$ with $-(x + 3)$. So we have,

$$|x + 3| \leq 5$$
$$-(x + 3) \leq 5$$
$$x + 3 > -5$$
$$x > -8$$

Hence, the solution set for case 2 is $(-\infty, -3) \cap (-8, \infty) = (-8, -3)$

Hence, the solution set for the inequality $|x + 3| \leq 5$ is

$[-3, -2] \cup (-8, -2) = (-8, -2)$ <u>Answer</u>

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<u>Mathematical Induction.</u>

Show that for each positive integer n, $1 + 2 + 3 + \ldots + n = n(n + 1) / 2$

Proof: For each positive integer n, let S(n) be the statement $1 + 2 + 3 + \ldots + n$

$= n(n + 1) / 2$

Basis step: $S(1)$ is the statement $1 = 1(1 + 1) / 2$. Thus $S(1)$ is true.

Inductive step: We suppose that $S(k)$ is true and prove that $S(k + 1)$ is true. Thus, we assume that

$1 + 2 + 3 + \ldots + k = k(k + 1) / 2$

and prove that
$1 + 2 + 3 + \ldots + k + k + 1 = (k + 1)(k + 1 + 1) / 2$
If we add $k + 1$ to both sides of the equality in $S(k)$, then on the left side of the sum, we obtain the left side of equality in $S(k + 1)$. Our hope is that the right of the sum equals the right side of $S(k + 1)$. Let us check: Adding $K + 1$ to both sides of $S(k)$ we get:

$1 + 2 + 3 + \ldots + k + k + 1$
$= k(k + 1) / 2 + (k + 1)$
$= (k + 1)(k / 2 + 1)$
$= (k + 1)(k / 2 + 2 / 2)$
$= (k + 1)(k + 2) / 2$
$= (k + 1)(k + 1 + 1) / 2$

Hence, if $S(k)$ is true, then $S(k + 1)$ is true.
Therefore, $1 + 2 + 3 + \ldots + n = n(n + 1) / 2$ for each positive integer n.

<u>Solving Inequalities</u>
– Solve $|2x + 3| + 4 < 5$. Graph the solution set.
First rewrite the inequality by subtracting 4 from each side $|2x + 3| < 1$
The inequality $|2x + 3| < 1$ says that $2x + 3$ is less than 1 units from 0.
$2x + 3 < 1$
$2x < -2$
$x < -1$
AND
$2x + 3 > -1$
$2x > -4$
$x > -2$
The solution set is $\{x | -2 < x < -1\}$

1) Prove that if $(a \rightarrow \sim b) \rightarrow (c \wedge d)$, $\sim d \vee b$, then $\sim a \rightarrow b$

    We assume $\sim a$ and prove b

1.	$\sim a$	assumption
2.	$(a \rightarrow \sim b) \rightarrow (c \wedge d)$	assumption
3.	$(\sim a \vee \sim b) \rightarrow (c \wedge d)$	2,Law of Implication

4. ~a→(~av~b) 1,Law of Addition

5. (~av~b) 1,4,Rule of Modus Ponens

6. c^d 3,5,Rule of Modus Ponens

7. c and d 6,Definition of Conjunction

8. ~dvb assumption

9. b 7,8,Disjunctive Syllogism

2.) Solve the inequality 4< l4x-1l ≤ 9

From 4 < l4x-1l ≤9 , we have

4 < l4x-1l and l4x-1l ≤9

®We consider 4 < l4x-1l we have

4x-1>4 or 4x-1<-4

x>5/4 or x<-3/4

The Solution set of 4 < l4x-1l is (-∞,-3/4)U(5/4,∞)

® We consider l4x-1l≤9 we have

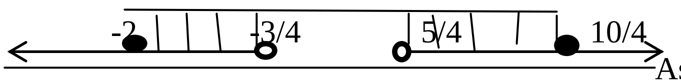
-9≤4x-1≤9

-8≤4x≤10

-2≤x≤10/4

The Solution set of l4x-1l≤9 is [-2,10/4]

Hence,the solution set of the inequality 4< l4x-1l ≤ 9 is [-2,3/4)U(5/4,10/4]#



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1. Prove mathematical induction

Use mathematical induction to prove that

$$S_n : 3 + 3^2 + 3^3 + .. + 3^n = \frac{3(3^n-1)}{2}$$

is true for every positive integer *n*.

<u>Proof</u> Let S_n be the statement

$$3 + 3^2 + 3^3 + .. + 3^n = \frac{3(3^n-1)}{2}$$

Where n is any positive integer.

1) S_1 is " $3^1 = \frac{3(3^1-1)}{2}$" , that is "3 = 3" which is true.
2) Assume that S_k is true and show that S_{k+1} is true.

That is we assume

$$3 + 3^2 + 3^3 + .. + 3^k = \frac{3(3^k-1)}{2}$$

Where k is any positive integer.

We need to show

$$3 + 3^2 + 3^3 + .. + 3^{k+1} = \frac{3(3^{k+1}-1)}{2}$$

Since $3 + 3^2 + 3^3 + .. + 3^k = \frac{3(3^k-1)}{2}$ is true
So we add 3^{k+1} both sides,
And we get $3 + 3^2 + 3^3 + .. + 3^k + 3^{k+1} = \frac{3(3^k-1)}{2} + 3^{k+1}$

$$= \frac{1(3^{k+1})-3+2(3^{k+1})}{2}$$

$$= \frac{3(3^{k+1}-1)}{2}$$

Hence, S_{k+1} is true.

∴By mathematical induction, we conclude that

$$3 + 3^2 + 3^3 + .. + 3^n = \frac{3(3^n-1)}{2}$$

For every positive integer n.

2. $\sqrt{\sqrt{x+1}+x^2} \le 1-x$

Solution. Since $x+1 \ge 0$ and $1-x \ge 0$, $-1 \le x \le 1$. Then, we consider

$$\left(\sqrt{\sqrt{x+1}+x^2}\right)^2 \le (1-x)^2$$

$$\sqrt{x+1}\cancel{+x^2} \le 1-2x\cancel{+x^2}$$

$$\left(\sqrt{x+1}\right)^2 \le (1-2x)^2$$

$$x+1 \le 1-4x+4x^2$$

$$4x^2-5x \ge 0$$

$$x(4x-5) \ge 0$$

Thus, $x \le 0$ and $x \ge \frac{5}{4}$. Also, from Line 2, $1-2x \ge 0$. That is, $x \le \frac{1}{2}$.

Hence, the set of all solutions is $\left((-\infty,0] \cup \left[\frac{5}{4},\infty\right)\right) \cap \left(-\infty,\frac{1}{2}\right] \cap [-1,1] = [-1,0]$.

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1. $(|x|+x-1)(2-|x-1|) > 0$

Solution. This problem can be considered in 3 cases.

<u>Case I</u> $x < 0$ Then, we have

$$\left(\cancel{-x}\,\cancel{+x}-1\right)\left(2+\left(x-1\right)\right)>0$$
$$x+1<0$$
$$x<-1$$

Thus, the solution of this case is $(-\infty,-1)$.

Case II $0\le x<1$ Then, we have

$$(x+x-1)(2+x-1)>0$$
$$(2x-1)(x+1)>0$$

Thus, the solution of this case is $[0,1)\cap\left((-\infty,-1)\cup\left(\frac{1}{2},\infty\right)\right)=\left(\frac{1}{2},1\right)$.

Case III $x\ge 1$ Then, we have

$$(2x-1)(2-x+1)>0$$
$$(2x-1)(x-3)<0$$

Thus, the solution of this case is $[1,\infty)\cap\left(\frac{1}{2},3\right)=[1,3)$.

Hence, the set of all solutions is $(-\infty,-1)\cup\left(\frac{1}{2},1\right)\cup[1,3)=(-\infty,-1)\cup\left(\frac{1}{2},3\right)$.

2. Show that x^n-y^n is divisible by $x-y$ for all positive integers n.

Solution. Basis step: Obviously, $x-y$ is divisible by $x-y$.

Inductive step: By going through one step of division, we have

$$\frac{x^n-y^n}{x-y}=x^{n-1}+\frac{y\left(x^{n-1}-y^{n-1}\right)}{x-y}.$$

Since $x^{n-1}-y^{n-1}$ is divisible by $x-y$ (by the induction hypothesis), x^n-y^n is divisible by $x-y$. Hence, we complete the proof.

1. If x∧y , (y∨z) ∧ (k∧x) ,and x→~z then k∧~z

Proof

1. x∧y assumption
2. x and y 1, Definition of conjunction
3. x→~z assumption

4. $\sim z$ — 2,3, Rule of Modus Ponens
5. $(y \vee z) \wedge (k \wedge x)$ — assumption
6. $(y \vee z)$ and $(k \wedge x)$ — 5, Definition of conjunction
7. k and x — 6, Definition of conjunction
8. $k \wedge \sim z$ — 4,7, Definition of conjunction

2. $\mathbf{x+2 > |x^2+x+2|}$

Since $|x^2+x+2| < x+2$ then $-(x+2) < (x^2+x-2) < x+2$, hence

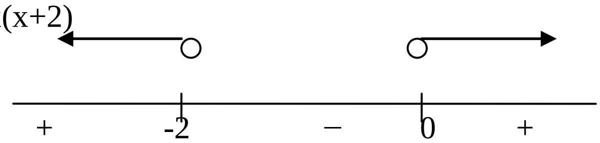
<u>Case1</u> $\quad -(x+2) < (x^2+x-2)$

$$-x-2 < x^2+x-2$$

$$0 < x^2+2x$$

$$0 < x(x+2)$$

The critical values are -2 and 0



The solution of this case is $(-\infty,-2)$ U $(0,\infty)$

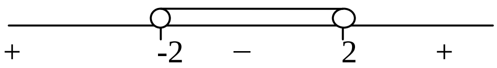
<u>Case2</u> $\quad (x^2+x-2) < x+2$

$$x^2-4 < 0$$

$$(x-2)(x+2)<0$$

The critical values are -2 and 2

The solution of this case is (-2,2)



Therefore the solution set of the inequality $x+2 > |x^2+x+2|$ is $(-\infty,-2)$ U $(0,\infty) \cap (-2,2) = (0,2)$

<u>Induction</u>

$$2^2 + 5^2 + 8^2 + \cdots + (3n-1)^2 = \underline{1}\, n(6n^2+3n-1).$$

Solution. Let $P(n)$: $2^2 + 5^2 + 8^2 + \cdots + (3n - 1)^2 = \frac{1}{2} n(6n^2 + 3n - 1)$.

Firstly, LHS of $P(1) = 2^2 = 4$

$=$ So $P(1)$ is true $(6.1 + 3.1 - 1) =$

RHS of $P(1)$.

Now assume $P(k)$ is true, for some natural number k, i.e.

$2^2 + 5^2 + 8^2 + \cdots + (3k - 1)^2 = k(6k^2 + 3k \quad 1)$

and deduce $P(k + 1)$. We could follow an approach similar to the previous exercise; instead, we will demonstrate another technique: that of expanding an expression in k in powers of $k + 1$ by replacing k by $k + 1 - 1$.

LHS of $P(k + 1) = 2^2 + 5^2 + 8^2 + \cdots$

$+ (3k - 1)^2 + \; 3(k + 1) - 1 \; ^2 = -1$

$k(6k + 3k - 1) + 9(k + 1)^2 - 6(k + 1) + 1$, (by inductive assumption)

$= k \; 3k(2k + 1) - 1 \; + 9(k + 1) - 6(k + 1) + 1$

$= k \; 3 \; (k + 1) - 1 \;\; 2(k + 1) - 1 \; - 1 + 9(k + 1)^2 - 6(k + 1) + 1$

$= k \; 3 \; 2(k + 1) - 3(k + 1) + 1 \; - 1 + 9(k + 1)^2$

$- 6(k + 1) + 1$

$= \frac{1}{2} (k + 1) - 1 \;\; 6(k + 1)^2 - 9(k + 1) + 2 \; + \frac{1}{2} \; 18(k + 1)^2 - 12(k + 1) + 2$

$= \frac{1}{}$ $(k + 1)$ $6(k + 1)^2 - 9(k + 1) + 2$ $-$ $6(k + 1)^2 -$
$9(k + 1) + 2 +$ $(k + 1)$ $18(k + 1) - 12$ $+ 2$

$= \frac{1}{}$ $(k + 1)$ $6(k + 1)^2 - 9(k + 1) + 2 -$ $(k + 1)$ $6(k +$
$1) - 9$ $- 2 +$ $(k + 1)$ $18(k + 1) - 12$ $+ 2$

$=(k + 1)6(k + 1)^2 -$ $9(k + 1) + 2 -$ $6(k + 1) + 9$

$=$ $_2$ $(k + 1)$ $6(k + 1)$

$=$ RHS of $P\,(k + 1)$.

So $P\,(k + 1)$ is true, *if* $P\,(k)$ *is true*.

$-18(k + 1) - 12$

$+ 3(k + 1) - 1$

Conclusion: Hence, by induction $P\,(n)$ is true for all natural numbers n.

Inequality

The solution set of the inequality $|x|^3 - 2x^2 - 4|x| + 3 < 0$ is ______________.

Solution Notice that $|x| = 3$ is a root of the equation $|x|^3 - 2x^2 - 4|x| + 3 = 0$. Then the original inequality can be rewritten as $(|x| - 3)(|x|^2 + |x| - 1) < 0$, that is

$$(|x|-3)\left(|x|-\frac{-1+\sqrt{5}}{2}\right)\left(|x|-\frac{-1-\sqrt{5}}{2}\right)<0.$$

Since $|x| - \frac{-1-\sqrt{5}}{2} > 0$, then $\frac{-1+\sqrt{5}}{2} < |x| < 3$.

So the solution set is $\left(-3, -\frac{\sqrt{5}-1}{2}\right) \cup \left(\frac{\sqrt{5}-1}{2}, 3\right)$.

Reference:
http://school.maths.uwa.edu.au/~gregg/Academy/2011/inductionprobswsoln.pdf
http://mathitude.perso.sfr.fr/PDF/Mathematical_Olympiad_in_China-Problems_and_Solutions.pdf

5504640797
Nichaphat Surawattanon

1. Prove that if (a∧~b), $(a \to c) \land (d \to c)$,and e→ $\sim(b \to c)$, then c∧$\sim e$

Proof:

1. $(a \land\sim b)$	Assumption
2. a and $\sim b$	1, Definition of Conjunction
3. $a \to (a \lor d)$	Law of Addition
4. a $\lor$ d	3, 4, Rule of Modus Ponens
5. $(a \to c) \land (d \to c)$	Assumption
6. $(\sim a \lor c) \land (\sim d \lor c)$	5, Law of Implication, Rule of Substitution
7.$(\sim a \land d) \lor c$	6, Distributive Laws, Rule of Substitution
8.$\sim(a \lor d) \lor c$	7, De Morgan's Laws, Rule of Substitution
9.$(a \lor d) \to$ c	8, Law of Implication, Rule of Substitution
10. c	4, 9, Rule of Modus Ponens
11. e $\to$ $\sim(b \to c)$	Assumption
12. $(b \to c) \to \sim e$	11, Law of Contraposition, Rule of Substitution

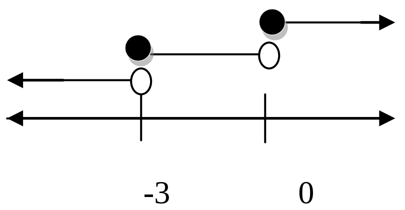
13. $(\sim b \vee c) \rightarrow \ \sim e$ — 12, Law of Implication, Rule of Substitution
14. $\sim b \ \rightarrow \ (\sim b \vee c)$ — Law of Addition
15. $(\sim b \vee c)$ — 2, 14, Rule of Modus Ponens
16. $\sim e$ — 13, 15, Rule of Modus Ponens
17. $c \wedge \sim e$ — 10, 16, Definition of Conjuction

2. $\frac{|3x+9|}{x^2+x+3} > \frac{1}{|x|}$

Note

$|3x+9| = \begin{cases} 3x+9\ ;x \geq -3 \\ -(3x+9)\ ;x < -3 \end{cases}$ and $|x| = \begin{cases} x\ ;x \ \geq 0 \\ -x\ ;x \ < 0 \end{cases}$ and $|x| = \begin{cases} x\ ;x \geq 0 \\ -x\ ;x \ < 0 \end{cases}$

Here are the cases for solving the inequality.



Case I When $x < \ -3$, we substitute $|3x+9|$ with $-(3x+9)$ and $|x|$ with $-x$, so we have

$$\frac{|3x+9|}{x^2+x+3} > \frac{1}{|x|}$$

$$\frac{-(3x+9)}{x^2+x+3} > \frac{1}{-x}$$

$$\frac{(-3x-9)(-x)}{x^2+x+3} < 1$$

$$\frac{3x^2+9x}{x^2+x+3} < 1$$

$$\frac{3x^2+9x}{x^2+x+3} - 1 < 0$$

$$\frac{3x^2+9x-(x^2+x+3)}{x^2+x+3} < 0$$

$$\frac{2x^2+8x-3}{x^2+x+3} < 0$$

Consider $x^2 + x + 3$, we see that $x^2 + x + 3$ $cannot\ be\ factored\ because\ b^2 - 4ac = -11 < 0$ and since a = 1 > 0 for all x. Therefore, we can only find the value of x that satisfy $2x^2 + 8x - 3 < 0$

Thus, from $2x^2 + 8x - 3$, $we\ have\ to\ use\ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ to solve for x.

That is,

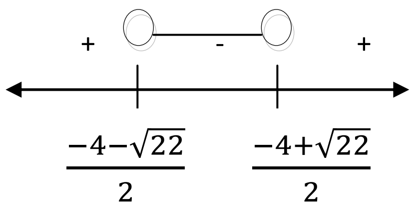
$$X = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$X = \frac{-8 \pm 2\sqrt{22}}{4}$$

$$X = \frac{-8+2\sqrt{22}}{4}, \frac{-8-2\sqrt{22}}{4}$$

$$X = \frac{-4+\sqrt{22}}{2}, \frac{-4-\sqrt{22}}{2}$$

Thus, the critical value are $\frac{-4+\sqrt{22}}{2}$ and $\frac{-4-\sqrt{22}}{2}$



Therefore, the solution set for this case is $\left(\frac{-4-\sqrt{22}}{2}, \frac{-4+\sqrt{22}}{2}\right) \cap (-\infty, -3)$ which is $\left(\frac{-4-\sqrt{22}}{2}, -3\right)$.

<u>Case II</u> When -3 $\leq x < 0$, we substitute |3x + 9|with3x + 9 and |x| with –x, so we have

$$\frac{|3x+9|}{x^2+x+3} > \frac{1}{|x|}$$

$$\frac{3x+9}{x^2+x+3} > \frac{1}{-x}$$

$$\frac{(3x+9)(-x)}{x^2+x+3} < 1$$

$$\frac{-3x^2-9x}{x^2+x+3} - 1 < 0$$

$$\frac{-3x^2-9x-(x^2+x+3)}{x^2+x+3} < 0$$

$$\frac{-4x^2-10x-3}{x^2+x+3} < 0$$

Consider x^2+x+3, we see that x^2+x+3 *cannot be factored because* $b^2-4ac = -11 < 0$ and since a = 1 > 0 for all x. Therefore, we can only find the value of x that satisfy $-4x^2-10x-3 < 0$

Thus, from $-4x^2-10x-3$, *we have to use* $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve for x.

$$x = \frac{-(-10)\pm\sqrt{(-10)^2-4(-4)(-3)}}{2(-4)}$$

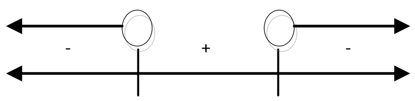
$$x=\frac{10\pm\sqrt{52}}{-8}$$

$$x=\frac{10\pm2\sqrt{13}}{-8}$$

$$x=\frac{5\pm\sqrt{13}}{-4}$$

$$x=\frac{5+\sqrt{13}}{-4}, \frac{5-\sqrt{13}}{-4}$$

Thus, the critical values are $\frac{5+\sqrt{13}}{-4}$ and $\frac{5-\sqrt{13}}{-4}$



$$\frac{5+\sqrt{13}}{-4} \quad \frac{5-\sqrt{13}}{-4}$$

Therefore, the solution set for this case is $\left[\left(-\infty, \frac{5+\sqrt{13}}{-4}\right) \cup \left(\frac{5-\sqrt{13}}{-4}, \infty\right)\right] \cap [-3, 0)$ which is $\left[-3, \frac{5+\sqrt{13}}{-4}\right) \cup \left(\frac{5-\sqrt{13}}{-4}, \infty\right)$

<u>Case III</u> When $x \geq 0$, we substitute $|3x+9|$ with $3x+9$ and $|x|$ with x, so we have

$$\frac{|3x+9|}{x^2+x+3} > \frac{1}{|x|}$$

$$\frac{3x+9}{x^2+x+3} > \frac{1}{x}$$

$$\frac{(3x+9)(x)}{x^2+x+3} - 1 > 0$$

$$\frac{3x^2+9x-(x^2+x+3)}{x^2+x+3} > 0$$

$$\frac{2x^2+8x-3}{x^2+x+3} > 0$$

Consider x^2+x+3, we see that x^2+x+3 *cannot be factored because* $b^2-4ac = -11 < 0$ and since a = 1 > 0 for all x. Therefore, we can only find the value of x that satisfy $2x^2+8x-3 > 0$

Thus, from $2x^2+8x-3$, *we have to use* $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve for x.

That is,

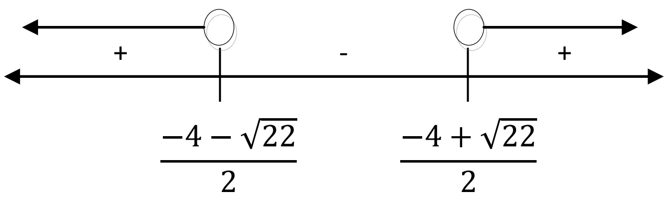
$$X = \frac{-8 \pm \sqrt{8^2-4\times2\times(-3)}}{2\times2}$$

$$X = \frac{-8\pm2\sqrt{22}}{4}$$

$$X = \frac{-8+2\sqrt{22}}{4}, \frac{-8-2\sqrt{22}}{4}$$

$$X = \frac{-4+\sqrt{22}}{2}, \frac{-4-\sqrt{22}}{2}$$

Thus, the critical value are $\frac{-4+\sqrt{22}}{2}$ and $\frac{-4-\sqrt{22}}{2}$



Therefore, the solution set for this case is $\left[\left(-\infty, \frac{-4-\sqrt{22}}{2}\right) \cup \left(\frac{-4+\sqrt{22}}{2}, \infty\right)\right] \cap [0,\infty)$ which is $[0,\infty)$.

Therefore, the solution set of the inequality $\frac{|3x+9|}{x^2+x+3} > \frac{1}{|x|}$ is $\left(\frac{-4-\sqrt{22}}{2}, -3\right) \cup [-3, \frac{5+\sqrt{13}}{-4}) \cup \left(\frac{5-\sqrt{13}}{-4}, \infty\right) \cup [0, \infty)$ which is $\left(\frac{-4-\sqrt{22}}{2}, \frac{5+\sqrt{13}}{-4}\right) \cup \left(\frac{5-\sqrt{13}}{-4}, \infty\right)$

TU152 Assignment

1.Prove that if $p \lor \sim q$, $\sim r \land (p \to s)$,and $q \lor r$ then s

1)	$\sim r \land (p \to s)$	Assumption
2)	$\sim r$ and $p \to s$	1, Definition of conjunction
3)	$q \lor r$	Assumption
4)	q	2,3, Disjunctive Syllogism
5)	$p \lor \sim q$	Assumption
6)	$\sim p \to \sim q$	5, Law of implication
7)	p	4,6, Modus Tollens
8)	s	2,7, Modus Ponens

2. Slove the inequality $x^2 - x - 2 \leq x + 46$

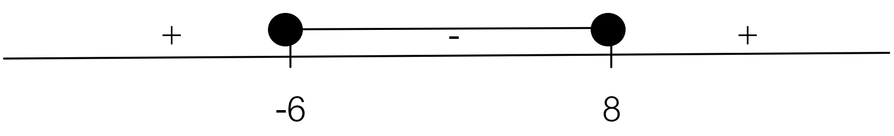
Solution $\quad x^2 - x - 2 \leq x + 46$

$$x^2 - 2x - 48 \leq 0$$

$$(x + 6)(x - 8) \leq 0$$

$\therefore$ The critical values are -6 , 8

Check $\quad x^2 - 2x - 48$ whether it is positive or negative in each interval



Therefore, The solution set is [-6 , 8] Answer

Fundamental Math

1.Prove that 1 + 2 + 3 + . . . + n = $\frac{n(n+1)}{2}$ for any integer n ≥1

Proof:

STEP 1: For n=1 is true, since 1 = $\frac{1(1+1)}{2}$

STEP 2: Suppose is true for some n = k ≥1, that is

$$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2}$$

STEP 3: Prove that is true for n = k + 1, that is

$$1 + 2 + 3 + \ldots + k + (k+1) = \frac{(k+1)(k+2)}{2}$$

So we have $1 + 2 + 3 + \ldots + k + (k+1) = \frac{k(k+1)}{2} + (k+1) =$

$$(k+1)\left(\frac{k}{2} + 1\right) = \frac{(k+1)(k+2)}{2}$$

2.Solve |3x – 8| < 19. Graph the solution set

The inequality |3x – 8| < 19 says that 3x – 8 is less than 19 units from 0.

3x – 8 < 19

3x < 27

x < 9

AND

3x – 8 > –19

3x > –11

x > -11/3

The solution set is {x| -11/3 <x <9

EXAMPLE $1^2 + 4^2 + 7^2 + \ldots\ldots + (3n-2)^2 = \frac{1}{2}n(6n^2 - 3n - 1)$;

Solution. Let P(n) : $1^2 + 4^2 + 7^2 + \ldots\ldots + (3n-2)^2 = \frac{1}{2}n(6n^2 - 3n - 1)$;

Firstly, LHS of P(1) = $1^2 = 1$

$$= \frac{1}{2}(1)\,(6(1)^2 \cdot 3(1) \cdot 1) = \text{RHS of } P(1):$$

So P(1) is true.

Now assume P(k) is true, for some natural number k, i.e.

$$1^2 + 4^2 + 7^2 + \ldots\ldots + (3k - 2)^2 = \frac{1}{2}\, k\,(6k^2 - 3k - 1)$$

and deduce P(k + 1):

$$\begin{aligned}
\text{LHS of } P(k + 1) &= 1^2 + 4^2 + 7^2 + \ldots\ldots + (3k - 2)^2 + (3(k + 1) - 2)^2 \\
&= \frac{1}{2}\, k(6k^2 - 3k - 1) + 9k^2 + 6k + 1 \text{ (by inductive assumption)} \\
&= \frac{1}{2}\,(6k^3 - 3k^2 - k + 18k^2 + 12k + 2) \\
&= \frac{1}{2}\,(6k^3 + 15k^2 + 11k + 2) \\
&= \frac{1}{2}\,(k + 1)(6k^2 + 9k + 2) \\
&= \frac{1}{2}\,(k + 1)(6(k + 1)^2 - 12k - 6 + 9k + 2) \\
&= \frac{1}{2}\,(k + 1)(6(k + 1)^2 - 3k - 4) \\
&= \frac{1}{2}\,(k + 1)(6(k + 1)^2 - 3(k + 1) - 1) \\
&= \text{RHS of } P(k + 1)
\end{aligned}$$

So P (k + 1) is true, if P(k) is true.

Hence, by induction P(n) is true for all natural numbers n.

<u>Example</u> Find the solution set of the inequality $\frac{3}{2}(1-x) > \frac{1}{4} - x$

multiplied both side by 4.

$6\,(1-x) > 1-4x$

$6 - 6x > 1 - 4x$

$-6x + 4x > 1 - 6$

$-2x > -5$

$X < \frac{5}{2}$

<u>Check</u>: Take $x = 0$,

LHS = $\frac{3}{2}$ $(1-0)$ = $\frac{3}{2}$

RHS = $\frac{1}{4}$

It is TRUE that $\frac{3}{2} > \frac{1}{4}$,

<u>Check</u>: Take $x = 3$

LHS = $\frac{3}{2}(1-3) = -3$

RHS= $\frac{1}{4} - 3 = -2\frac{3}{4}$

It is NOT true that $-3 > -2\frac{3}{4}$ and so $x = 3$ is not true.

We can be sure our answer ($x < \frac{5}{2}$) is correct.

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Lapon Wangsermwong

Exmaple 1

Assume for some integer k, P(k): $1+2+3+...+k = \frac{k(k+1)}{2}$

Show: P(k+1): $1+2+3+...+(k+1) = \frac{(k+1)((k+1)+1)}{2}$

$$1+2+3+\ldots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{k^2 + 3k + 2}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)((k+1)+1)}{2}$$

Thus P(k+1) is true when P(k) is true, and therefore P(n) is true for all natural number

Example 2

Solve the inequality $2\left|\frac{2x}{3}+1\right| \geq 4$

$2\left|\frac{2x}{3}+1\right| \geq 4$

This gives

$\left|\frac{2x}{3}+1\right| \geq 2$

So either...

$\frac{2x}{3}+1 \leq -2$ OR $\frac{2x}{3}+1 \geq 2$

Multiply both sides by 3

$2x+3 \leq -6$

$2x \leq -9$

$x \leq -9/2$

OR

$2x+3 \geq 6$

$2x \geq 3$

$x \geq 3/2$

Solution: $x \leq -9/2$ or $x \geq 3/2$

1. Use Mathematical induction to prove that $2+4+6+\ldots+2n = n(n+1)$ for every positive integer n

Proof Let S_n be the statement $2+4+6+\ldots+2n = n(n+1)$ for every positive integer n

1. S_1 is " $2 = 1(1+1) = 2$" which is true
2. Assume S_k is true and we need to show that S_{k+1} is true
   That is we assume $2+4+6+\ldots+2n = n(n+1)$
   We need to show $2+4+6+\ldots+2(n+1) = (n+1)(n+2)$

Since $2+4+6+\ldots+2n = n(n+1)$

$2+4+6+\ldots+2n = n^2+n$

Then $2+4+6+\ldots+2n+(2n+1) = n^2+n+2n+1$

$2+4+6+\ldots+2n+(2n+1) = n^2+3n+1$

$2+4+6+\ldots+2n+(2n+1) = (n+2)(n+1)$

Hence, S_{k+1} is true.

Therefore, by Mathematical Induction we conclude that $2+4+6+\ldots+2n = n(n+1)$ for every positive integer n

2. Find the solution set of the equality $|X+6| - |X-3| \leq 7$

Solution

$|X+6|$ {X+6; $X+6 \geq 0$ or $X \geq -6$

{-X+6; $X+6<0$ or $X<-6$

And

$|X-3|$ { X-3; $X-3 \geq 0$ or $X \geq 3$

{-X-3; $X-3<0$ or $X<-3$

We have 3 cases

Case1 When $X<-6$, we substitute $|X+6|$ with $-(X+6)$ and substitute $|X-3|$ with $-(X-3)$

$-(X+6)-[-(X-3)] \leq 7$

$-X-6+X-3 \leq 7$

$-9 \leq 7$

Which is impossible. Hence, the solution set for case1 is $(-\infty,-6) \cap \{\ \} = \{\ \}$

Case2 When $-6 \leq X < 3$, we substitute $|X+6|$ with $X+6$ and substitute $|X-3|$ with $-(X-3)$

$X+6-[-(X-3)] \leq 7$
$X+6+X-3 \leq 7$
$2X+3 \leq 7$
$X \leq 2$
The solution set for case2 is $[-6,3)\cap[2,\infty) = [2,3)$
<u>Case3</u> When $X\geq3$, we substitute $|X+6|$ with X+6 and substitute $|X-3|$ with X-3
$(X+6)-(X-3) \leq 7$
$3 \leq 7$
Which is true for all values of X. Hence, the solution set ofcase3 is $[3,\infty)\cap R = [3,\infty)$
Therefore, the solution set of the inequality $|X+6| - |X-3| \leq 7$ is $\{\ \}U[2,3)U[3,\infty) = [2,\infty)$

1. Prove that if $(r \to\sim s) \to (q \to\sim w), s \to (p \vee\sim t)$, and $(t \to p) \to\sim$ t, then

$[\,(\sim w \vee r) \wedge q\,] \to (w \to\sim t)$

Since $[\,(\sim w \vee r) \wedge q\,] \to (w \to\sim t) \equiv \sim (w \to\sim t) \to \sim [(\sim w \vee r) \wedge q]$ by law of contraposition,

we can assume $\sim (w \to\sim t)$ and prove $\sim [(\sim w \vee r) \wedge q]$

1. $\sim (w \to\sim t)$	Assumption
2. $\sim (\sim w \vee\sim t)$	1, Law of implication, Law of double negation, Rule of substitution
3. $w \wedge t$	2, De Morgan’s law, Law of double negation, Rule of substitution
4. w *and* t	3, Definition of conjunction
5. $(t \to p) \to\sim t$	Assumption
6. $\sim (t \to p)$	4, 5, Rule of Modus Tollens, Law of double negation, Rule of substitution
7. $\sim (\sim t \vee p)$	6, Law of implication, Rule of substitution
8. $\sim (p \vee\sim t)$	7, Commutative law, Rule of substitution
9. $s \to (p \vee\sim t)$	Assumption
10. $\sim s$	8, 9, Rule of Modus Tollens
11. $\sim s \to (\sim s \vee\sim r)$	Law of addition
12. $\sim s \vee\sim r$	10, 11, Rule of Modus Ponens

13. $\sim r \vee \sim s$	12, Commutative law, Rule of substitution
14. $r \rightarrow \sim s$	13, Law of implication, Law of double negation, Rule of substitution
15. $(r \rightarrow \sim s) \rightarrow (q \rightarrow \sim w)$	Assumption
16. $q \rightarrow \sim w$	14, 15, Rule of Modus Ponens,
17. $\sim q$	4, 16, Rule of Modus Tollens, Law of double negation, Rule of substitution
18. $\sim q \rightarrow [\sim q \vee (w \wedge \sim r)]$	Law of addition
19. $\sim q \vee (w \wedge \sim r)$	17, 18, Rule of Modus Ponens
20. $(w \wedge \sim r) \vee \sim q$	19, Commutative law, Rule of substitution
21. $\sim (\sim w \vee r) \vee \sim q$	20, De Morgan's law, Law of double negation, Rule of substitution
22. $\sim [(\sim w \vee r) \wedge q]$	21, De Morgan's law, Rule of substitution

2. Find the solution set of the inequality $\frac{8x^3+120x^2+600x+1000}{x^3-64} \leq 0$

$$\frac{(2x)^3+3(2x)^2(10)+3(2x)(10)^2+10^3}{x^3-4^3} \leq 0$$

$$\frac{(2x+10)^3}{(x-4)(x^2-4x+16)} \leq 0$$

Consider $x^2-4x+16$, we have $b^2-4ac = (-4)^2-4(1)(16) = -48 < 0$

$\therefore x^2-4x+16$ cannot be factored. And since a = 1 > 0, $x^2-4x+16 > 0$ for all x

So, to satisfy the inequality $\frac{(2x+10)^3}{(x-4)(x^2-4x+16)} \leq 0$, $\frac{(2x+10)^3}{(x-4)}$ need to be less than or equal to 0

That is, $\frac{(2x+10)^3}{(x-4)} \leq 0$.

$\therefore$ The critical values are -5, 4. Also noted that x ≠ 4 because at x = 4 the inequality is not defined.

$\frac{(2x+10)^3}{(x-4)}$ + - +

Hence, the solution set for the inequality $\frac{8x^3+120x^2+600x+1000}{x^3-64} \leq 0$ is [-5, 4)

Homework

1. If p→q, (p□q)→r , p→(r→s) and (r□s) → t then p→t

Solution we assume that p is true and show t is also true

1. p	Assumption
2. p→q	Assumption
3. q	1,2 Law of Modus Ponens
4. p□q	1,3 Definition of conjunction
5. (p□q)→ r	Assumption
6. r	4,5 Law of Modus Ponens
7. p→(r→s)	Assumption
8. r→s	1,7 Law of Modus Ponens
9. s	6,8 Law of Modus Ponens
10. r□s	6,9 Definition of conjunction
11. (r□s) → t	Assumption
12. t	10,11 Law of Modus Ponens

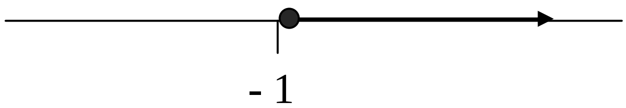
Hence, t is true

2.) Solve the inequality $2|x+2| - 1 \geq |x+2|$

Solution we consider $|x+2|$ in two case $x+2 \geq 0$; $x \geq -2$ and $-(x+2) < 0$; $x < -2$

case I when $x \geq -2$ we substitute $|x+2|$ with $x+2$

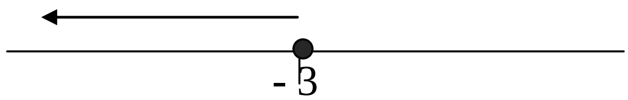
$$2(x+2) - 1 \geq x+2$$
$$2x + 4 - 1 \geq x+2$$
$$x \geq -1$$



The solution set of this case $[-2, \infty) \cap [-1, \infty) = [-1, \infty)$

Case II when $x < -2$ we substitute $|x+2|$ with $-(x+2)$

$$-2(x+2) - 1 \geq x+2$$
$$-2x - 4 - 1 \geq -x - 2$$
$$x \leq -3$$



Therefore, the solution set of this case is $(-\infty, -2) \cap (-\infty, -3] = (-\infty, -3]$

Hence, the solution set of $2|x+2| - 1 \geq |x+2| = (-\infty, -3] \cup [-1, \infty)$.

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1. Prove that if

$\sim r \rightarrow (s \rightarrow \sim t)$ *and* $\sim r \vee w,\ \sim p \rightarrow s, \sim w, t$ *then* p

$\sim r \vee w$	Assumption
$\sim w$	Assumption
$\sim r$	1, 2 Disjunctive syllogism
$\sim r \rightarrow (s \rightarrow \sim t)$	Assumption
$(s \rightarrow \sim t)$	3, 4 Rule of Modus Ponens
t	Assumption
$\sim s$	5, 6 Rule of Modus Tollens
$\sim p \rightarrow s$	Assumption
p	7, 8 Rule of modus Tollens

2. Find the solution set of the inequality $|3x+2| - |x-4| \le 5$

$|3x+2| = 3x+2;\ x \ge -\frac{2}{3}$ and $|x-4| = x-4;\ x \ge 4$

$-3x-2;\ x < -\frac{2}{3}$ $-x+4;\ x < 4$

Case 1 when $x < -\frac{2}{3}$ we substitute $|3x+2|$ with $-3x-2$ and $|x-4|$ with $-x+4$ into the inequality so we have

$$|3x+2| - |x-4| \le 5$$

That is
$$-3x-2+x-4 \le 5$$
$$-2x \le 11$$
$$X \ge -\frac{11}{2}$$

The solution set of case 1 is $\left[-\frac{11}{2},\ \infty\right) \cap (-\infty, -\frac{2}{3}) = \left[-\frac{11}{2}, -\frac{2}{3}\right)$

Case 2 when $-\frac{2}{3} \le x < 4$ we substitute $|3x+2|$ with $3x+2$ and $|x-4|$ with $-x+4$ into the inequality so we have

$$|3x+2| - |x-4| \le 5$$

That is
$$3x+2+x-4 \le 5$$
$$4x-2 \le 5$$
$$4x \le 7$$
$$X \le \frac{7}{4}$$

The solution set of case 2 is $\left[-\frac{2}{3},\ 4\right) \cap \left(-\infty\ , \frac{7}{4}\right] = \left[-\frac{2}{3}, \frac{7}{4}\right]$

Case 3 when $x \ge 4$ we substitute $|3x+2|$ with $3x+2$ and $|x-4|$ with $x-4$ into the inequality so we have

$$|3x+2| - |x-4| \le 5$$

That is
$$3x+2-x+4 \le 5$$
$$2x+6 \le 5$$
$$2x \le -1$$
$$X \le -\frac{1}{2}$$

The solution set of case 3 is $\left[-\infty, -\frac{1}{2}\right] \cap [4, \infty) = x \in \emptyset$

Therefore, the solutions set of the inequality $|3x+2| - |x-4| \le 5$ is

$\left[-\frac{11}{2}, -\frac{2}{3}\right) \cup \left[-\frac{2}{3}, \frac{7}{4}\right] = \left[-\frac{11}{2}, \frac{7}{4}\right]$

Logic & Method of proof

$\sim(q \wedge (r \vee t))$, $(t \rightarrow (\sim q \vee p))$ and q then p

1. q assumption
2. ~(q⋀ (r⋁t)) assumption
3. ~q ⋁ ~ (r⋁t) 2, De morgan's law
4. ~ (r⋁t) 1,3, Disjunctive Syllogism
5. ~r ⋀ ~t 4, De morgan's Law
6. ~r and ~t 5, Definition of conjunction
7. (t ⟶ (~q⋁p)) assumption
8. (~q ⋁ p) 6,7, Modus Tollen
9. p 1,8, Disjunctive Syllogism

Inequalities Problem

(x^2+x-56)(4x^2-x+3) ≦ 0

Since 4x^2-x+3 cannot be factored becuase b^2-4ac = (-1)^2-4(4)(3)= -48 ≦ 0

From 4x^2-x+3, we have a=4>0 meaning that 4x^2-x+3 > 0 for all x. Therefore, x^2+x-56 must be less or equal to 0 in order to satisfy the equality.

x^2+x-56 ≦ 0

(x+8)(x-7) ≦ 0

the critical number is -8 and 7

Hence the solution set of this equality is [-8,7]

Logic and Method of proof problem

(π ϖ θ) ⟶ r , ~(r v s), p then q

1. p assumption
2. ~ (r v s) assumption
3. ~r ⋀ ~s 2, De Morgan's Law

4. ~r and ~s   3, Definition of conjunction
5. (p∧q) → r   assumption
6. ~ (p∧q)   4,5, Modus Tollens
7. ~p v ~q   6, De morgan's law
8. q   1,7, Disjunctive syllogism

Inequalities problem

x^2-2x □ -24

x^2-2x+24 □ 0

(x-6)(x+4) □ 0

the critical number is 6 and -4

Hence the solution set is (-∞,-4] ∪ [6, ∞)

<u>Questions</u>:

Mathematical induction:
1.) If [~(c^b)vcva]----->(c^d), then d.

Solving inequalities:
2.) Find the solution set of this inequality: (168x-1)>(168x-1)

Solutions:

1.) If [~(c^b)vcva]----->(c^d), then d.

Proof:

1.) cv~c	Law of Excluded Middle
2.) (cv~c)---->[(cv~c)v(av~b)]	Law of Addiction
3.) (cv~)v(av~b)	1,2, Rule of Modus Ponens
4.) [(cv~c)v(av~b)]<---->[(~cv~b)vcva]	Associative Laws and Commutative Laws
5.) (~cv~b)vcva	4,5, Rule of Substitution
6.) [(~cv~b)vcva]<---->[~(c^b)vcva]	De Morgan's Laws
7.) ~(cvb)vcva	5,6, Rule of Substitution
8.) [~(cvb)vcva]---->(c^d)	Given
9.) c^d	7,8, Rule of Modus Ponens
10.) d	9, Definition of Conjunction

Hence, d.

2.) Find the solution set of this inequality: (168x-1)>(168x-1)

Since -1 = -1 , the solution set of this inequality is an empty set.

<u>Problem</u>

Prove that $\sum_{i+1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for all natural number by using mathematical induction

1. Let S_n is $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

n=1 Hence S_1 is $1^2 = \frac{1(2)(3)}{6} = 1$

2. We assume that S_k is true and we have to prove that S_{k+1} is true for all natural number

Let natural number is $\{x|x > 0 \text{ } and \text{ } x \in I\}$

We assume $\sum_{k=1}^{n} i^2 = 1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$ for all k natural number

Add $(k+1)^2$ in both sides $1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)+6(k^2+2k+1)}{6}$$

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{2k^3+3k^2+k+6k^2+12k+6}{6}$$

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{2k^3+9k^2+13k+6}{6}$$

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

Hence S_{k+1} is true

By mathematical induction, we can conclude that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ is true for all n natural nuProblem

Find the solution set of $||x| + 1| \geq 5$

CASE I when $|x| + 1 \geq 0$ $\quad$ $|x| + 1 \geq 5$

$|x| \geq -1$ $\quad$ $|x| \geq 4$

$x \in R$ $\quad$ $(-\infty, -4] \cup [4, \infty)$

Solution set of CASE I is $R \cup (-\infty, -4] \cup [4, \infty) = (-\infty, -4] \cup [4, \infty)$

CASE II when $|x| + 1 < 0$ $\quad$ $-(|x| + 1) \geq 5$

$|x| < -1$ $\quad$ $|x| + 1 \leq -5$

$\emptyset$ $\quad$ $|x| \leq -5$

$\emptyset$

Solution set of CASE II is $\emptyset \cap \emptyset = \emptyset$

So, the solution set of $||x| + 1| \geq 5$ is $\{(-\infty, -4] \cup [4, \infty)\} \cup \emptyset = (-\infty, -4] \cup [4, \infty)$

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TU 152 ASSIGNMENT

1) ~x v (y->z) , ~y -> ~x and x then z

1)	~x v (y->z)	Assumption
2)	x -> (y -> z)	1, Law of implication
3)	x	Assumption

4) Y -> z — 2,3 Modus Ponens

5) x<-> ~(~x) — 3, Law of double negation

6) ~(~x) — 3,5 Rule of Substitution

7) ~y -> ~x — Assumption

8) y — 6,7 Rule of Modus Tollens

9) z — 4,8 Modus Ponens

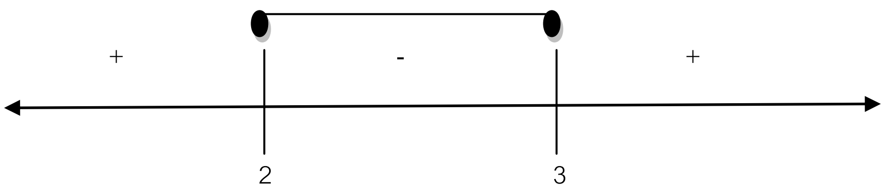
2) $X^2 – 5x + 6 \leq 0$

$X^2 – 5x + 6 \leq 0$

$X^2 – 2x – 3x + 6 \leq 0$

$(x-3)(x-2) \leq 0$

Therefore, the critical values are 3 , 2



Hence the solution set is [2 , 3]

Tu152

Question **Topic: method of prove**

If ~p→~q, p→(r□s), q□t, ~t then ~r→s

Assumption ~t and prove ~r

1. ~t — assumption
2. q□t — assumption
3. q — 1,2 disjunctive syllogism
4. ~p→~q — assumption
5. p — 3,4 rules of modus tollen
6. p→(r□s) — assumption
7. (r□s) — 5,6 rules of modus ponens
8. (~r→s) — 7 law of implication

Topic: Inequality

Find $l(lx-1l)^2-1l <50$

Solution

Since lal <b is equal to –b < a < b

$-50 < (lx-1l)^2-1 <50$

$-49 < (lx-1l)^2 < 51$

$\cancel{-49} < (lx-1l)^2 < 51$

Since $(lx-1l)^2$ cannot be negative so -49 is not possible

$(lx-1l)^2 < 51$

$lx-1l < \sqrt{51}$

$-\sqrt{51} < x-1 < \sqrt{51}$

$-\sqrt{51}+1 < x < \sqrt{51}+1$

Therefore the solution set is $(1-\sqrt{51}, 1+\sqrt{51})$

1. Prove that if (e □ f) → ~ (a → c) ,and ~ a, then ~ f

1)	f	Assumption
2)	f → (f □ e)	1, Law of Addition
3)	(f □ e)	2, Rule of Modus Ponens
4)	(e □ f)	3, Commutative Laws
5)	(e □ f) → ~ (a → c)	Assumption
6)	~ (a → c)	5, Rule of Modus Ponens
7)	~ (~ a □ c)	6, Law of Implication
8)	~ (~ a) □ ~ c	7, De Morgan's Law
9)	a □ ~ c	8, Law of Double Negation
10)	a and ~ c	9, Definition of Conjunction
11)	~ a	Assumption
12)	a □ ~ a	10,11, Definition of Conjunction

Since (a □ ~ a) is a contradiction, therefore the assumption that f is true is always false. F is false ,hence , ~ f is always true.

∴ if (e □ f) → ~ (a → c) ,and ~ a, then ~ f.

2. $| 2 + 3x | + | x - 4 | \leq 5$

1. $| 2 + 3x |$ could be divided into 2 cases

1) $| 2 + 3x | = 2 + 3x$; $2 + 3x \geq 0 = x \geq \frac{-2}{3}$

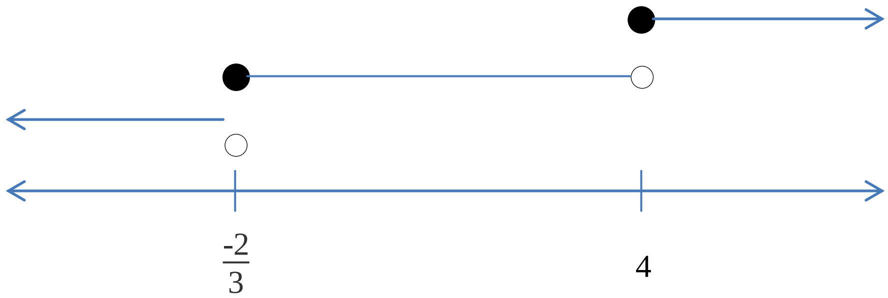
2) $| 2 + 3x | = - (2 + 3x)$; $2 + 3x < 0 = x < \frac{-2}{3}$

2. $| x - 4 |$ could be divided into 2 cases

1) $| x - 4 | = x - 4$; $x - 4 \geq 0 = x \geq 4$

2) $| x - 4 | = - (x - 4)$; $x - 4 < 0 = x < 4$

Therefore, we have 2 points on the number line which are $\frac{-2}{3}$ and 4



Hence, we have 3 gaps of x ranging from $x < \frac{-2}{3}$, $\frac{-2}{3} \leq x < 4$, and $4 \leq x$.

<u>Case I</u> when $x < \frac{-2}{3}$

We substitute $| 2 + 3x |$ with $- (2 + 3x) = - 2 - 3x$ since $x < \frac{-2}{3}$ and we substitute $| x - 4 |$ with $- (x - 4) = - x + 4$ since $x < \frac{-2}{3}$ which is always less than 4.

We get
$$- 2 - 3x - x + 4 \leq 5$$
$$-3x - x \leq 5 - 4 + 2$$
$$-4x \leq 3$$
$$x \geq \frac{-3}{4}$$

Therefore, the solution set for Case I is $(- \infty, \frac{-2}{3}) \cap [\frac{-3}{4} , \infty) = [\frac{-3}{4} , \frac{-2}{3})$

<u>Case II</u> when $\frac{-2}{3} \leq x < 4$

We substitute $| 2 + 3x |$ with $2 + 3x$ since $x \geq \frac{-2}{3}$ and we substitute $| x - 4 |$ with $- (x - 4) = - x + 4$ since $x < 4$.

We get
$$2 + 3x - x + 4 \leq 5$$
$$2x \leq 5 - 4 - 2$$
$$2x \leq - 1$$
$$x \leq \frac{-1}{2}$$

Hence, the solution set for Case II is $(-\infty, \frac{-1}{2}] \cap [\frac{-2}{3}, 4) = [\frac{-2}{3}, \frac{-1}{2}]$

<u>Case III</u> when $x \geq 4$

We substitute $| 2 + 3x |$ with $2 + 3x$ since $x \geq \frac{-2}{3}$ and we substitute $| x - 4 |$ with $x - 4$ since $x \geq 4$.

We get
$$2 + 3x + x - 4 \leq 5$$
$$4x \leq 5 + 4 - 2$$
$$4x \leq 7$$
$$x \leq \frac{7}{4}$$

Therefore, the solution set for Case III is $(-\infty, \frac{7}{4}] \cap [4, \infty) = [\frac{7}{4}, 4]$

We combine all solutions of three cases together to get the final solution set.
That is $[\frac{-3}{4}, \frac{-2}{3}) \cup [\frac{-2}{3}, \frac{-1}{2}] \cup [\frac{7}{4}, 4] = [\frac{-3}{4}, \frac{-1}{2}] \cup [\frac{7}{4}, 4]$

Hence, the solution set for the inequality $| 2 + 3x | + | x - 4 | \leq 5$ is $[\frac{-3}{4}, \frac{-1}{2}] \cup [\frac{7}{4}, 4]$

1)

Let $[o \rightarrow (m \rightarrow g)] \wedge [(\sim m \rightarrow \sim o) \wedge o]$, prove that g is true.

<u>Proof</u>	1. $[o \rightarrow (m \rightarrow g)] \wedge [(\sim m \rightarrow \sim o) \wedge o]$	Assumption
	2. $o \rightarrow (m \rightarrow g)$ and $(\sim w \rightarrow \sim o) \wedge o$	1, Def^{n} of Conjunction
	3. $(\sim m \rightarrow \sim o)$ and o	2, Def^{n} of Conjunction
	4. $m \rightarrow g$	2,3, Modus Ponens
	5. $o \leftrightarrow \sim(\sim o)$	Double Negation
	6. $\sim(\sim o)$	3,5, Substitution
	7. $\sim(\sim m)$	3,6, Modus Tollens
	8. $\sim(\sim m) \leftrightarrow m$	Double Negation
	9. o	7,8, Substitution
	10. g	4,9, Modus Ponens

2)

$$\left|\frac{x-\sqrt{x^2-1}}{2}\right| > 1$$

We separate the inequality into 2 inequalities.

1. $\frac{x-\sqrt{x^2-1}}{2} > 1$

We get
$$x - \sqrt{x^2 - 1} > 2$$
$$x - 2 > \sqrt{x^2 - 1}$$
$$x^2 - 4x + 4 > x^2 - 1$$
$$-4x + 5 > 0$$
$$x < \frac{-5}{-4}$$
$$x < \frac{5}{4}$$

And

2. $\frac{x-\sqrt{x^2-1}}{2}$ < -1

We get $x-\sqrt{x^2-1}<-2$

$x+2<\sqrt{x^2-1}$

$x^2+4x+4<x^2-1$

$4x+5<0$

$x<\frac{-5}{4}$

Therefore, the solution set for the inequality $\left|\frac{x-\sqrt{x^2-1}}{2}\right|>1$ is

$$\left(-\infty,\frac{-5}{4}\right)\cup\left(\frac{5}{4},\infty\right)$$

1. Prove that if ~ p → ~ q , p → (r v s) , q v t and ~ t , then r v s

Solution

1.~ t Assumption

2.q v t Assumption

3. q 1,2 disjunctive syllogism

4.~ p → ~ q Assumption

5. p 3,4 modus tolllens

6. p → (r v s) Assumption

7.r v s 5,6 Modus tollens

2. $|x^2-3| \leq |2x|$

$|x^2-3|^2 \leq |2x|^2$

$x^2-3 \leq 2x$

$x^2-2x-3 \leq 0$

$(x-3)(x+1) \leq 0$

The critical values are -1 , 3

Hence, the solution set is {-1,3}

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1) Method of proofs

Prove that if p V (p Λ ~q) and p → q then q

1. p → q — Assumption
2. ~ p V q — 1 and Law of Implication
3. ~ (p Λ ~q) — 2 and De Morgan's Laws
4. p V (p Λ ~q) — Assumption
5. p — 3 , 4 and Disjunctive Syllogism
6. q — 1 , 5 and Rule of Modus Ponens

2) Inequality

Find the solution set of the inequality $6 \leq |4x-2| < 14$

<u>Consider</u> $6 \leq |4x-2|$ we have

$4x - 2 \geq 6$	or	$4x - 2 \leq -6$
$4x \geq 8$	or	$4x \leq -4$
$x \geq 2$	or	$x \leq -1$

The solution set of $6 \leq |4x-2|$ is $(-\infty , -1] \cup [2 , \infty)$

<u>Consider</u> $|4x-2| < 14$ we have

$$-14 < 4x-2 < 14$$

$$-12 < 4x < 16$$

$$-3 < x < 4$$

The solution set of $|4x-2| < 14$ is $(-3 , 4)$

Hence , The solution set of the inequality $6 \leq |4x-2| < 14$ is $(-3 , -1] \cup [2 , 4)$

Method of Proofs

Prove that if p→ (q→r), p, ˜t→q, then r→t
Solution:

1. r — Assumption
2. p — Assumption

3. $p \to (q \to r)$ Assumption
4. $q \to r$ 2,3 Modus Ponens
5. $\tilde{}q$ 1,4 Modus Tonens
6. $\tilde{}t \to q$ Assumption
7. t 5,6 Modus Tonens

Inequality

Find the solution set of inequality $(x^{2-}8)/x \leq 2$

Solution:

$(x^{2-}2x-8)/x \leq 0$

$(x-4)(x+2)/x \leq \mathbf{0}$

Critical Point= (-2,0, 4)

The solution set is $(-\infty,-2) \cup (0,4)$

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Question1 : Methods of Proof

Suppose the $[p \to (q \to r)]$, ~pvs, $\sim t \to q$, and ~s. Prove that $\sim r \to t$

1. ~r assumption
2. ~pvs assumption
3. ~s assumption
4. ~p 2,3, Disjunctive Syllogism
5. $\sim p \to (q \to r)$ assumption
6. $q \to r$ 5,4, Rule of Modus Ponens
7. ~q 1,6, Rule of Modus Tollens
8. $\sim t \to q$ assumption
9. t 8,7, Rule of Modus Tollens

Question2 : Inequality

Find the solution of $\frac{|x|}{|x|-1} \leq 2$

Case 1

$\frac{|x|}{|x|-1} \leq 2$ when $x \geq 0$

$\frac{x}{x-1} - 2 \leq 0$

$\frac{x-2x+2}{x+1} \leq 0$

$\frac{-x+2}{x-1} \leq 0$

Multiply $(x-1)^2$ both sides of the inequality

$$(x-1)^2 \bullet \frac{-x+2}{x-1} \le 0$$
$$(x-1)(-x+2) \le 0$$
$$(x-1)(x-2) \ge 0$$
The solution set for case1 is $[0,1) \cap [2,\infty)$

Case2

$\frac{-x}{-x-1} \le 2$ when x<0

$$\frac{-x}{-x-1} \le 2$$
$$\frac{(-1)(x)}{(-1)(x+1)} \le 2$$
$$\frac{x}{x+1} \le 2$$
$$\frac{x}{x+1} - 2 \le 0$$
$$\frac{x-2x-2}{x+1} \le 0$$
$$\frac{-x-2}{x+1} \le 0$$
Multiply $(x+1)^2$ both sides of the inequality
$$(x+1)^2 \bullet \frac{(-x-2)}{(x+1)} \le 0$$
$$(x+1)(-x-2) \le 0$$
$$(x+1)(x+2) \ge 0$$
The solution set for case2 is $(-\infty,2] \cap (-1,0)$

Therefore the solution set of $\frac{|x|}{|x|-1} \le 2$ is $(-\infty,-2] \cup (-1,1) \cup [2,\infty)$

Proof/Induction

1 Prove that the following identity:

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n = \frac{(n-1) \cdot n \cdot (n+1)}{3}$$

is true for positive integer n

<u>Solution</u>

Let S_n be the statement

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n = \frac{(n-1) \cdot n \cdot (n+1)}{3}$$

Where n is every positive integer

1) S_1 is "$0 \cdot 1 = \frac{(1-1) \cdot 1 \cdot (1+1)}{3}$" ; that is "0=0" which is true.
2) Assume that S_k is true and show that S_{k+1} is true.
   That is, we assume

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (k-1) \cdot k = \frac{(k-1) \cdot k \cdot (k+1)}{3}$$

Where k is any positive integer

We need to show that

$$S_{k+1} = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k \cdot (k+1) = \frac{(k) \cdot (k+1) \cdot (k+2)}{3}$$

Since $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (k-1) \cdot n = \frac{(k-1) \cdot k \cdot (k+1)}{3}$ is true,

So we add $k \cdot (k+1)$ to both sides, we have

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (k-1) \cdot k + k \cdot (k+1) = \frac{(k-1) \cdot k \cdot (k+1)}{3} + k \cdot (k+1)$$

$$= \frac{(k-1) \cdot k \cdot (k+1) + 3[k \cdot (k+1)]}{3}$$

$$= \frac{k \cdot (k+1)[(k-1) + 3]}{3}$$

$$= \frac{k \cdot (k+1) \cdot (k+2)}{3}$$

Hence, S_{k+1} is true.

∴ By mathematical induction, we conclude that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n = \frac{(n-1) \cdot n \cdot (n+1)}{3}$$

is true for positive integer n Proven

Inequality

2. Find the solution set of the inequality $\left|\frac{2x}{x-5}\right| > 4$

Solution

Consider $\left|\frac{2x}{x-5}\right| > 4$,

$$\frac{2x}{x-5} > 4$$

$$\frac{2x}{x-5} - 4 > 0$$

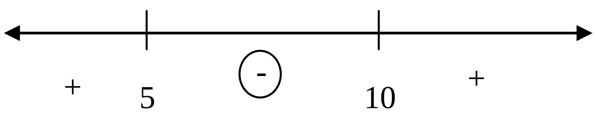
$$\frac{2x - 4(x-5)}{x-5} > 0$$

$$\frac{2x - 4x + 20}{x-5} > 0$$

$$\frac{-2x + 20}{x-5} > 0$$

$$\frac{x-10}{x-5} < 0$$

The critical values are 5 and 10.



or

$$\frac{2x}{x-5} < -4$$

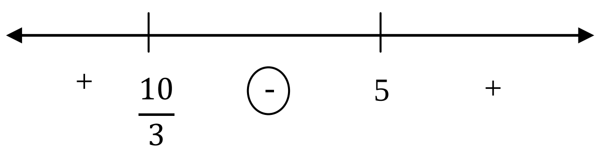
$$\frac{2x}{x-5} + 4 < 0$$

$$\frac{2x + 4(x-5)}{x-5} < 0$$

$$\frac{6x - 20}{x-5} < 0$$

$$\frac{3x - 10}{x-5} < 0$$

The critical values are $\frac{10}{3}$ and 5.



Then the solution of $\left|\frac{2x}{x-5}\right| > 4$ is $(5,10) \cup \left(\frac{10}{3}, 5\right) = \left(\frac{10}{3}, 5\right)$ Ans

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1. Prove that if $[(a\to b)\wedge(a\to c)]$ then $[a\to(b\wedge c)]$

Proof

1.$[(a\to b)\wedge(a\to c)]$	Assumption
2.$[(\sim a\vee b)\wedge(\sim a\vee c)]$	1, Law of implication
3.$[\sim a\vee(b\wedge c)]$	2, Distributive law
4.$[a\to(b\wedge c)]$	3, Law of implication

Hence $[a\to(b\wedge c)] \leftrightarrow [(a\to b)(a\to c)]$

2.

$$\frac{X^3 + X^2 - 6X}{-X^2 + 3X - 10} < -1$$

Solution

$$\frac{X(X^2+X-6)}{(-X+5)(X-2)} < -1$$

$$\frac{X(X+3)(X-2)}{(-X+5)(X-2)} < -1$$

$$\frac{X(X+3)}{(-X+5)} < -1$$

Critical Number are X = 0,-3,5

```
                          0----------->
      +        -        +          -
 <--------|---------|----------|----------->
         -3         0          5
```

Therefore the solution set of the inequality is $(5,\infty)$

Prove (p v q), (~q), (~r→ ~p), ~(~r^~s) then s

1. p v q   Assumption
2. ~q   Assumption
3. p   1, 2, Disjunctive Syllogism
4. ~r→ ~p   Assumption
5. ~r   3, 5, Modus Tollens, Double Negation, Substitution
6. ~(~r^~s)   Assumption
7. r v s   6, De Morgan's law
8. s   5, 7, Disjunctive Syllogism

Solve the inequality $1<|9x+5| \le 10$

Sol From $1<|9x+5| \le 10$, we get

$1<|9x+5|$ and $|9x+5| \le 10$

Consider $1<|9x+5|$

$1< 9x+5$ or $-1> 9x+5$

$\frac{-3}{9} < x$ or $\frac{-6}{9} > x$

The solution set of the inequality $1<|9x+5|$ is $(-\infty,\frac{-6}{9}) \cup (\frac{-3}{9},\infty)$

Consider $|9x+5| \le 10$

$-10 \le |9x+5| \le 10$

$-10 -5 \le 9x \le 10-5$

$\frac{-15}{9} \le x \le \frac{5}{9}$

The solution set of the inequality $|9x+5| \le 10$ is $\left[\frac{-15}{9},\frac{5}{9}\right]$

Therefore, the solution set of inequality $1<|9x+5| \le 10$

is $\left[\frac{-15}{9},\frac{-6}{9}\right) \cup \left(\frac{-3}{9}, \frac{5}{9}\right]$

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Question 1: Let n be positive integer and $n \ge 2$, prove that $2^n (n+1) > 2^{(n+1)}$

Proof: S_n be the statement "$2^n (n+1) > 2^{(n+1)}$ where n is any positive integer and $n \ge 2$"

1) S_2 is $2^2 (2+1) > 2^{(2+1)}$

$4 (3) > 2^3$

$12 > 8$, which is true.

2) Assume that S_k is true and true and show S_{k+1} is true

That is, we assume $2^k (k+1) > 2^{(k+1)}$

Where S_k is a positive integer and $n \ge 2$,

We need to show $2^{k+1} [(k+1)+1] > 2^{[(k+1)+1]}$

$2^{k+1} (k+2) > 2^{(k+2)}$

$2\times2^k\times(k+2) > 2\times2^{(k+1)}$

We divided both sides by 2, $2^k\times(k+2) > 2^{(k+1)}$

$$2^k \times (k+2) > 2^k \times (k+1) > 2^{(k+1)}$$
$$2^k \times (k+1) > 2^{(k+1)} \text{ because } (k+1) < (k+2)$$

Hence, S_{k+1} is true.

Therefore, by Mathematical Induction, we can conclude that $2^n (n+1) > 2^{(n+1)}$ where n is any positive integer and $n \geq 2$.

Question 2: Find the solution set of the inequality $\sqrt{n-4} \geq \sqrt{3n}$

$$(\sqrt{n-4})^2 \geq (\sqrt{3n})^2$$
$$|n-4| \geq |3n|$$

$|n-4| \rightarrow n-4, \ when\ n \geq 4$

$\quad\quad \rightarrow 4-n, when\ n < 4$

$|3n| \quad \rightarrow 3n, \ when\ n \geq 0$

$\quad\quad \rightarrow -3n, when\ n < 0$

The critical points are 0 and 4.

Hence, we can divide it in to three cases.

Case 1: when $n < 4$

We substitute $|n-4|$ with $4-n$ and $|3n|$ with $-3n$.

Hence, $4-n-(-3n) \geq 0$

$$4-n+3n \geq 0$$
$$4+2n \geq 0$$
$$2n \geq -4$$
$$n \geq -2$$

Hence, the solution set of Case 1 is (-∞, 0) ^ $[-2, \infty)$ which is [2,0).

Case 2: when $0 \leq n < 4$

We substitute $|n-4|$ with $4-n$ and $|3n|$ with $3n$.

Hence,$4-n-(3n) \geq 0$

$$4-4n \geq 0$$
$$-4n \geq -4$$
$$n \leq 1$$

Hence, the solution set of Case 2 is [0,4) ^ (-∞,-1] which is □.

Case 3: when $n \geq 4$

We substitute $|n-4|$ with $n-4$ and $|3n|$ with $3n$.

Hence,$n-4-(3n) \geq 0$

$$-2n-4 \geq 0$$
$$-2n \geq 4$$
$$n \leq -2$$

Hence, the solution set of Case 3 is [4,∞) ^ (-∞,-2] which is □.

Therefore, the solution set of $\sqrt{n-4} \geq \sqrt{3n}$ is [2,0).

Method of Proofs

1. If $(p \wedge q) \rightarrow s$, $\sim r \rightarrow \sim s$, ~(r □ ~q) then $\sim p$

<u>Solution</u>

1. ~(r □ ~q)	Assumption
2. $\sim r \wedge q$	De Morgan's law
3. ~r and q	Definition of conjunction
4. $\sim r \rightarrow \sim s$	Assumption
5. ~s	3,4 Modus pollens
6. $(p \wedge q) \rightarrow s$	Assumption
7. $\sim(p \wedge q)$	5,6 Modus tollens
8. ~p □ ~q	De Morgan's law
9. ~p	3,8 Disjunctive Syllogism

Solve for Inequality

2. Solve the inequality 1< l3x+5l<6

<u>Solution:</u> From question we have

l3x+5l >1 and l3x+5l<6

consider l3x+5l >1 first

$3x+5 > 1$ $\quad$ $3x+5 < -1$

$3x > -4$ $\quad$ $3x < -6$

$x > -4/3$ $\quad$ $x < -2$

The solution set of l3x+5l >1 is $(-\infty,-2) \cup (-4/3,\infty)$

now consider l3x+5l<6

$-6 < 3x+5 < 6$

$-11 < 3x < 1$

$-11/3 < x < 1/3$

The solution set of l3x+5l<6 is $(-11/3,1/3)$

Hence, the solution set of 1< l3x+5l<6 is

$(-\infty,-2) \cup (-4/3,\infty) \cap (-11/3,1/3) = (-11/3,-2) \cup (-4/3,1/3)$ ans

Question1

Prove that $p \wedge q, (q \vee r) \rightarrow (s \wedge p)$, and $p \rightarrow \sim r$, then $s \wedge \sim r$

1. $p \wedge q$	Assumption
2. p and q	1, Definition of Conjunction
3. $p \rightarrow \sim r$	Assumption
4. ~r	2,3 and Modus Ponens
5. $q \rightarrow (q \vee r)$	2, Law of addition
6. $q \vee r$	2,5 and Modus Ponens
7. $(q \vee r) \rightarrow (s \wedge p)$	Assumption
8. $s \wedge p$	6,7 and Modus Ponens
9. s and p	8, Definition of conjunction
10. $s \wedge \sim r$	8,9 and Definition of conjunction

Question2

Find the solution set of the inequality $\frac{3|1-x|}{x-1} < -x$

Case1: When $x \leq 1$, we subsitute $|1-x|$ with 1-x , so we have

$$\frac{3|1-x|}{x-1} < -x$$
$$\frac{3(1-x)}{x-1} < -x$$
$$\frac{3(1-x)}{x-1} + x < 0$$
$$\frac{3-3x+x(x-1)}{x-1} < 0$$
$$\frac{3-3x+x^2-x}{x-1} < 0$$
$$\frac{x^2-4x+3}{x-1} < 0$$
$$\frac{(x-1)(x-3)}{(x-1)} < 0$$

where x $\neq$ 1

The critical values are 1 and 3

Hence, the solution set for case1 is $(-\infty,1] \cap (-\infty,1) \cup (1,3) = (-\infty,1)$

Case2 When x>1, we substitute $|1-x|$ with –(1-x) , so we have

$$\frac{3|1-x|}{x-1} < -x$$
$$\frac{-3(1-x)}{x-1} < -x$$
$$\frac{-3(1-x)}{x-1} + x < 0$$
$$\frac{-3+3x+x(x-1)}{x-1} < 0$$
$$\frac{x^2+2x-3}{x-1} < 0$$
$$\frac{(x-1)(x-3)}{(x-1)} < 0; x \neq 1$$

The critical values are 1 and -3

Hence, the solution set for case 2 is $(1,\infty) \cap (-\infty,-3) = \varnothing$

Therefore, the solution set for the inequality $\frac{3|1-x|}{x-1} < -x$ is $(-\infty,1) \cup \varnothing = (-\infty,1)$

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1.Proof

Prove that if (Av ~ B), and C →~(A v D) , then B→~C

We assume B and prove ~C

1. B Assumption
2. (Av ~B) Assumption

3. A 1,2 Substitution, double negation, Disjunctive syllogism
4. C→~(A vD) Assumption
5. A→(A v D) 3 law of addition
6. (A v D) 3,5 law of addition
7. ~C 4,6 modus tollen, doublenegation, substitution

2. Solving the inequality
Solving | 2-5x | = |x + 4| – 1

| 5x-2 | = |x + 4| – 1

case 1 x<-4 →1

-(5x-2) = -(x+4)-1

-5x+2 = -x-4-1
-4x=-7
X=7/4
X€ {7/4} →2
Take 1∩2 = X€ϕ→3

case 2 -4≤x<2/5 →4

-(5x-2)=-(x+4)-1

-5x+2=x+3
-6x=1
x = -1/6
x€{-1/6} →5
take 4∩5 = X€{-1/6} →6

Case 3 x≥ 2/5 →7
5x-2 = x+4-1
4x = 5
X = 5/4
X€{5/4} →8
Take 7∩8 =X€{5/4}→9

Therefore set of equation =3 U 6 U 9 U =-{1/5,5/4}

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Proof/Induction

1. Prove that for every positive integer n, 3 divides n^3+5n.

Proof: Let S_n be the statement "3 divides n^3+5n" where n is positive n integer.
1. S_1 is 3 divides $1^3 + 5(1)$, that is 3/6 which is true
2. Assume S_k is true, that is $3/k^3 + 5k$ we need to know that S_{k+1} is true that is to show $3/(k^3+5k) + 5(k+1)$ [or 3x where x is an integer]

Since $3/(k^3+5k)$, then $k^3 + 5k = 3m$ where m is an integer, we have

$$(k+1)^3 + 5(k+1) = k^3 + 3k^2 + 3k + 5k + 5$$
$$= (k^3 + 5k) + (3k^2 + 3k + 6)$$
$$= 3m + 3(k^2 + 3k + 6) \rightarrow 3(m+k^2+k+2)$$

Since m is integer and k is a positive integer, hence $m+k^2+k+2$ is also an integer so 3 divides

$(k+1)^3 + 5(k+1)$

∴ We conclude that by mathematical induction, for every positive integer n, 3 divides

n^3+5n.

<u>Inequality</u>

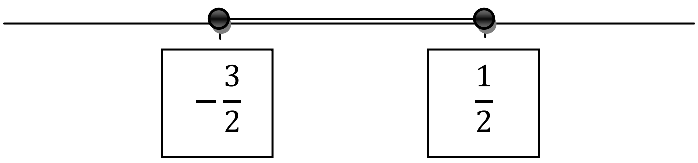
2. Find the solution set of the inequality $|2x+3| \le 4$

$$|2x+3| = \begin{cases} 2x+3 \ ; \ 2x+3 \ge 0 \\ -(2x+3) \ ; \ -2x-3 < 0 \end{cases} \quad \text{or} \quad |2x+3| = \begin{cases} 2x+3 \ ; \ x \ge -\frac{3}{2} \\ -2x-3 \ ; \ x < \frac{3}{2} \end{cases}$$

<u>Case I</u>: When $x \ge -\frac{3}{2}$, we substitute 2x+3 into the inequality so we have

$$|2x+3| \le 4$$
$$2x+3 \le 4$$
$$2x \le 1$$
$$X \le \frac{1}{2}$$

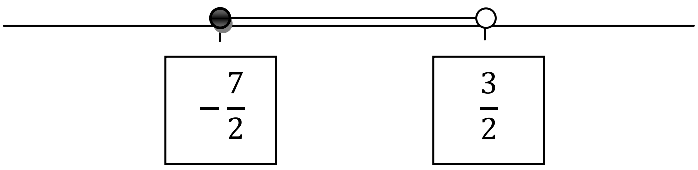
The solution set of case I is $(-\infty, \frac{1}{2}] \cap [-\frac{3}{2}, \infty) = [-\frac{3}{2}, \frac{1}{2}]$



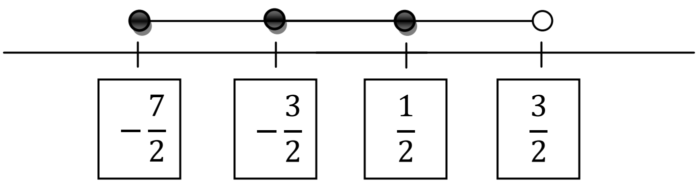
<u>Case II</u>: When $x < \frac{3}{2}$, we substitute -2x-3 into the inequality so we have

$$|2x+3| \le 4$$
$$-2x-3 \le 4$$
$$-2x \le 7$$
$$X \ge -\frac{7}{2}$$

The solution set of case II is $[-\frac{7}{2}, \infty) \cap (-\infty, \frac{3}{2}) = [-\frac{7}{2}, \frac{3}{2})$



∴ Therefore, the solution set of $|2x+3| \le 4$ is $[-\frac{3}{2}, \frac{1}{2}] \cup [-\frac{7}{2}, \frac{3}{2}) = [-\frac{7}{2}, \frac{3}{2})$



<u>**Question**</u>

Proof

1. Prove that if (~p→r), ~(q→p), and p↔~q, then r

Proof

1. ~(q→~p)	Given
2. ~(~q□~p)	Law of Implication
3. q□p	1,2 De Morgan’s Law
4. q and p	3 Definition of Conjunction
5. p↔~q	Given
6. (p→~q) □ (~q→p)	5 Law of Equivalence
7. (q→~p) □ (~p→q)	Law of Contraposition
8. (~q□~p) □ (p□q)	Law of Implication
9. (~q□~p) and (p□q)	8 Definition of Conjunction
10. ~p	4,9 Disjunctive Syllogism
11. (~p→r)	Given
12. r	10,11 Rule of Modus Ponens

Solving the inequality

1. Find the solution set of the inequality $\left| |x-1|-1 \right| \cdot \left| |x-1|+1 \right| < 48$

$\left| |x-1|-1 \right| \cdot \left| |x-1|+1 \right| < 48$

$\left| |x-1|^2 - 1^2 \right| < 48$

$-48 < \left| |x-1|^2 - 1 \right| < 48$

$-47 < |x-1|^2 < 49$

But any number that is squared must be more than or equal to 0. Hence,

$|x-1|^2 < 49$

$|x-1| < \sqrt{49}$

$-\sqrt{49} < x-1 < \sqrt{49}$

$1-\sqrt{49} < x < 1+\sqrt{49}$

$-6 < x < 8$

Therefore, the solution set of the inequality | **|x-1|-1| ▪| |x-1|+1| < 48 is**

(-6,8)

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1. Prove that if $[\sim r \to (s \to \sim t)] \wedge (\sim r \vee w) \wedge (\sim p \to s) \wedge \sim w$, then $(t \to p)$

1. $\sim r \vee w$	Assumption
2. $\sim w$	Assumption
3. $\sim r$	1, 2, Disjunctive Syllogism
4. $\sim r \to (s \to \sim t)$	Assumption
5. $s \to \sim t$	3, 4, Rule of Modus Ponens
6. $\sim p \to s$	Assumption
7. $\sim p \to \sim t$	5, 6, Law of Syllogism
8. $t \to p$	7, Law of Contraposition

Hence, we have proved that $[\sim r \to (s \to \sim t)] \wedge (\sim r \vee w) \wedge (\sim p \to s) \wedge \sim w$ then, $(t \to p)$

2. Solve the inequality $\frac{1}{(x-3)} \geq \frac{1}{x}$.

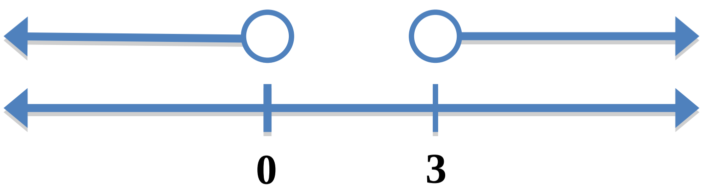
$$\frac{1}{(x-3)} - \frac{1}{x} \geq 0$$

$$\left(\frac{1 \times x}{(x-3) \times x}\right) - \left(\frac{1 \times (x-3)}{x \times (x-3)}\right) \geq 0$$

$$\frac{x-(x-3)}{(x-3) \times x} \geq 0$$

$$\frac{3}{(x-3) \times x} \geq 0$$

$\therefore$ The critical values are 3 and 0



$\therefore$The solution set is $(-\infty, 0) \cup (3, \infty)$

<u>Mathematic of Proof Solution Problem</u>

Prove that if $x \rightarrow (y \rightarrow z)$, $\sim y \rightarrow \sim x$, and x, then z.

Solution:

1.	$\sim u \wedge \sim t$	Given
2.	$\sim u$ and $\sim t$	1. And Def$^{\underline{n}}$ of conjunction
3.	$u \vee (r \rightarrow t)$	Given
4.	$r \rightarrow t$	2,3 and Disjunctive Syllogism
5.	$\sim r$	2,4 and Rule of Modus Tollens

Mathematic of inequality solution problem

Solve: $|5x - 4| = 11$

Solution: Use the theorem stated above to rewrite
The equation.

$|X| = b$
$X = 5x - 4$ and $b = 11$

$5x - 4 = 11, 5x - 4 = -11$

Solve each equation using the
Addition Principle and the
Multiplication Principle.

$5x = 15, 5x = -7$
$x = 3, x = -(7/5)$

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Prove that if $\sim (p \wedge q)$, $[(w \vee m) \rightarrow \sim q] \wedge w$, $m \rightarrow q$, and q, then $(\sim m \leftrightarrow \sim p)$.

Solution: Since $(\sim m \leftrightarrow \sim p) \leftrightarrow [(\sim m \rightarrow \sim p)$ □ $(\sim p \rightarrow \sim m)]$, by law of equivalence,
we can prove $(\sim m \rightarrow \sim p)$ □ $(\sim p \rightarrow \sim m)$ instead of $(\sim m \leftrightarrow \sim p)$.

To prove (~ m → ~ p) ⍰ (~ p → ~ m), we have to prove that ~ m → ~ p is true and also
~ p → ~ m is true.

Prove 1 : ~ m → ~ p

We have to assume ~ m and then show ~ p.

1.	~ m	Assumption
2.	~ (p ⍰ q)	Assumption
3.	~ p ⍰ ~q	2, De Morgan’s Laws, Double Negation, rule of substitution
4.	q	Assumption
5.	~p	3, 4, Disjunctive Syllogism

Hence ~ m → ~ p is true

Prove 2 : ~ p → ~ m

We have to assume ~ p and then show ~ m.

1.	~ p	Assumption
2.	~ (p ⍰ q)	Assumption
3.	~ p ⍰ ~q	2, De Morgan’s Laws, Double Negation, rule of substitution
4.	[(w ⋁ m) → ~ q] ⋀ w	Assumption
5.	(w ⍰ m) → ~ q and w	Definition of conjunction
6.	w → (w ⍰ m)	Law of Addition
7.	w ⍰ m	3, 4, Rule of Modus Ponens
8.	~ q	3, 5, Rule of Modus Ponens
9.	m → q	Assumption
10.	~ m	8, 9, Rule of Modus Tollens

Hence ~ p → ~ m is true.

Therefore, (~ m ↔ ~ p) is true.

Inequality

Find the solution set of the inequality $| x-3 | < | 2x-7|$

Solution:

Square on both sides

$$(x-3)^2 < (2x-7)^2$$
$$x^2 - 6x + 9 < 4x^2-28x+49$$
$$0 < 3x^2 + 34x + 40$$
$$0 < (3x+4)(x+10)$$

The critical values are $-\frac{4}{3}$ and -10

Hence, the solution set of this case is $(-10, -\frac{4}{3})$

1. Prove that for any integer $n \geq 1$,

$$1 + 3 + 5 + . . . + (2n - 1) = n^2$$

Proof

Let S_n be " $1 + 3 + 5 + . . . + (2n - 1) = n^2$ "

1. S_1 is " $1 = 1^2$ " that is " $1 = 1$ " which is true.
2. Assume S_k and show S_{k+1}

   That is, we assume

$$1 + 3 + 5 + . . . + (2k - 1) = k^2 \quad \text{(Assumption)}$$

where $k \geq 1$.

We need to show

$$1 + 3 + 5 + . . . + (2k - 1) + (2k + 1) = (k+1)^2$$

Since $1 + 3 + 5 + . . . + (2k - 1) = k^2$ is true, so we add $2(k + 1)-1$ both sides and we get

$$1 + 3 + 5 + . . . + (2k - 1) + \{2(k + 1)-1\} = k^2 + \{2(k + 1)-1\}$$

$$1 + 3 + 5 + . . . + (2k - 1) + (2k + 2 - 1) = k^2 + (2k + 2 - 1)$$

$$1 + 3 + 5 + . . . + (2k - 1) + (2k + 1) = k^2 + (2k + 1)$$

$$1 + 3 + 5 + . . . + (2k - 1) + (2k + 1) = k^2 + 2k + 1$$

$$1 + 3 + 5 + . . . + (2k - 1) + (2k + 1) = (k + 1)(k + 1)$$

$$1 + 3 + 5 + . . . + (2k - 1) + (2k + 1) = (k + 1)^2$$

Hence, S_{k+1} is true.

$\therefore$ By Mathematical Induction, we conclude that

$$1 + 3 + 5 + . . . + (2n - 1) = n^2 \text{ for any integer } n \geq 1.$$

2. Find the solution set of the inequality $\left|\frac{2x-1}{x-1} - 5\right| > 5$

Solution

Note : $x-1 \neq 0$
$X \neq 1$

$$\left|\frac{2x-1}{x-1}-5\right|>5$$
$$\left|\frac{(2x-1)-5(x-1)}{x-1}\right|>5$$
$$\left|\frac{2x-1-5x+5}{x-1}\right|>5$$
$$\left|\frac{-3x+4}{x-1}\right|>5$$
$$\frac{|-3x+4|}{|x-1|}>5$$
$$|-3x+4|>5|x-1|$$
$$|-3x+4|>|5x-5|$$
$$(-3x+4)^2>(5x-5)^2$$
$$(-3x+4)^2-(5x-5)^2>0$$
$$\{(-3x+4)-(5x-5)\}\{(-3x+4)+(5x-5)\}>0$$
$$(-3x+4-5x+5)(-3x+4+5x-5)>0$$
$$(-8x+9)(2x-1)>0$$

$\therefore$ The critical values are $\frac{1}{2}$ and $\frac{9}{8}$.

		$\frac{1}{2}$		$\frac{9}{8}$	
(-8x+9)(2x-1)	-		+		-

Hence, the solution set is $(\frac{1}{2}, \frac{9}{8}) - \{1\} = (\frac{1}{2}, 1) \cup (1, \frac{9}{8})$.

1. Induction

Ex. Let n be positive integer and n>3, prove that 2n+4<5 power n
Proof: Let Sn be a statement "2n+4<5 power n" where n is any positive integer and n>3.

1) S4 is "2(4)+4<5 power 4". That is 12<625. Which is true.
2) Assume Sk is true and show that Sk+1 is true. That is, we assume

2k+4<5 power k

Where k is any positive integer and k>3.
We need to show

2(k+1) +4<5 power k+1
= 2k+6<5 power k+1

Since "2k+4<5 power k" is true, so we multiply 5 to both sides and we get

5(2k+4)<5 power k times with 5
10K+20<5 power k+1
Since 2k+6 is less than 10k+20 because 2 is less than 10 and 6 is less than 20, therefore,
2k+6<5 power k+1
Hence, Sk+1 is true.
By Mathematical Induction, we conclude that 2n+4<5 power n

For any positive integer n and n>3

2. Solving the inequality

Ex. Find the solution set of the inequality x power 3 + x power 2 – 7x ≥-x

Solution: x power 3 + x power 2 – 7x + x ≥ 0
X power 3 + x power 2 - 6x≥ 0
X(x power 2 + x – 6) ≥0
X(x+3)(x-2) ≥0
Critical values = 0, -3, 2
Hence, the solution set of the inequality is [-3, 0] U [2, ∞)

TU152
Method of Proving

If $p \wedge\sim q, \sim s \vee r \vee q \vee x, and\ (\sim x \rightarrow y) \rightarrow\sim t$
show $[(p \rightarrow s) \wedge\sim r] \rightarrow\sim t$

1. $p \wedge q$	Assumption
2. $\sim s \vee r \vee q \vee x$	Assumption
3. $(\sim x \rightarrow y) \rightarrow\sim t$	Assumption
4. $(p \rightarrow s) \wedge\sim r$	Assumption
5. $p\ and\ q$	1, Definition of conjunction
6. $p \rightarrow s\ and\ \sim r$	4, Definition of conjunction
7. s	5,6, Rule of Modus Ponens
8. $\sim r \wedge\sim q$	5,6, Definition of conjunction
9. $(\sim r \wedge\sim q) \leftrightarrow\sim (r \vee q)$	De Morgan's Law
10. $\sim (r \vee q)$	8,9, Rule of substitute
11. $(\sim s \vee r \vee q \vee x) \leftrightarrow [(r \vee q) \vee (\sim s \vee x)]$	Commutative Law, Associative Law
12. $(r \vee q) \vee (\sim s \vee x)$	10,11, Rule of substitution
13. $\sim s \vee x$	10,12, Disjunctive Syllogism
14. $\sim (\sim s) \leftrightarrow s$	Law of Double Negetion
15. $\sim (\sim s)$	7,14, Rule of substitution
16. x	13,15, Disjunctive Syllogism
17. $[(\sim x \rightarrow y) \rightarrow\sim t] \leftrightarrow [\sim (\sim x \rightarrow y) \vee\sim t]$	Law of Implication
18. $\sim (\sim x \rightarrow y) \vee\sim t$	3,17, Rule of substitution
19. $(\sim x \wedge\sim y) \vee\sim t$	18, Law of Implication, Double negation
20. $(\sim x \vee\sim t) \wedge (\sim y \vee\sim t)$	19, Distribution Law

21. $\sim x \vee \sim t$ *and* $\sim y \vee \sim t$ — 20, Definition of conjunction

22. $\sim t$ — 16,21, Disjunctive Syllogism

Since $[(p \to s) \wedge \sim r]$ is true, then $\sim t$ is true

Solving the inequality

$$\frac{|x-1|-1}{|x-1|} \leq \frac{2}{3}$$

<u>Solution</u>

$$3|x-1|-3 \leq 2|x-1|$$

Let $a = |x-1|$;

$$3a-3 \leq 2a$$
$$a \leq 3$$
$$|x-1| \leq 3$$

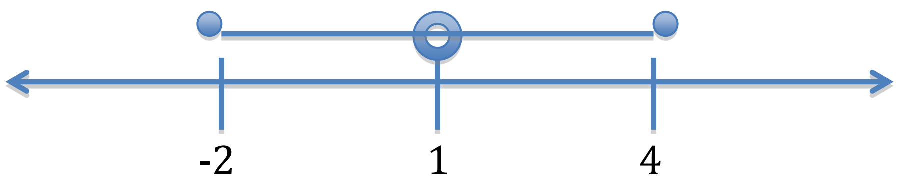
$|x| \leq d$

$-d \leq x \leq d;$

$$-3 \leq |x-1| \leq 3$$
$$-3 \leq x-1 \leq 3$$
$$-2 \leq x \leq 4$$

<u>Note</u> : From the inequality $\frac{|x-1|-1}{|x-1|} \leq \frac{2}{3}$, $x \neq 1$ because if x=1 the inequality does not exist

The critical values are -2, 1, 4



Therefore, the solution set of inequality $\frac{|x-1|-1}{|x-1|} \leq \frac{2}{3}$

Is $[-2,1) \cup (1,4]$

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1. Inequality

   $|2-5x| \geq 4-x$
   $2-5x \geq 4-x$ or $2-5x \leq 4-x$
   $-4x \geq 2$ or $-6x \leq -6$
   $x \leq -2$ or $x \geq 1$
   So, the solution set of the inequality $|2-5x| \geq 4-x$ is $(-\infty,-2] \cup [1,\infty)$

2. Proofs
   ~S→(~Q→R), S→R, (PvQ) →~R and P, then Q
   1. ~S→(~Q→R) Given
   2. S→R Given
   3. (PvQ) →~R Given
   4. P Given
   5. PvQ 4, law of addition
   6. ~R 5,3, modus ponens
   7. ~S 2,6, modus tollens
   8. ~Q→r 7,1, modus ponens
   9. Q 6,8, modus tollens

1.Prove that if aVbVc, ~(dΛe) -> ~(fVa), and (c->a) Λ(~bVf), then d
Assume ~d

1. ~d Assumption
2. ~d ->(~dV~e) Law of Addition
3. ~dV~e 1,2,Rule of Modus Ponens
4. (~dV~e) <-> ~(dΛe) De Morgan's Rule
5. ~(dΛe) 3,4,Rule of Substitution
6. ~(dΛe) -> ~(fVa) Given
7. ~(fVa) 5,6,Rule of Modus Ponens
8. ~(fVa) <-> (~fΛ~a) De Morgan's Rule
9. ~fΛ~a 7,8,Rule of Substitution

10. ~f and ~a — 9, Definition of Conjunction
11. (c->a) Λ (~bVf) — Given
12. (c->a) and (~bVf) — 11, Definition of Conjunction
13. ~c — 10,12, Rule of Modus Tollens
14. ~b — 10,12, Disjunctive Syllogism
15. aVbVc — Given
16. a — 13,14,15, Disjunctive Syllogism
17. a Λ ~a — 10,16, Definition of conjunction

Since, a and ~a is contradiction, so we conclude d.

2. $\dfrac{X}{X+4} \geq \dfrac{2}{X-2}$

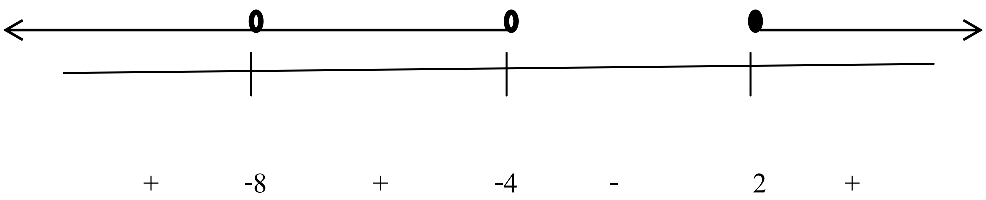
$= \dfrac{X}{X+4} - \dfrac{2}{X-2} \geq 0$

$= \dfrac{X(X-2)-2(X+4)}{(X-2)(X+4)} \geq 0$

$= \dfrac{X^2 - 2X - 2X - 8}{(X-2)(X+4)} \geq 0$

$= \dfrac{(X+8)^2}{(X-2)(X+4)} \geq 0$

So, the critical points are -8, 2 and -4



Hence, the solution set are (-∞,-8) U (-8,-4) U (2 ,∞) Ans

Method of Proof

Prove that if q→p and r∧q then ~r →(p∧q)

1. q→p — Assumption
2. r∧q — Assumption
3. r and q — 2, Definition of Conjunction
4. q — 1,3, Rule of Modus Ponens
5. (q→p)↔(~q∨p) — 1, Law of Implication

6. ~q∨p 5, Rule of Substitution
7. p 3, 6, Disjunctive Syllogism
8. (p∧q) 3, 7, Definition of Conjunction
9. (p∧q) → [(p∧q) ∨ ~r] 8, Law of addition
10. p∧q) ∨ ~r 8, 9, rule of Modus Ponens,
11. ~r →(p∧q) 10, Law of Implication, Rule of Substitution

Therefore if q→p and r∧q then ~r →(p∧q) is true

Inequality

Solve the inequality $-25 \leq \frac{|2x+5|}{2} \leq 22$

$-25 \leq \frac{|2x+5|}{2} \leq 22$

$-50 \leq |2x+5| \leq 44$

We have $-50 \leq |2x+5|$ and $|2x+5| \leq 44$

Consider $-50 \leq |2x+5|$ by property of the absolute value

$2x+5 \leq -(-50)$ or $2x+5 \geq -50$

$2x \leq 45$ $2x \geq -55$

$x \leq \frac{45}{2}$,$(-\infty,22.5]$ $x \geq -\frac{55}{2}$, $[-27.5,\infty)$

Hence, the solution set of the inequality -50 ≤|2x+5} is (-∞,22.5]U [-27.5,∞)= R

Consider $|2x+5| \leq 44$

$-44 \leq 2x+5 \leq 44$ by property of the absolute value

$-49 \leq 2x \leq 39$

$-\frac{49}{2} \leq x \leq \frac{39}{2}$, $[-24.5,19.5]$

Hence, the solution set of the inequality |2x+5|≤ 44 is [-24.5,19.5]

∴ The solution set of the inequality $-25 \leq \frac{|2x+5|}{2} \leq 22$ is R∩ [-24.5,19.5] = [-24.5,19.5]

1. ***Logic and prove***

$(\sim r \rightarrow t) \wedge (s \vee t)$ find contradition

1. $\sim[(\sim r \rightarrow t) \wedge (s \vee t)] \equiv \sim(\sim r \rightarrow t) \vee \sim(s \vee t)$ De Morgan's Laws

2. $(\sim r \wedge \sim t) \vee \sim(s \vee t)$ Law of Negation for Implication

3. $(\sim r \wedge \sim t) \vee (\sim s \wedge \sim t)$ De Morgan's Laws

4. $(\sim r \vee \sim s) \wedge \sim t$ Distributive Laws

5. $\sim(r \wedge s) \wedge \sim t$ De Morgan's Laws

2.Inequlity problem

Solve the inequality 3|1−x|−4≥|1−x| for x.

$$3|1-x| - 4 \geq |1-x|$$

If we subtract $|1-x|$ from both sides of the inequality, then add 4 to both sides of the inequality, we get

$$3|1-x| - |1-x| \geq 4.$$

On the left, we have like terms.

Note that $3|1-x|-|1-x| = 3|1-x|-1|1-x| = 2|1-x|$.

$$2|1-x| \geq 4.$$

$$|1-x| \geq 2$$

$$1-x \leq -2 \text{ or } 1-x \geq 2.$$

We can solve each of these inequalities independently. First, subtract 1 from both sides of each inequality, then multiply both sides of each resulting inequality by -1, reversing each inequality as you go.

$$1-x \leq -2 \ or \ 1-x \geq 2$$

$$-x \leq -3 \quad -x \geq 1$$

$$x \geq 3 \quad x \leq -1$$

The solution set is (-∝,-1]U [3,∝)

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Method of proof

If $\sim q \to (p \vee r)$ and $(p \to q) \vee s$,then $(\sim r \wedge \sim s) \to q$

Proof : we have to assume $(\sim r \wedge \sim s)$, then prove q.

1. $(\sim r \wedge \sim s)$	assumption
2. $\sim r\ and\ \sim s$	1, definition of conjunction
3. $(p \to q) \vee s$	assumption
4. $p \to q$	2,3 Disjunctive syllogism
5. $\sim p \vee q$	4, law of implication
6. $\sim q \to (p \vee r)$	assumption
7. $q \vee (p \vee r)$	6, law of implication double negation
8. $(q \vee p) \vee r$	7, associative law
9. $q \vee p$	2,8 disjunctive syllogism
10. $(q \vee p) \wedge (\sim p \vee q)$	5,9 definition of conjunction
11. $q \vee (p \wedge \sim p)$	10 distributive law
12. $\sim (p \wedge \sim p)$	law of contradiction
13. q	11,12 Disjunctive syllogism

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Solving inequalities.

Find the solution set of the inequality : $|x| \geq \frac{2}{|x+1|}$

solve :: First, we can rearrange this inequality into

$$|x||x+1| \geq 2$$

because the denominator is always positive.

And as the property of absolute value |ab|=|a||b|,

$$|x^2 + x| \geq 2$$

$$x^2 + x \leq -2 \qquad \text{or} \qquad x^2 + x \geq 2$$

Case 1 :: $x^2 + x \leq -2$

Rearrange it $x^2 + x + 2 \leq 0$

We cannot factor this inequality because $b^2 - 4ac = -7$ [a=1 b=1 c=2] which is less than 0 and since a=1 which is greater than 0,so $x^2 + x + 2$ must be greater than 0 for all x.

Therefore the solution set of case 1: $x^2 + x \leq -2$ is an empty set

Case 2 :: $x^2 + x \geq 2$

Rearrange it $x^2 + x - 2 \geq 0$

And then we have to factor it

$$(x+2)(x-1) \geq 0$$

The critical values are -2 and 1 and we have to consider 3 ranges-($x \leq -2, -2 \leq x \leq 1, 1 \leq x$) and find the range(s) that have values satisfying this inequality.

Therefore the solution set of case 2: $x^2 + x \geq 2$ is $(-\infty,-2] \cup [1,\infty)$

The solution set of the inequality $|x| \geq \frac{2}{|x+1|}$ is $\varphi \cup \{(-\infty,-2] \cup [1,\infty)\}$ or

$(-\infty,-2] \cup [1,\infty)$

5504640912

<u>Solve</u> $3|x-1| - 2x > 2|3x-1|$

$$|x-1| \begin{cases} x-1 ; & x \geq 1 \\ -(x-1); & x < 1 \end{cases} \qquad |3x-1| \begin{cases} 3x-1 ; & x \geq \frac{1}{3} \\ -(3x-1) ; & x < \frac{1}{3} \end{cases}$$

We have 3 cases

When $X < \frac{1}{3}$, When $\frac{1}{3} \leq X < 1$, When $X \geq 1$

<u>Case 1</u>

$3(-x-1)-2x > 2(-3x+1)$

$-5x - 3 > -6x+2$

$X > 5$

Since $X > 5$ doesn't intersect with $X < \frac{1}{3}$ interval, so it's $\emptyset$

<u>Case 2</u>

$3(-x+1)-2x > 2(3x-1)$

$-5x+3 > 6x-2$

$5 > 11x$

$\frac{11}{5} > x$

Since $\frac{11}{5} > x$ doesn't intersect with $\frac{1}{3} \leq X < 1$ interval, so it's $\emptyset$

<u>Case3</u>

$3(x-1)-2x > 2(3x-1)$

$X - 3 > 6x-2$

$-1 > 5x$

$$\frac{-1}{5} > x$$

Since $\frac{-1}{5} > x$ doesn't intersect with $X \geq 1$ interval, so it's $\emptyset$

Since all of the solution is $\emptyset$, hence the final solution is $\emptyset$

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Question Prove that if (~a v b) → c, ~c v d, and d →~ (e v ~e) then a

Solution: we assume ~a to prove for the contradictions

1. ~a	assumption
2. ~a → (~a v b)	law of addition
3. ~a v b	1,2 Rule of Modus Ponens
4. (~a v b) → c	Given
5. C	3,4 Rule of modus ponens
6. C ↔ ~(~c)	law of double negation
7. ~(~c)	5,6 rule of substitution
8. ~c v d	given
9. d	7,8 disjunctive syllogism
10. d →~(e v ~e)	given
11. ~(e v ~e)	9,10 rule of modus Ponen
12. ~(e v ~e) ↔ (~e^~(~e))	Morgan's laws
13. ~e ^ ~(~e)	11,12 rule of substitution

Since ~e ^ ~ (~e) is contradiction, hence a is true.

Question 2 $x/5 - 2 > 2/3\,(x+3)$

Multiply throughout by 5:

$x-10 > 10/3(x+3)$

Multiply throughout by 3:

$3x-30 > 10(x+3)$

$3x-30 > 10x+30$

$3x > 10x+60$

$-7x > 60$

$x < -60/\,7 \approx -8.57$

$|5x-2| \geq |2x+1|$

<u>Note</u> $|5x-2| = 5x-2 \; ; x \geq \frac{2}{5}$ or $-(5x-2) \; ; x < \frac{2}{5}$

$|2x+1| = 2x+1 \; ; x \geq -\frac{1}{2}$ or $-(2x+1) \; ; x < -\frac{1}{2}$

<u>Case 1</u> When $x < -\frac{1}{2}$, we substitute $-(2x+1)$ in $|2x+1|$ and $-(5x-2)$ in $|5x-2|$.

$$|5x-2| \geq |2x+1|$$
$$-5x+2 \geq -2x-1$$
$$-3x \geq -3$$
$$X \leq 1$$

Hence, the solution set for case 1 is $(-\infty, -\frac{1}{2}) \cap (-\infty, 1) = (-\infty, -\frac{1}{2})$

<u>Case 2</u> When $-\frac{1}{2} \leq x < \frac{2}{5}$, we substitute $2x+1$ in $|2x+1|$ and $-(5x-2)$ in $|5x-2|$.

$$|5x-2| \geq |2x+1|$$
$$-5x+2 \geq 2x+1$$
$$-7x \geq -1$$
$$X \leq \frac{1}{7}$$

Hence, the solution set for case 2 is $[-\frac{1}{2}, \frac{2}{5}) \cap (-\infty, \frac{1}{7}] = [-\frac{1}{2}, \frac{1}{7}]$

<u>Case 3</u> When $x \geq \frac{2}{5}$, we substitute $5x-2$ in $|5x-2|$ and $2x+1$ in $|2x+1|$.

$$|5x-2| \geq |2x+1|$$
$$5x+2 \geq 2x+1$$
$$3x \geq -1$$
$$X \geq -\frac{1}{3}$$

Hence , the solution set for case 3 is $[\frac{2}{5}, \infty) \cap [-\frac{1}{3}, \infty) = [\frac{2}{5}, \infty)$

$\therefore$ The solution set of inequality $|5x-2| \geq |2x+1|$ is $(-\infty, -\frac{1}{2}) \cup [-\frac{1}{2}, \frac{1}{7}] \cup [\frac{2}{5}, \infty) = (-\infty, \frac{1}{7}] \cup [\frac{2}{5}, \infty)$

Prove that If $(p \to q)$ ^ $(s \to \sim r)$, $(\sim r \to \sim p)$ then $p \to q$

1. p — Assumption
2. $p \to (p \vee s)$ — 1 , Law of Addition
3. p □ s — 1 , 2 , Rules of Modus Ponens
4. $(p \to q)$ □ $(s \to \sim r)$ — Assumption
5. q □ $\sim r$ — Constructive Dilemma
6. $\sim r \to \sim p$ — Assumption
7. r — 1 , 6 , Rule of Modus Tollens , Double Negation , Rule of Substitution
8. q — 5, 7, Disjunctive Syllogism

$\therefore$ Therefore, if $(p \to q)$ ^ $(s \to \sim r)$, $(\sim r \to \sim p)$ then $p \to q$.

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INDUCTION PROOF

Proof that if (a→ ~ b) V c and bVd then ~c → (a→d)

1. ~c — Assumption
2. (a→ ~ b) V c — Assumption
3. a→ ~ b — 1 , 2 , Disjunctive Syllogism
4. b V d — Assumption
5. ~b → d — 4 , Law of Implication
6. a→ d — 3 , 5 , Hypothetical Syllogism

Therefore, if (a→ ~ b) V c and bVd then ~c → (a→d).

INEQUALITY

$|2+3x| - |x - 4| \leq 5$

<u>Note</u> $|2+3x| = 2+3x$; $x \geq -\frac{2}{3}$ or $-(2+3x)$; $x < -\frac{2}{3}$

$|x - 4| = x - 4$; $x \geq 4$ or $-(x-4)$; $x < 4$

Case 1 When $x < -\frac{2}{3}$, we substitute $-(2+3x)$ in $|2 + 3x|$ and substitute $-x+4$ in $|x - 4|$

$$|2 + 3x| - |x - 4| \leq 5$$
$$-(2+3x) - (- x + 4) \leq 5$$
$$-2- 3x + x - 4 \leq 5$$
$$-2x - 6 \leq 5$$
$$-2x \leq 11$$
$$x \geq -\frac{11}{2}$$

Hence, the Solution set of case 1 is $(-\infty, -\frac{2}{3}) \cap [-\frac{11}{2}, \infty) = [-\frac{11}{2}, -\frac{2}{3})$

Case 2 When $-\frac{2}{3} \leq x < 4$, we substitute $2+3x$ in $|2 + 3x|$ and substitute $-x + 4$ in $|x - 4|$

$$|2 + 3x| - |x - 4| \leq 5$$
$$2+3x - (- x + 4) \leq 5$$
$$2 + 3x + x - 4 \leq 5$$
$$4x - 2 \leq 5$$
$$4x \leq 7$$
$$x \leq \frac{7}{4}$$

Hence, the Solution set of case 2 is $[-\frac{2}{3}, 4) \cap (-\infty, \frac{7}{4}] = [-\frac{2}{3}, \frac{7}{4}]$

Case 3 When $x \geq 4$, we substitute $2+3x$ in $|2 + 3x|$ and substitute $x - 4$ in $|x - 4|$

$$|2 + 3x| - |x - 4| \leq 5$$
$$2+3x - x + 4 \leq 5$$
$$2x + 6 \leq 5$$
$$x \leq -\frac{1}{2}$$

Hence, the Solution set of case 3 is $[4, \infty) \cap (-\infty, -\frac{1}{2}] = \varphi$

Therefore, the solution set for the inequality $|2 + 3x| - |x - 4| \leq 5$ is $[-\frac{11}{2}, -\frac{2}{3}) \cup [-\frac{2}{3}, \frac{7}{4}] \cup \varphi$
$= [-\frac{11}{2}, \frac{7}{4}]$

Prove s v ~r , ~p → (r ^ s) , p → q then ~s → q
So we assume ~s and prove q

1. ~s Assumption
2. ~s → (~s v ~r) 1, Law of addition
3. ~s v ~r 1, 2, Modus Ponens
4. s v ~r Assumption
5. r 1, 4, Disjunctive syllogism
6. ~ (s ^ r) 3, De Morgan's law
7. ~ (r ^ s) 6, Commulative law
8. ~p → (r ^ s) Assumption
9. p 7, 8, Modus Tollens
10. p → q Assumption
11. q 9, 10, Modus Ponens

Therefore , q is true then ~s → q is true

Solve the inequality $|4x + 3| - |x + 3| \leq 6$

$|4x + 3| = \{ 4x+3 ; x \geq \frac{-3}{4}$

$-(4x+3) ; x < \frac{-3}{4} \}$

$|x + 3| = \{ x+3 ; x \geq -3$

$-(x+3) ; x < -3$

<u>Case1</u> when $x < -3$, we substitute $-x -3$ into $|x + 3|$ and $-4x -3$ into $|4x + 3|$. So we have

$$(-4x -3) – (-x - 3) \leq 6$$

$-3x \le 6$

$X \le -2$

Hence, the solution set for case1 is $[-2,\infty)$

<u>Case2</u> when $-3 < x < \frac{-3}{4}$, we substitute $-(4x+3)$ into $|4x+3|$ and $x+3$ into $|x+3|$. So we have

$-(4x+3) - (x+3) \le 6$

$-5x - 6 \le 6$

$X \le 0$

Hence, the solution set for case2 is $[0, \infty)$

<u>Case3</u> when $x \ge \frac{-3}{4}$, we substitute $4x + 3$ into $|4x + 3|$ and $x+3$ into $|x+3|$. So we have

$(4x+3) - (x+3) \le 6$

$3x \le 6$

$X \le 2$

Hence, the solution set for case3 is $(-\infty, 2]$

Therefore, the solution set for inequality for $|4x+3| - |x+3| \le 6$ is $[-2,2]$

1. Solve the inequality $|2x+7| \le |x+9| - 3$

$|2x+7| = \{2x+7, x \ge -\frac{7}{2}$

$\{-(2x+7), x < -\frac{7}{2}$

$|x+9| = \{x+9, x \ge -9$

$\{-(x+9), x < -9$

CASE 1

When $x < -9$, we substitute $|x+9|$ with $-(x+9)$ and $|2x+7|$ with $-(2x+7)$. So we have

$-(2x+7) \le -(x+9)-3$

$-2x-7 \le -x-9-3$

$-2x+7 \le -x-12$

$-2x+x \le -12-7$

$-x \le -19$

$x \ge 19$

The solution set is $(-\infty,-9) \cup [19,\infty)$

CASE2

When $-9 \le x \le -\frac{7}{2}$, we substitute $|2x+7|$ with $-(2x+7)$ and $|x+9|$ with $(x+9)$. So we have

$-(2x+7) \le x+9-3$

$-2x-7 \le x+6$

$-7-6 \le x+2x$

$-13 \le 3x$

$$X \geq -\frac{3}{13}$$

The solution set is $[-9 \leq x \leq -\frac{7}{2}) \cup [-\frac{3}{13}, \infty)$

CASE3

When $x > -\frac{7}{2}$, we substitute $|2x+7|$ with $2x+7$ and $|x+9|$ with $x+9$. So we have

$$(2x+7) \leq x+9-3$$
$$(2x+7) \leq x+6$$
$$(2x+7) \leq x+9-3$$
$$(2x+x) \leq 6-7$$
$$x \leq -1$$

The solution set is $[-\frac{7}{2}, -\infty) \cap [-1, \infty) = [-\frac{7}{2}, -1]$

2. P v (q → t), ~ p ^ ~ t then ~q

1.	~p ^~t	assumption
2.	~p and ~ t	Definition of Conjunction
3.	P v (q →t)	Assumption
4.	q → t	2,3, disjunctive syllogism
5.	~q	2,4, Modus Tollens

Question 1

For all integers n≥1 we have

$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} = 2^n$

Proof: Putting a = b = 1 in the Theorem above, we get

$\mathbf{(1+1)^n}$

$\mathbf{=1^n +}$

$\mathbf{\binom{n}{1}(1^{n-1})(1) + \binom{n}{2}(1^{n-2})(1^2) + \ldots + \binom{n}{n-2}(1^2)(1^{n-2}) + \binom{n}{n-1}(1)(1^{n-1}) + 1^n,}$

Hence

$\mathbf{2^n = 1 + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-2} + \binom{n}{n-1} + 1}$

Therefore by property 1 we get

$\mathbf{2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-2} + \binom{n}{n-1} + \binom{n}{n}}$

Question 2

Solve | x^2 +3x−1| < 3

$-3 < x^2 + 3x - 1 < 3$

Now we need to break this up into 2 parts:

Left part

$-3 < x^2 + 3x - 1$

$0 < x^2 + 3x + 2$

$x^2 + 3x + 2 > 0$

$(x+2)(x+1) > 0$

Critical values for the left side are:

$x = -2 \text{ and } x = -1$

This gives:

$x < -2 \text{ and } x > -1$

Right part

$x^2 + 3x - 1 < 3$

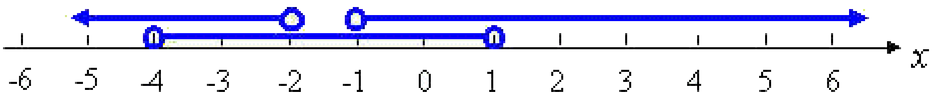
$x^2 + 3x - 4 < 0$

$(x+4)(x-1) < 0$

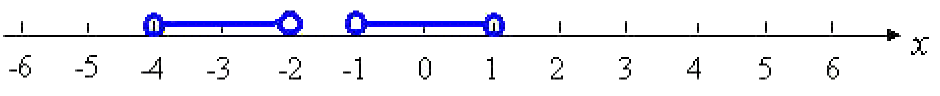
Critical values: $x = -4$ and $x = 1$

This gives: $-4 < x < 1$

We need to graph these to see the result:



Considering where these areas intersect, we can see that there are 2 areas:



This gives us our final solution: -4 < *x* < -2 and -1 < *x* < 1

TU152 Homework

Inequality

Find the solution set of the inequality $|x+2| + |x| - |x-4| \geq 5$.

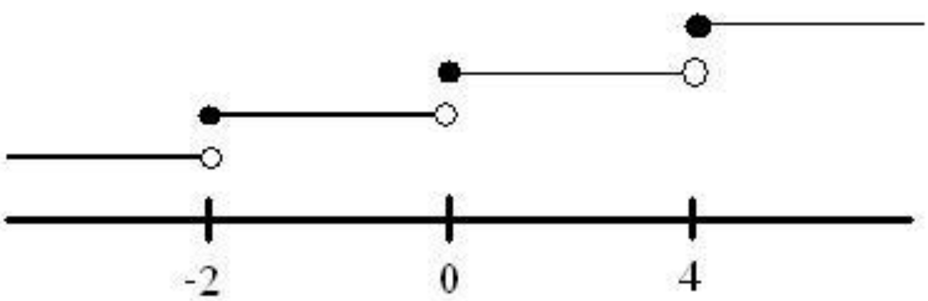
Solve

Note that

$|x+2| = \begin{cases} x+2\ ; x \geq -2 \\ -x-2\ ; x < -2 \end{cases}$

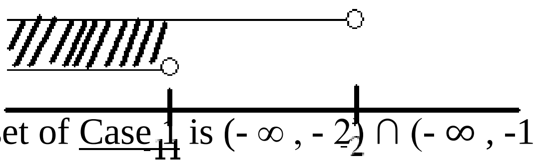
$|x| = \begin{cases} x\ ; x \geq 0 \\ -x\ ; x < 0 \end{cases}$

$|x-4| = \begin{cases} x-4\ ; x \geq 4 \\ 4-x\ ; x < 4 \end{cases}$



Case 1 x < -2 , we substitute $|x+2|$ with $-x-2$, $|x|$ with – x and $|x-4|$ with 4 – x. So we have $|x+2| + |x| - |x-4| \geq 5$

$$- x - 2 + (- x) - (- x + 4) \geq 5$$
$$- x - 2 - x + x - 4 \geq 5$$
$$- x - 6 \geq 5$$
$$- x \geq 11$$
$$x \leq -11$$

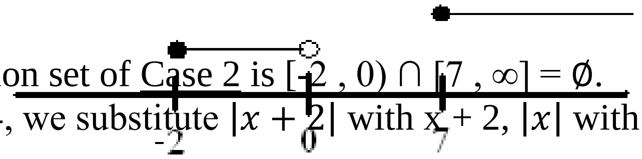


Hence, the solution set of Case 1 is (- ∞ , - 2) ∩ (- ∞ , -11) = *(- ∞ , -11)*

Case 2 - 2 ≤ x < 0 , we substitute $|x+2|$ with x + 2, $|x|$ with - x and $|x-4|$ with 4 – x.

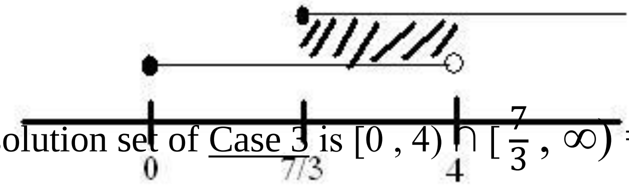
So we have $|x+2| + |x| - |x-4| \geq 5$

x + 2 - x + x - 4 ≥ 5
x - 2 ≥ 5
x ≥ 7



Hence, the solution set of Case 2 is [-2 , 0) ∩ [7 , ∞] = ∅.
Case 3 $0 \le x < 4$, we substitute $|x+2|$ with x + 2, $|x|$ with x and $|x-4|$ with 4 – x.

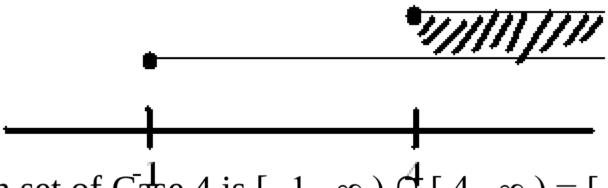
So we have $|x+2| + |x| - |x-4| \ge 5$
x + 2 – x + x – 4 ≥ 5
3x – 2 ≥ 5
3x ≥ 7
$x \ge \frac{7}{3}$



Hence, the solution set of Case 3 is [0 , 4) ∩ $[\frac{7}{3}, \infty) = [\frac{7}{3}, 4)$.

Case 4 $x \ge 4$, we substitute $|x+2|$ with x + 2, $|x|$ with x and $|x-4|$ with x – 4.

So we have $|x+2| + |x| - |x-4| \ge 5$
x + 2 + x – x + 4 ≥ 5
x + 6 ≥ 5
x ≥ -1



Hence, the solution set of Case 4 is [-1 , ∞) ∩ [4 , ∞) = [4 , ∞).
Thereby, the original solution set of the inequality $|x+2| + |x| - |x-4| \ge 5$ is $(-\infty, -11] \cup [4, \infty) \cup \emptyset \cup [\frac{7}{3}, \infty) = (-\infty, -11) \cup (\frac{7}{3}, \infty)$.

Method of Proof
Let a → ~ b and ~ (a ^ b) → (c ^ ~ d) be true, prove ~ d.
Solve

1) a → ~ b	Assumption
2) ~ a v ~ b	1 , Law of Implication
3) ~ (a ^ b)	2 , De Morgan's Law
4) ~ (a ^ b) → (c ^ ~ d)	Assumption
5) c ^ ~ d	3 , 4 , Rule of Modus Ponens
6) c and ~ d	5 , Definition of Conjunction

TU152 Homework

Proof

Prove that if ~q→ (p□r) and ~ (p→q)→s, then ~q→(r□s)

First, we need to show ~q→(r□s). Assume ~q prove (r□s)

1) ~q	Assumption
2) ~q→(p□r)	Assumption
3) p□r	1,2 Rule of Modus Ponens
4) ~(p→q)→s	Assumption
5) ~[~(p→q)]□s	4, Law of implication
6) (~p□q)□s	Law of double negation, Rule of substitution, Law of implication
7) (~p□s)□q	6, Associative law
8) (p→s)□q	7, Law of implication
9) p→s	1,8, Disjunctive Syllogism
10) r□p	3, Commutative law
11) ~r→p	9, Law of implication
12) (~r→p)^(p→s)	9,11, Definition of conjunction
13) [(~r→p)^(p→s)]→[~r→s]	12, Law of syllogism
14) ~r→s	12, 13, Rule of Modus Ponens
15) r□s	14, Law of implication

Inequality

$|2x-3|-4 \leq |x+1|$

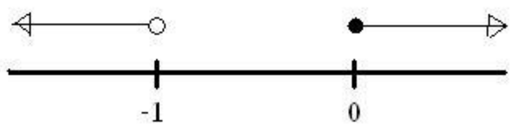
Note $|2x-3|$: 1) $2x-3$; $2x-3 \geq 0$; $x \geq \frac{3}{2}$

: 2) $-2x+3$; $2x-3<0$; $x< \frac{3}{2}$

And $|x+1|$: 1) $x+1$; $x+1 \geq 0$; $x \geq -1$
: 2) $-x-1$; $x+1 < 0$; $x < -1$

<u>Case 1</u> When $x<-1$, we substitute $|2x-3|$ with $-2x+3$ and $|x+1|$ with $-x-1$,
so we have $|2x-3|-4 \leq |x+1|$
That is $-2x+3-4 \leq -x-1$

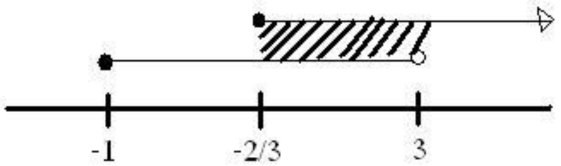
$$-2x+3+x+1 \leq 4$$
$$-x \leq 0$$
$$x \geq 0$$



The solution set for case 1 is $\emptyset$

<u>Case 2</u> When $-1 \leq x < \frac{3}{2}$, we substitute $|2x-3|$ with $-2x+3$ and $|x+1|$ with $x+1$,
so we have $|2x-3|-4 \leq |x+1|$
That is $-2x+3-4 \leq x+1$

$$-2x+3-x-1 \leq 4$$
$$-3x+2 \leq 4$$
$$-3x \leq 2$$
$$x \geq -\frac{2}{3}$$



The solution set for case 2 is $[-\frac{2}{3},3)$
<u>Case 3</u> When $x \geq \frac{3}{2}$, we substitute $|2x-3|$ with $2x-3$ and $|x+1|$ with $x+1$,

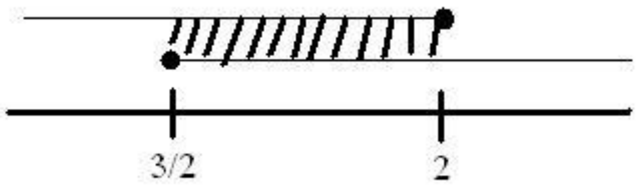
so we have $|2x-3|-4 \leq |x+1|$

That is $2x-3-4 \leq x+1$

$2x+3-x-1 \leq 4$

$x+2 \leq 4$

$x \leq 2$



The solution set for case 3 is $[\frac{3}{2},2]$

Hence, the solution for the inequality $|2x-3|-4 \leq |x+1|$ is $\emptyset \cup [-2/3,3) \cup [3/2,2] = [-2/3,3)$

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Method of Proofs

Prove that if $(q\to r)\to t$, $p\to\sim t$, $\sim[(r\to w) \wedge (s\vee y)]$, then $(q\vee y)\to\sim p$

<u>Solution</u>

Since $(q\vee y)\to\sim p \equiv p\to\sim(q\vee y)$ Law of Contraposition

$\equiv p\to(q\wedge\sim y)$ De Morgan's laws

Then we will assume p, then prove $(q\wedge\sim y)$

1)	p	assumption
2)	$p\to\sim t$	assumption
3)	$\sim t$	1),2), Rule of Modus Ponens
4)	$(q\to r)\to t$	assumption
5)	$\sim(q\to r)$	3),4), Rule of Modus Tollens
6)	$\sim(\sim q\vee r)$	5), Law of Implication
7)	$q\wedge\sim r$	6), De Morgan's laws
8)	q and $\sim r$	7), Definition of Conjunction
9)	$\sim[(r\to w) \wedge (s\vee y)]$	assumption
10)	$\sim(r\to w) \vee \sim(s\vee y)$	9), De Morgan's laws
11)	$\sim(\sim r\vee w) \vee (\sim s\wedge\sim y)$	10), Law of Implication, De Morgan's laws
12)	$(\sim r\vee w) \to (\sim s\wedge\sim y)$	11), Law of Implication
13)	$\sim r\to(\sim r\vee w)$	8), Law of Addition

14) (~r∨w) 8),13), Rule of Modus Ponens
15) (~s∧~y) 12),14), Rule of Modus Ponens
16) ~s and ~y 15), Definition of Conjunction
17) q∧~y 8),16), Definition of Conjunction

Hence, p→(q∧~y) is true and equivalent to (q∨y)→~p ;therefore, (q∨y)→~p is true.

The linear inequality

Linsey bought 200 units of book, romance and suspense novel which cost 1,200 baht. She will sell the romance novel at the price of 5 baht and sell the suspense novel at the price of 8 baht. Find the highest quantity of romance novel that can be bought if she wants to make the net profit more than 250 bath (Suppose there is no fixed cost)

Solution

- Let x be the highest quantity of romance novel that can be bought
- From the total units of the book bought is 200 the quantity of suspense novel is 200-x
- Total Cost = Total fixed cost + Total variable cost
  = 0 + 1,200
  = 1,200
- Total Revenue = Selling Price × Quantity of units sold
  = 5(x) + 8(200-x)
  = 5x + 1,600 -8x
  = -3x + 1,600
- Net Profit = Total Revenue - Total Cost
  =-3x + 1,600-1,200
  =-3x +400

Therefore, the inequality is

-3x + 400 > 250
-3x > -150
X < 50

Therefore, the highest quantity of romance novel that can be bought is 49.

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$$(h \rightarrow a) \rightarrow p$$
$$(y\ v\ i) \rightarrow a$$
$$y$$
$$\overline{\therefore p}$$

Assume r true to prove for contradiction

1. (h→a) →p Assumption

2. □p Assume
3. □(h→a) 1,2, Rule of Modus Tollens
4. □ (□h v a) 3, Implication law
5. h⍰ □a 4, De Morgan's law
6. h 5, Definition of conjunction
7. □a 5, Definition of conjunction
8. (y v i)→a Assumption
9. □(y v i) 7,8, Rule of Modus Tollens
10. □y □ □i 9, De Morgan's law
11. □y 10, Definition of conjunction
12. □i 10, Definition of conjunction
13. y Assumption
14. □y ⍰ y 11,13, definition of conjunction

$$\left|\frac{4x}{3} - 5\right| \geq 7$$

$$\left|\frac{4x}{3} - 5\right| \geq 7 \;\text{ OR }\; \left|\frac{4x}{3} - 5\right| \leq -7$$

$$4x - 15 \geq 21 \qquad 4x - 15 \leq -21$$

$$4x \geq 36 \qquad 4x \leq -6$$

$$x \geq 9 \qquad x \leq -1.5$$

So the solution is: x ≤ -1.5 or x≥9.

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Nontrakarn Horpattaporn

$$(h \to a) \to p$$
$$(y\ v\ i) \to a$$
$$y$$
$$\overline{\qquad\qquad\qquad}$$
$$\therefore p$$

Assume □r true to prove for contradiction

15. (h→a) →p Assumption
16. □p Assume
17. □(h→a) 1,2, Rule of Modus Tollens
18. □ (□h v a) 3, Implication law
19. h⍰ □a 4, De Morgan's law
20. h 5, Definition of conjunction
21. □a 5, Definition of conjunction
22. (y v i)→a Assumption
23. □(y v i) 7,8, Rule of Modus Tollens
24. □y ⍰ □i 9, De Morgan's law
25. □y 10, Definition of conjunction
26. □i 10, Definition of conjunction
27. y Assumption

28. □y ⍰ y 11,13, definition of conjunction

$\left|\frac{4x}{3} - 5\right| \geq 7$

$\left|\frac{4x}{3} - 5\right| \geq 7$ OR $\left|\frac{4x}{3} - 5\right| \leq -7$

$4x - 15 \geq 21$ $\quad$ $4x - 15 \leq -21$

$4x \geq 36$ $\quad$ $4x \leq -6$

$x \geq 9$ $\quad$ $x \leq -1.5$

So the solution is: x ≤ -1.5 or x≥9.

$$\frac{3x^2 - x^3}{-2} \geq 0$$

Multiply by -2 in the both sides of equation

$$(-2) * (\frac{3x^2 - x^3}{-2}) \leq (-2)(0)$$

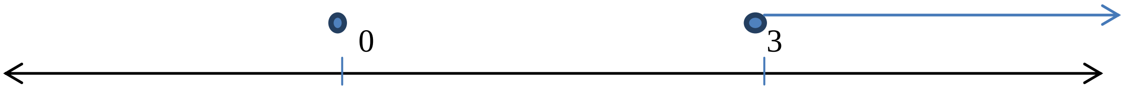
$$3x^2 - x^3 \leq 0$$

Multiply by -1 in the both sides of equation

$$x^3 - 3x^2 \geq 0$$

$$x^2(x - 3) \geq 0$$

Critical values are 0 and 3



$x^2(x - 3)$ substitute by -1, the answer is negative.
$x^2(x - 3)$ substitute by 1, the answer is negative.
$x^2(x - 3)$ substitute by 4, the answer is positive.
Hence, the answer is {0} U [3,∞)

Show that ~p→(q→ $\sim r$), ~p v s , ~t→ q, ~s, and r then t

1. ~s Assumption
2. ~p v s Assumption
3. ~p and s 2, Definition of conjunction
4. ~p 2, 3 Definition of conjunction
5. S 2, 3 Definition of conjunction
6. ~p→(q→ $\sim r$) Assumption
7. q→ $\sim r$ 4,6 Law of Modus Ponens
8. ~t→ q Assumption
9. ~t→ $\sim r$ 7,8 Law of hypothetical syllogism
10. r Assumption
11. ~(~r) Law of double negation

12. ~(~t) 9,11 Law of Modus Tollens
13. t 12 De Morgan's Laws

Therefore, t is true.

Induction Problem

1. Prove Bernoulli's Inequality which states " If $x \geq -1$ then $(1+x)^n \geq 1+nx$ for all natural numbers n "

Solution Let S_n be the statement "$(1+x)^n \geq 1+nx$, if $x \geq -1$" where n is all natural numbers

1. S_1 is "$(1+x)^1 \geq 1+x$ " that is "$1+x \geq 1+x$ " which is true
2. Assume S_k is true and show S_{k+1} is true. That is, we assume $(1+x)^k \geq 1+kx$, if $x \geq -1$ is true where k is all natural numbers.
   We need to show $(1+x)^{k+1} \geq 1+(k+1)x$

Since $(1+x)^k \geq 1+kx$ is true
Multiply $(1+x)$ both sides
We have $(1+x)(1+x)^k \geq (1+x)(1+kx)$ by inductive assumption $1+x \geq 0$ since $x \geq -1$

$$(1+x)^{k+1} \geq kx^2+(kx+x)+1$$
$$(1+x)^{k+1} \geq kx^2+(k+1)x+1 \geq 1+(k+1)x$$
$$(1+x)^{k+1} \geq 1+(k+1)x \text{ (Since,} k > 0 \; and \; x^2 \geq 0 \; so \; that \; kx^2 \geq 0)$$

Hence, S_{k+1} is true
∴ By Mathematical induction, we can conclude that $(1+x)^n \geq 1+nx$, if $x \geq -1$ is true where n is all natural numbers.

Inequality Problem

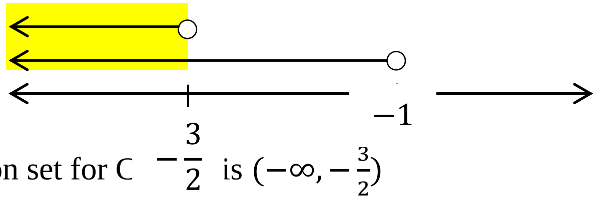
2. Find the solution of the inequality $\left|x+\frac{3}{2}\right| > \frac{|x|}{2x}$

Solution Note $\left|x+\frac{3}{2}\right| \begin{cases} x+\frac{3}{2}; x+\frac{3}{2} \geq 0 \to x \geq -\frac{3}{2} \\ -(x+\frac{3}{2}); x+\frac{3}{2} < 0 \to x < -\frac{3}{2} \end{cases}$ and $|x| \begin{cases} x; x \geq 0 \\ -x; x < 0 \end{cases}$

CASE1 When $x < -\frac{3}{2}$, ... ute $\left|x+\frac{3}{2}\right|$ with $-(x+\frac{3}{2})$ that is $-x-\frac{3}{2}$ and $|x|$ with $-x$

$$\left|x+\frac{3}{2}\right| \geq \frac{|x|}{2x}$$
$$-x-\frac{3}{2} > \frac{-x}{2x}$$
$$-x-\frac{3}{2} > \frac{-1}{2}$$
$$-x-\frac{3}{2}+\frac{1}{2} > 0$$

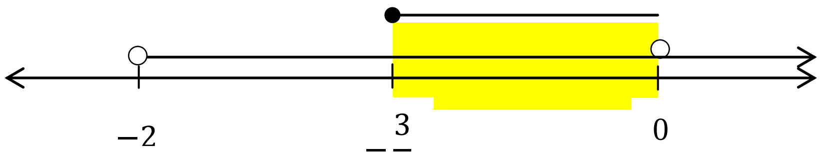
$$-x-\frac{2}{2}>0$$
$$-x-1>0$$
$$-1>x$$
$$x<-1$$



Hence, the solution set for C[ASE1] is $(-\infty,-\frac{3}{2})$

<u>CASE2</u> When $-\frac{3}{2}\le x<0$; we substitute $\left|x+\frac{3}{2}\right|$ with $x+\frac{3}{2}$ and $|x|$ with $-x$

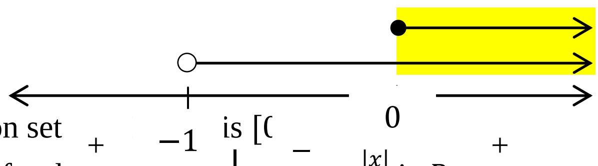
$$\left|x+\frac{3}{2}\right|\ge\frac{|x|}{2x}$$
$$x+\frac{3}{2}>\frac{-x}{2x}$$
$$x+\frac{3}{2}>\frac{-1}{2}$$
$$x>-\frac{3}{2}-\frac{1}{2}$$
$$x>-\frac{4}{2}$$
$$x>-2$$



Hence, the solution set for CASE2 is $[-\frac{3}{2},0)$

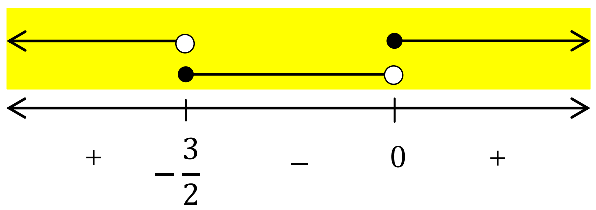
<u>CASE3</u> When $x\ge 0$; we substitute $\left|x+\frac{3}{2}\right|$ with $x+\frac{3}{2}$ and $|x|$ with x

$$\left|x+\frac{3}{2}\right|\ge\frac{|x|}{2x}$$
$$x+\frac{3}{2}>\frac{x}{2x}$$
$$x+\frac{3}{2}>\frac{1}{2}$$
$$x>\frac{1}{2}-\frac{3}{2}$$
$$x>-\frac{2}{2}$$
$$x>-1$$



Hence, the solution set [for CASE3] is $[0$

∴The solution set for the inequality $\left|x+\frac{3}{2}\right|>\frac{|x|}{2x}$ is R



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1) Prove that if ~p->r, ~r v s and ~s, then p

Note: We want to prove that p is true, so we assume that p is false. Therefore ~p is true. We want to prove for contradiction.

1)	~p	Assumption
2)	~p->r	Assumption
3)	r	1,2, Modus Ponens
4)	~r v s	Assumption
5)	s	3,4, Disjunctive syllogism
6)	~s	Assumption
7)	s Λ~s	5,6, Definition of conjunction

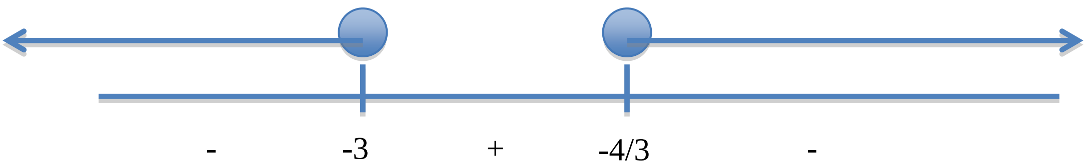
Since s Λ~s is a contradiction, ~p is false. Therefore p is true.

2) Find the solution set of the inequality $(-x^2+2x-11)(3x^2+13x+12) > 0$

Since $(-x^2+2x-11)$ cannot be factored because $b^2-4ac = -40$ which is less than 0. From $-x^2+2x-11$, we have a=-1 which is less than 0. Hence,
$-x^2+2x-11 < 0$ for all value of x. Therefore, $3x^2+13x+12$ must be less than 0 to satisfy the inequality.

$$3x^2+13x+12 < 0$$
$$(x+3)(3x+4) < 0$$

Therefore, the critical values are -3 and -4/3.



Hence, the solution set is (-∞,-3) u (-4/3, ∞)

Example : Prove that $if\ (\sim a \vee c) \leftrightarrow (b \rightarrow d), \sim d \vee e\ then\ a \vee (\sim e \rightarrow \sim b)$

Since $a \vee (\sim e \rightarrow \sim b) \equiv a \vee (b \rightarrow e)\ From\ law\ of\ Contraposition$

$\equiv \sim a \rightarrow (b \rightarrow e)\ From\ Law\ of\ Implication$

$\equiv \sim (b \rightarrow e) \rightarrow a\ From\ Law\ of\ Contraposition$

$So, assume \sim (b \rightarrow e)\ and\ prove\ for\ a$

1. $\sim (b \rightarrow e)$ $\quad Assumption$
2. $\sim (\sim b \vee e)$ $\quad 1, Law\ of\ Implication, Rule\ of\ Substitution$
3. $b \wedge \sim e$ $\quad 2, De\ Morgan's\ Law, Law\ of\ Double\ Negation$
4. $b\ and \sim e$ $\quad 3, Definition\ of\ Conjunction$
5. $\sim d \vee e$ $\quad Assumption$
6. $\sim d$ $\quad 4, 5, Disjunctive\ Syllogism$
7. $b \wedge \sim d$ $\quad 4, 6, Definition\ of\ Conjunction$
8. $\sim (\sim b \vee d)$ $\quad 7, De\ Morgan's\ Laws,$ $Law\ of\ Double\ Negation$
9. $\sim (b \rightarrow d)$ $\quad 8, Law\ of\ Implication, Rule\ of\ Substitution$
10. $(\sim a \vee c) \leftrightarrow (b \rightarrow d)$ $\quad Assumption$
11. $[(\sim a \vee c) \rightarrow (b \rightarrow d)] \wedge [(b \rightarrow d) \rightarrow (\sim a \vee c)]$ $\quad 10, Law\ of\ Equivalence$

12. $[(\sim a \vee c) \rightarrow (b \rightarrow d)]\ and\ [(b \rightarrow d)$

$\rightarrow (\sim a \vee c)]$ 11, $Definition\ of\ Conjunction$

13. $\sim(\sim a \vee c)$ 9, 12, $Rule\ of\ Modus\ Tollens$

14. $a \wedge \sim c$ 13, $Morgan's\ Laws,$

$Law\ of\ Double\ Negation$

15. $a\ and\ \sim c$ 14, $Definition\ of\ Conjunction$

Example : If the solution set of the inequality $|\mathrm{x}^2 + \mathrm{x} - 18|$

$< x + 18$

are (a, b) or (c, d)So, what is the value of $\mathrm{a} + \mathrm{b} + \mathrm{c} + \mathrm{d}$

Solution

$|\mathrm{x}^2 + \mathrm{x} - 18| < x + 18$

Case I	Case II
$-\mathrm{x} - 18 < \mathrm{x}^2 + \mathrm{x} - 18$	$\mathrm{x}^2 + \mathrm{x} - 18 < x + 18$
$0 < \mathrm{x}^2 + 2\mathrm{x}$	$\mathrm{x}^2 - 36 < 0$
$0 < \mathrm{x}(\mathrm{x} + 2)$	$(\mathrm{x} - 6)(\mathrm{x} + 6) < 0$
The solution set of case I	The solution set of case II
is $(-\infty, -2) \cup (0, \infty)$	is $(-6, 6)$

Therefore, the solution set of inequality $|\mathrm{x}^2 + \mathrm{x} - 18| < x + 18\ is$

$[(-\infty, -2) \cup (0, \infty)] \cap (-6, 6) = (-6, -2) \cup (0, 6)$

$(\mathrm{a}, \mathrm{b}) = (-6, -2)$

$(\mathrm{c}, \mathrm{d}) = (0, 6)$

So, $\mathrm{a} + \mathrm{b} + \mathrm{c} + \mathrm{d} = -6 - 2 + 0 + 6 = -2$

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Midterm Exam

Question1: Prove that $s \to q$, $r \vee s$, and q, then $r \to \sim s$
Assume $\sim(r \to \sim s)$ to prove for contradiction

1. $s \to q$ Assumption
2. r v s Assumption
3. q Assumption
4. $\sim(r \to \sim s)$ Assumption
5. $\sim(\sim r \vee \sim s)$ 4, Law of implication, Rule of substitution
6. $r \wedge s$ 5, De Morgan's law, Rule of substitution
7. r and s 6, Definition of conjunction
8. q 1, 7, Rule of Modus Ponens
9. $q \wedge \sim q$ 3, 8, Definition of conjunction

Since $q \wedge \sim q$ is contradiction, hence $\sim(r \to \sim s)$ is true

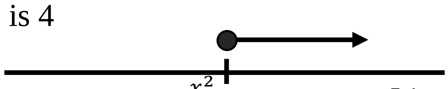
Question2: Find the solution set of the inequality $\frac{x^2}{-x^2+x-4} \leq -1$
Solution:

$$\frac{x^2}{-x^2+x-4}+1 \leq 0$$
$$\frac{x^2+(-x^2+x-4)}{-x^2+x-4} \leq 0$$
$$\frac{x-4}{-(x^2+x-4)} \leq 0$$

Consider$-x^2+x-4$, we have b^2 - 4ac = -15 < 0
Hence we cannot factor $-x^2+x-4$
Since a=-1<0, then $-x^2+x-4<0$ for all x
So we can find the values of x that satisfy $x-4$x

$$x-4 \geq 0$$
$$x \geq 4$$

Therefore the critical values is 4



Hence the solution set of the inequality $\frac{x^2}{-x^2\cdot}$ 4 ≤ -1 is $[4, \infty)$

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Proof/ Induction
If (x v ~y)→z and x, then z
Using law of contraposition, assume that z is false.

1) $\sim z$ Assumption
2) (x v ~y)→z Assumption
3) ~(x v ~y) 1, 2 Rule of Modus Tollens
4) ~ x ^ y 3, De Morgan's Law, Double Negation, Rule of Substitution
5) ~x and y 4, Definition of Conjunction
6) x Assumption
7) $\sim x$^x 5,6 Definition of Conjunction

Therefore, since $\sim x$ ^x is contradiction, hence ~ z is false and thus, z is true.

2. Solving the inequality

Find the solution set of the inequality $\frac{|2x-1|}{|x+2|}=1$

<u>Note</u>

$|2x-1|\begin{cases} 2x-1; x\geq \frac{1}{2} \\ -(2x-1); x<\frac{1}{2}\end{cases}$ $\quad\quad$ $|x+2|\begin{cases} x+2\,; x\geq -2 \\ -(x+2); x<-2\end{cases}$

←-----------○ $\quad\quad\quad\quad$ ●--------------→

●----------------------------○

Case1(-∞,-2) $\quad$ -2 $\quad$ Case2 $\left[-2,\frac{1}{2}\right)$ $\quad$ $\frac{1}{2}$ $\quad$ Case3 $\left[\frac{1}{2}\right.$

Case 1

When x< -2, we substitute $|2x-1|$ with –(2x-1) and $|x+2|$ with -(x+2).

So we have: $\frac{-(2x-1)}{-(x+2)} > 1$

$\frac{-2x+1}{-x-2} - 1 > 0$

$\frac{-2x+1+x+2}{-x-2} > 0$

$\frac{-x+3}{-x-2} > 0$

Thus, the critical values are -2, 3

←---------○ $\quad\quad\quad\quad$ ○------------→

+ $\quad$ -2 $\quad$ - $\quad$ 3 $\quad$ +

Hence, the solution set for Case 1 is (-∞,-2) ∩ ((-∞,-2) ∪ (3, ∞)) = (-∞,-2)

Case 2

When -2≤ x < $\frac{1}{2}$, we substitute $|2x-1|$ with –(2x-1) and $|x+2|$ with x+2.

So we have: $\frac{-(2x-1)}{X+2} > 1$

$\frac{-2x+1}{X+2} - 1 > 0$

$\frac{-2x+1-x-2}{X+2} > 0$

$\frac{-3x-1}{X+2} > 0$

Thus, the critical values are -2, -1/3

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-2 $\quad\quad$ -1/3

Hence, the solution set for case 2 is (-2,-1/3) ∩ [-2, ½) = [-2, -1/3)

Case 3

When x>1/2, we substitute $|2x-1|$ with 2x-1 and $|x+2|$ with x+2.

So we have: $\frac{2x-1}{X+2} > 1$

$\frac{2x-1}{X+2} - 1 > 0$

$\frac{2x-1-x-2}{X+2} > 0$

$\frac{x-3}{x+2} > 0$

Thus, the critical values are -2,3

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←-----○                        ○---------→
——————+————————————————+———————
   +  -2        -       3        +
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Hence, the solution set for case 3 is $[1/2, \infty) \cap ((3,\infty) \cup (-\infty,-2) = (3,\infty)$

Therefore, by case 1,2 and 3, the solution set of $\frac{|2x-1|}{|x+2|} = 1$ is $(-\infty,-2) \cup [-2, -1/3) \cup (3,\infty)$

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1. Prove that if (~x^y) -> ~z, ~(xvy) -> w, and ~x, then ~zvw.

In this example, we can either assume ~w and prove ~z or assume ~(~z) and prove w. In this case, we will assume ~(~z) and prove w.

1. ~(~z)	Assumption
2. (~x^y) -> ~z	Given
3. ~(~x^y)	1,2, Rule of Modus Tollens
4. ~(~x^y) <-> ~(~x^y)	De Morgan's Law
5. ~(~x^y)	3,4, Rule of Substitution
6. ~(~x) v ~y <-> xv~y	Law of Double Negation
7. xv~y	5,6, Rule of Substitution
8. ~x	Given
9. ~y	7,8, Disjunctive Syllogism
10. ~x^~y	8,9, Definition of Conjunction
11. (~x^~y) <-> ~(xvy)	De Morgan's Law
12. ~(xvy)	10,11, Rule of Substitution

13. ~(xvy) -> w Given
14. w 12,13, Rule of Modus Ponens

Hence, ~(~z) -> w
Since, ~(~z)->w <-> ~zvw, so we can conclude that ~zvw is true.

Inequality

|x-3|-|x+4|< 4

Note that |x-3| = x-3 ; x>=3
= -x+3 ; x< 3

|x+4| = x+4 ; x>= -4
= -x-4 ; x < -4

```
                         [------- >
      [-----------------)
< ---)
____-4______________3_____
```

So, there are 3 cases

Case I
If X<-4, we substitute |x-3|with –x+3, and |x+4| with –x-4.

-x+3-(-x-4)<4
-x+3+x+4-4<0
3<0 which was false. So solution of this case is empty set,

Case II
If -4<=x<3, we substitute |x-3| with –x+3, and |x+4| with x+4

-x+3-(x+4)<4
-x+3-x+-4-4<0
-5<2x
-5/2< x

(-∞,-5/2) ∩ [-4,3)
[-4,-5/2)

Case III
If X >=3 we substitute │x-3│ with x-3, and │x+4│ with x+4

X+3-(x-4)<4
X+3-x+4-4<0
3<0 which is false, so this solution was empty set

Hence, the solution of this inequality is ∅∪[-4,-5/2)∪∅ = [-4,-5/2)

1. Prove that if $\sim r \rightarrow \sim p, \sim q, r \rightarrow s, pVq\ then\ s$

pVq	assumption
$\sim q$	assumption
p	1, 2 disjunctive syllogism
$\sim r \rightarrow \sim p$	assumption
r	3, 4 Rule of modus tollens
$r \rightarrow s$	assumption
s	5,6 Rule of modus ponen

2. Solve for this function lx^2+3x-6l = l2x+6l
:. $(x^2+3x-6)^2 = (2x+6)^2$
$(x^2+3x-6)^2-(2x+6)^2 = 0$
$(x^2+3x-6-2x-6)(x^2+3x-6+2x+6) = 0$
$(x^2+X-12)(x^2+5x) = 0$
$(x+4)(x-3)(x)(x+5) = 0$
:. X = -4,3,0,-5
The critical point for this one is {-4,3,0,-5}

1 Using the given premises M □ -S and Q → ~ (M □ T), then S→ -Q
Solution

1. S assumption
2. M □ - S assumption
3. M 2, Disjunctive Syllogism
4. M→ (M□ T) 3, law of addition
5. (m□ T) 3,4, modus ponens
6. Q → ~(M□ T) assumption
7. –Q 5,6 modus tollens

2 solve the inequality X□2 – 2X> 0
X(X-2) > 0
So the critical value is
X=0 or X= 2

Therefore 0<x< 2

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Logic problem

$\sim$r→ $\sim s, \sim u, f \rightarrow t$ *and* qVu prove t

The solution

1. qVu *Assumption*
2. $\sim u$ *Asumption*
3. *q* *1, 2 Disjunctive syllogism*
4. $s \leftrightarrow \sim(\sim s)$ *3, Law of double negation*
5. $\sim(\sim r)$ *3, 4 Law of substitute*
6. $\sim r \rightarrow \sim s$ *Assumption*
7. $\sim(\sim r)$ *5, 6 Rule of modus Tollens*
8. $\sim(\sim r) \leftrightarrow f$ *Law of Double Negation*
9. *r* *7,8Law of substitute*
10. $r \rightarrow t$ *Assumption*
11. *t* *9, 10 Rule of Modus ponens*

Hence the final solution is t which is true.

The inequality problem

The inequality since both side are in absolute, we will square root in both side to find the answer of the inequality.

$$[x+3] \leq [x+2]$$
$$(x+3)^2] \leq (x+2)^2$$
$$(x^2+6x+9) \leq (x^2+4x+4)$$
$$6x+9 \leq 4x+4$$
$$2x \leq -5$$
$$x \leq \frac{-5}{2}$$

The solution set of $[x+3] \leq [x+2]$ *we have* $(-\infty, \frac{-5}{2}]$

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Logic

Prove that if (~p v q) → w, ~w v s, and w→(~s v p) prove p

Prove: assume $\sim p$ by contradiction

1. $\sim p$ Assumption
2. $\sim p \rightarrow (\sim p$ v q) Law of Addition
3. $\sim p$ v q 1,2 Rule of Modus Ponens
4. ($\sim p$ v q) → w Assumption
5. w 3,4 Rule of Modus Ponens
6. $\sim w$ v s Assumption
7. s 5,6 Rule of Disjunctive Syllogism
8. w → $(\sim s \ v \sim p)$ Assumption

9. $\sim w$ v ($\sim s\ v\ p$) 8, Law of implication
10. ($\sim w$ v $\sim s$) v p 9, Associative Laws
11. ~(w ∧ s) v p 10, De Morgan's Laws
12. (w ∧ s) → p 11, Law of implication
13. w ∧ s 3,7,Definition of conjunction
14. p 12, 13 Rule of Modus Ponens
15. p ∧ ~p 1,14 Definition of Conjunction
Since, p ∧ ~p is contradiction. Hence, p is tautology.

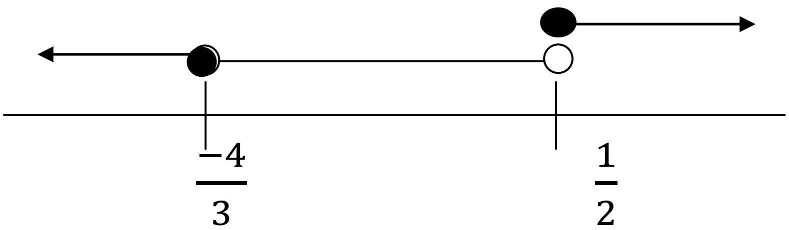
Inequality

Find the solution set of the inequality $\frac{|2x-1| - |3x+4|}{4x^2+x}$

Solution

<u>Note</u> 2x-1 = 2x-1 ; 2x-1 ≥ 0
= - (2x-1) ; 2x-1 < 0

And 3x+4 = 3x+4 ; 3x+4 ≥ 0
- (3x-4) = 3x+4 ; 3x+4 < 0



<u>CASE I</u> When x < - $\frac{4}{3}$ we substitute l2x-1l with -2x+1 and l3x+4l with -3x-4, so we have

$$\frac{|2x-1| - |3x+4|}{4x^2+x} \geq 0$$

$$\frac{-2x-1-(-3x+4)}{4x^2+x} \geq 0$$

$$\frac{x+5}{x(4x+1)} \geq 0 \quad ;\ x \neq 0, -\frac{1}{4}$$

The critical values are 0, - $\frac{1}{4}$, -5

∴ The solution set for case I is [-5, - $\frac{1}{4}$) ∪ $(0, \infty) \cap x < -\frac{4}{3}$ = [-5 , - $\frac{4}{3}$)

Case II

When $-\frac{4}{3} \le x \le \frac{1}{2}$, we substitute l2x-1l with -2x+1 and l3x+4l with 3x+4, so we have $\frac{-2x+1-(3x+4)}{4x^2+x} \ge 0$

$$\frac{-5x-3}{x(4x+1)} \ge 0$$

$$\frac{5x+3}{x(4x+1)} \le 0 \qquad ; x \ne 0, -\frac{1}{4}$$

The critical values are 0 , $-\frac{1}{4}$, $-\frac{3}{5}$

∴ The solution set for case II is $[-\infty\ , -\frac{3}{5}] \cup (\frac{3}{5}, 0) \cap -\frac{4}{3} \le x < \frac{1}{2}$

$= \left(-\frac{1}{4}, \frac{1}{2}\right)$

CASE III

When $x \ge \frac{1}{2}$, we substitute l2x-1l with 2x-1 and l3x+4l with 3x+4 , so we have $\frac{2x-1-(3x+4)}{4x^2+x} \ge 0$

$$\frac{-x-5}{x(4x+1)} \ge 0$$

$$\frac{x+5}{x(4x+1)} \le 0 \qquad ; x \ne 0, -\frac{1}{4}$$

The critical values are 0, $-\frac{1}{4}$, -5

∴ The solution set for case III is $(-\infty.5) \cup (\frac{1}{4}, 0) \cap x \ge \frac{1}{2} = \emptyset$

Hence, the solution set for the inequality $\frac{|2x-1|-|3x+4|}{4x^2+x}$ is

$\left[-5, -\frac{4}{3}\right) \cup \left(-\frac{1}{4}, \frac{1}{2}\right) \cup \emptyset = [-5, -\frac{4}{3}) \cup (-\frac{1}{4}, \frac{1}{2})$