1.2) $var(a_t) = E[(a_t - E(a_t))^2]$

$$= E[a_t^2]$$

$$= E[E(a_t^2|F_{t-1})]$$

$$= E[\sigma_t^2]$$

$$= E[\alpha_0 + \alpha_1 a_{t-1}^2]$$

$$= \alpha_0 + \alpha_1 E[a_{t-1}^2]$$

$$= \frac{\alpha_0}{1-\alpha_1}$$

$$= \frac{0.01}{1-0.1} = 0.0111 \quad \#$$

$$var(a_t|F_{t-1}) = \sigma_t^2 \quad \#$$

$$= 0.01 + 0.1 a_{t-1}^2$$

1.3) volatility forecast $\boxed{t=h}$

$$\sigma^2_h = \alpha_0 + \alpha_1 a^2_{h-1}$$

1 step : $\sigma^2_{h+1} = \alpha_0 + \alpha_1 a^2_h$

$$E[\sigma^2_{h+1} | F_h] = \alpha_0 + \alpha_1 E[\cancel{a^2_h | F_h}]^{0}$$

$$\sigma^2_h(1) = \alpha_0$$

$$= 0.02 \ \#$$

$r_h = 0.02 + a_h$

1 step ahead :

$r_{h+1} = 0.02 + a_{h+1}$

$$E[r_{h+1} | F_h] = 0.02 + E[\cancel{a_{h+1} | F_h}]^{0}$$

$$= 0.02 + 0$$

$$= 0.02 \ \#$$

1.4)

$$r_h(\infty) = E(r_t) = 0.002 \ \#$$

$$\sigma^2_h(\infty) = \lim_{l \to \infty} \sigma^2_h(\infty)$$

$$= \text{unconditional variance of } a_t = \frac{\alpha_0}{1-\alpha_1} = 0.0111 \ \#$$