Game Theory

Game Theory Overview

What is Game Theory?

Game Theory is the study of mathematical models of strategic interaction among rational decision-makers.

The key pioneers of game theory were mathematicians John von Neumann and **John Nash**.

What is a Game?

Games are defined as strategic situations in which there are multiple participants. Furthermore, the outcome of the decision any individual makes is dependent on the decisions made by all of the other participants.

Game Theory Overview

Is Sudoku a "game?"

No, not the way we defined "game." Sudoku is not a "game" because what you do when solving the game is <u>independent</u> of what anyone else does.

Is Chess a "game?"

Yes! Imagine that you are playing a game of chess with a friend. Whether you win or not will be dependent on the moves you make and the moves your friend makes.

Types of Games

Types of Games

- One-shot: a game that is played once.
- Repeated: a game that is played more than once.

- Simultaneous-move: players make decisions at the same time.
- Sequential-move: players takes turns to make decisions.

- 2-player game: a game played by two players
- n-player game: a game played by n players

Types of Games

Types of Games

- Perfect vs Imperfect Information
  “Perfect” means no hidden information.

- Complete vs Incomplete Information
  “Complete” means same common knowledge.

In our course, we will focus on 2-player game with perfect and complete information.

An Example

This is a **one-shot**, **simultaneous-move** game, called “Battle of the Sexes”.

		WOMAN	
		Boxing	Shopping
MAN	Boxing	2 , 1	0 , 0
	Shopping	0 , 0	1 , 2

Game Elements

Players: Agents participating in the game (Man, Woman)

Strategies: Actions that each player may take under any possible circumstance (Boxing, Shopping)

Outcomes: The various possible results of the game (four, each represented by one cell of matrix)

Payoffs: The benefit that each player gets from each possible outcome of the game (the numbers entered in each cell of the matrix)

For example, if both players choose Boxing, (2, 1) will be the outcome with 2 being the payoff of Man and 1 being the payoff of Woman.

Capacity Expansion Game

What is the likely outcome of this game?

We study Game Theory to answer such question.

Toyota

Honda

	Build a new plant	Do not Build
Build a new plant	16,16	20,15
Do not Build	15,20	18,18

Nash Equilibrium

Definition: A **Nash Equilibrium (NE)** occurs when each player chooses a strategy that gives him/her the highest payoff, given the strategy chosen by the other player(s).

In a simple term, a NE is a situation where

- **I am doing your best given what you are doing, AND**
- **you are doing your best given what I am doing.**

Definition: A **Best Response (BR)** of a player is the player's optimal strategy (giving the highest payoff) given any strategy chosen by the other player.

e.g. $BR_{Player\ 1}$(when Player 2 chooses Rock) = Paper

$BR_{Player\ 2}$(when Player 1 choose Scissors) = Rock

Nash Equilibrium

NE of Toyota vs Honda Game: **Each Firm Builds a New Plant**.

Consider Honda's perspective:

- If Toyota builds a new plant, Honda's best response is to build a new plant.
- If Toyota does not build a new plant, Honda's best response is to build a new plant.

Consider Toyoya's perspective:

- If Honda builds a new plant, Toyota's best response is to build a new plant.
- If Honda does not build a new plant, Toyota's best response is to build a plant.

Nash Equilibrium

NE of Toyota vs Honda Game: **Each Firm Builds a New Plant**.

When both build new plants, their BEST RESPONSES coincide.

That is, BR_{Toyota}(Honda chooses Build) = Build

BR_{Honda}(Toyota chooses Build) = Build

In other words, Honda is doing its best given what Toyota is doing, and Toyota is doing its best given what Honda is doing.

Why is the Nash Equilibrium plausible?

It is "**self enforcing**", and no player would want to deviate from the NE. If a player deviates, he/she will get a lower payoff.

Prisoner's Dilemma

Definition: A game situation in which there is a conflict between the collective interest of all of the players and the self-interest of individual players is called a **Prisoner's Dilemma**.

		PRISONER 2	
		Confess	Lie
PRISONER 1	Confess	-8 , -8	0 , -10
	Lie	-10 , 0	-1 , -1

Dominant and Dominated Strategies

Definition: A **dominant strategy** is a strategy that is better than any other strategy that a player might choose, no matter what strategy the other player follows.

Note: When a player has a dominant strategy, that strategy will be the player's Nash Equilibrium strategy.

Definition: A **dominated strategy** is a strategy such that the player has another strategy that gives a higher payoff, no matter what the other player does.

Note: Dominated strategies can be eliminated from consideration in order to solve the game more easily.

Dominant Strategy

Definition: A **Dominant Strategy Equilibrium** occurs when each player uses a dominant strategy.

		Toyota	
		Build a new plant	Do not Build
Honda	Build a new plant	16,16	20,15
	Do not Build	15,20	18,18

Dominated Strategy

Toyota

Honda

	Build Large	Build Small	Do Not Build
Build Large	0, 0	12, 8	18, 9
Build Small	8, 12	16, 16	20, 15
Do Not Build	9, 18	15, 20	18, 18

"Build Large" is dominated for each player, so this strategy will not be chosen by both players. We can eliminate it from our consideration.

Finding the NE

LEARNING-BY-DOING EXERCISE 14.1

Finding the Nash Equilibrium: Coke versus Pepsi

Table 14.6 shows Coke's and Pepsi's profits for various combinations of prices that each firm might charge.

Problem Find the Nash equilibrium in this game.

Price Competition between Coke and Pepsi*

		Coke			
		$10.50	**$11.50**	**$12.50**	**$13.50**
Pepsi	**$6.25**	66, 190	68, 199	70, 198	73, 191
	$7.25	79, 201	82, 211	85, 214	89, 208
	$8.25	82, 212	86, 224	90, 229	95, 225
	$9.25	75, 223	80, 237	85, 244	91, 245

(a)

	a	b	c
A	2,12	1,10	1,11
B	0,12	0,10	0,11
C	0,12	0,10	0,13

(b)

	a	b	c
A	1,1	−2, 0	4, −1
B	0,3	3,1	5,4
C	1,5	4,2	6,2

(c)

	N_2	C_2	J_2
N_1	73,25	57,42	66,32
C_1	80,26	35,12	32,54
J_1	28,27	63,31	54,29

(d)

	a	b	c	d	e
A	63, −1	28, −1	−2, 0	−2, 45	−3, 19
B	32, 1	2, 2	2, 5	33, 0	2, 3
C	54, 1	95, −1	0, 2	4, −1	0, 4
D	1, −33	−3, 43	−1, 39	1, −12	−1, 17
E	−22, 0	1, −13	−1, 88	−2, −57	−3, 72

Ans

(A,a)

(A,a), (C,a)

(J,C)

(B,c)

Nash Equilibrium Limitations

The Nash Equilibrium need not be unique.

The **Game of Chicken**, also known as the **Hawk–Dove Game**, is a model of conflict for two players in game theory.

The principle of the game is that while it is to both players' benefit if one player yields, the other player's optimal choice depends on what their opponent is doing: if the player opponent yields, they should not, but if the opponent fails to yield, the player should.

The game of chicken has TWO NEs, which show that one player will win and one player will lose, but we cannot say who will win.

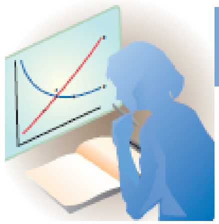
Nash Equilibrium Limitations

The Nash Equilibrium need not be unique.

The name "chicken" has its origins in a game in which two drivers drive towards each other on a collision course: one must swerve, or both may die in the crash, but if one driver swerves and the other does not, the one who swerved will be called a "chicken", meaning a coward.

		Driver A	
		Swerve	Drive Straight
Driver B	Swerve	2, 2	1, 3
	Drive Straight	3, 1	0, 0

Finding the NEs



LEARNING-BY-DOING EXERCISE 14.2

Finding All of the Nash Equilibria in a Game

Problem What are the Nash equilibria in the game in Table 14.10?

TABLE 14.10 What Are the Nash Equilibria?

		Player 2		
		Strategy *D*	**Strategy *E***	**Strategy *F***
	Strategy *A*	4, 2	13, 6	1, 3
Player 1	**Strategy *B***	3, 10	0, 0	15, 2
	Strategy *C*	12, 14	4, 11	5, 4

Nash Equilibrium Limitations

Nash Equilibrium need not exist.

Matching pennies is a game, played by 2 players (A and B):

- Each player has a penny and must secretly turn the penny to heads or tails.
- The players then reveal their choices simultaneously.
- If the pennies match (both heads or both tails), then one player (e.g. Player B) keeps both pennies.
- If the pennies do not match (one head and one tail), then the other player (e.g. Player A) keeps both pennies.

Matching Pennies is a **zero-sum** game, which does not have NE. Another example of a zero-sum game is Rock-Paper-Scissors.

Nash Equilibrium Limitations

Nash Equilibrium need not exist.

		Player A	
		Heads	Tails
Player B	Heads	(1, -1)	(-1, 1)
	Tails	(-1, 1)	(1, -1)

Pure Strategies and Mixed Strategies

Pure Strategy – A specific choice of a strategy from the player's possible strategies in a game.

Note: The games with the NE(s) so far has a pure strategy NE, i.e. players select their strategies from the choices they have.

Mixed Strategy – An assignment of a probability to each pure strategy. This allows for a player to randomly select a pure strategy.

Note: A game without a pure strategy NE, e.g. matching pennies, will always have <u>at least one NE that involves a mixed strategy</u>.

Finding the Mixed Strategy NE

This involves at least one player randomizing between the pure strategies, i.e. he/she will assign probabilities to the choices.

In matching pennies, each player will play Head with 50% probability and Tail with 50% probability.

Example: Matching Pennies

Repeated Prisoner's Dilemma

So far, we considered only "one-shot" games, where players cannot cooperate, cooperation can arise in "repeated" games.

Cooperation can result from <u>self-interested</u> behaviour on the part of each player under certain circumstances:

"Grim Trigger" Strategy – One-time cheating by one player triggers the grim prospect of a permanent breakdown in cooperation for the remainder of the game.

"Tit-for-Tat" Strategy – A strategy in which you do to your opponent in this period what your opponent did to you in the last period.

Repeated Prisoner's Dilemma

		Player 1	
		Cheat	**Cooperate**
Player 2	**Cheat**	5, 5	14, 1
	Cooperate	1, 14	10, 10

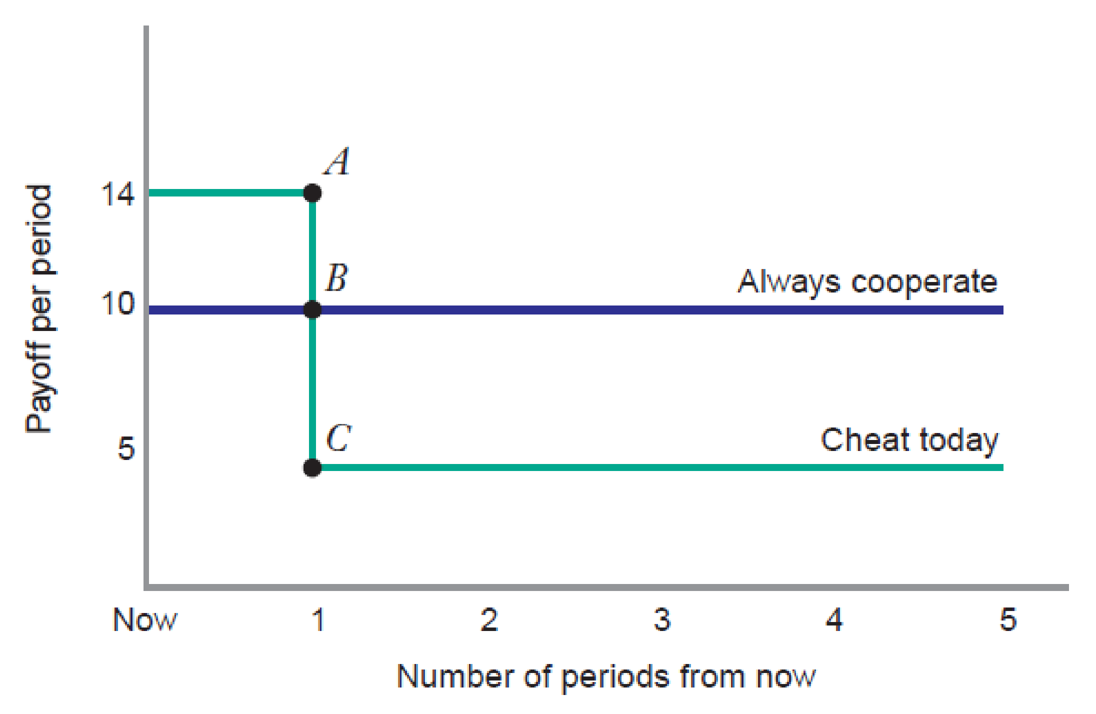


FIGURE 14.1 **Payoffs in the Repeated Prisoners' Dilemma under the "Grim Trigger" Strategy**
If Player 1 cheats today, he receives a stream of payoffs given by the light line. If he cooperates today and in the future he can ensure himself a stream of payoffs given by the dark line. The distance of line segment *AB* represents the one-time gain to Player 1 from cheating. The distance of line segment *BC* represents the reduction in each of Player 1's subsequent payoffs because Player 2 retaliates against Player 1's cheating.

Repeated Prisoner's Dilemma

Likelihood of cooperation increases under these conditions:

1. The players are patient.
2. Interactions between the players are frequent.
3. Cheating is easy to detect.
4. The one-time gain from cheating is relatively small.

Likelihood of cooperation diminishes under these conditions:

1. The players are impatient.
2. Interactions between the players are infrequent.
3. Cheating is hard to detect.
4. The one-time gain from cheating is large in comparison to the eventual cost of cheating.

Sequential-Move Games

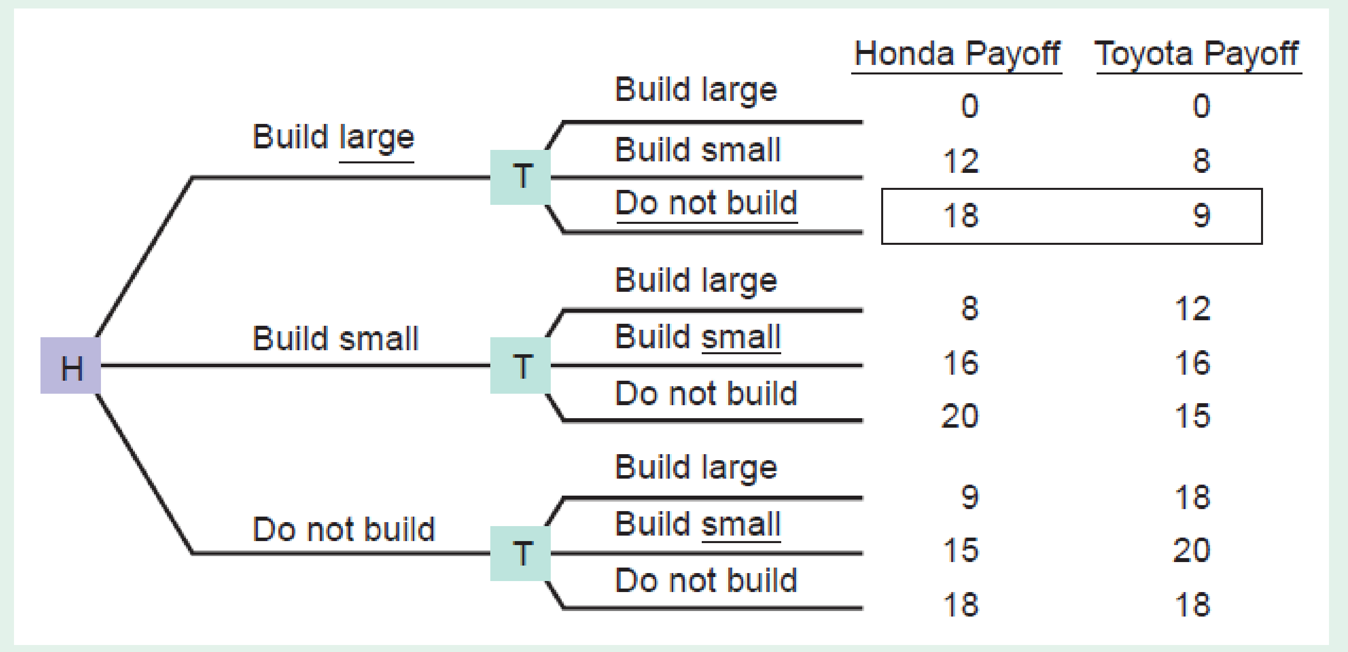
Sequential-Move Games are games in which one player (the first mover) takes an action before another player (the second mover). The second mover observes the action taken by the first mover before deciding what action it should take.

A **game tree** shows the different strategies that each player can follow in the game and the order in which those strategies get chosen.

Backward Induction is a procedure for solving a sequential-move game by starting at the end of the game tree and finding the optimal decision for the player at each decision point.

Sequential-Move Games

Here, Honda moves first. After Honda chooses its strategy, Toyota then decides its strategy.



Sequential-Move Games

With Backward-Induction, the NE of the sequential-move game is Build Large for Honda and Do Not Build for Toyota.

Recall (from Page 12) the NE of the simultaneous-move version of this game is Build Small for both companies.

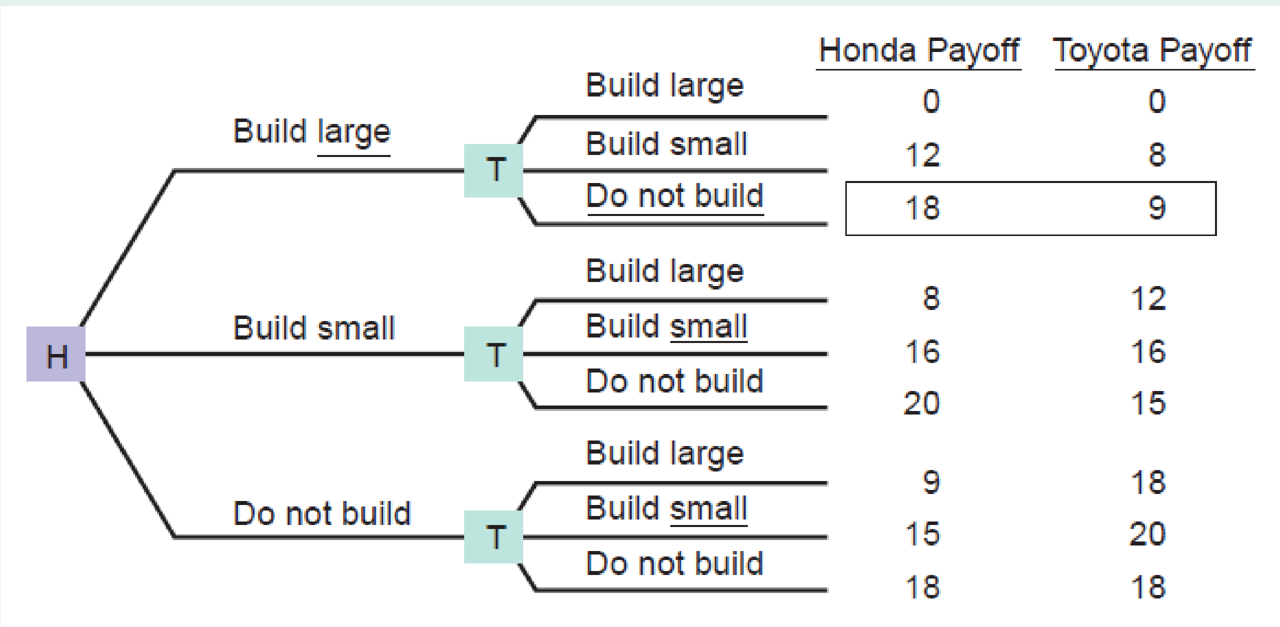


FIGURE 14.2 Game Tree for the Sequential-Move Capacity Expansion Game between Toyota and Honda
Honda moves first and can choose among three strategies: Toyota moves next (having observed Honda's move), also choosing among the same three strategies. Assuming that Toyota will always make its best (payoff-maximizing) response, Honda can maximize its own payoff by choosing "build large," as Toyota's best response will be "do not build."

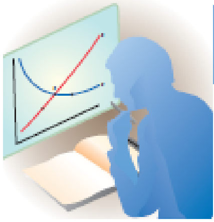
Sequential-Move Games

Sometimes, in sequential-move games, the first mover can adopt “**strategic moves**” which are actions taken in an early stage of a game that can alter the second mover’s behavior.

In the above example, the strategic move is for Honda to make commitment to “Build Large” when Toyota were to choose “Do Not Build”.

In short, strategic moves that seemingly limit options (i.e. by having the commitment) can actually make a player better off.

For a strategic move to work, it must be visible, understandable, and hard to reverse.



LEARNING-BY-DOING EXERCISE 14.3

An Entry Game

Avinash Dixit and Barry Nalebuff, authors of a delightful book on game theory, *Thinking Strategically*, have written, “It takes a clever carpenter to turn a tree into a table; a clever strategist knows how to turn a table into a tree.”[18] In this exercise, we illustrate their point in the context of a simple entry game.

Suppose you own a firm that is considering entry into the digital camera business, where you will compete head to head with Kodak (which, let’s say, currently has a monopoly). Kodak can react in one of two ways: It can start a price war or it can be accommodating. You can enter this business on a large scale or a small scale. Table 14.14 shows the payoffs you and Kodak are likely to get under the various scenarios that could unfold.

Problem Should you enter this business on a large scale or a small scale?

		Kodak	
		Accommodate	**Price War**
You	**Small**	4, 20	1, 16
	Large	8, 10	2, 12