

CHAPTER 10

Applications of Integration in Economics

Topics

- Total revenue function from marginal revenue function
- Total cost function from marginal cost function
- Profit function from MR-MC
- Utility function from marginal utility function
- Consumption and saving functions from marginal propensity functions
- Consumer surplus, producer surplus and total surplus
- First degree price discrimination

Indefinite Integral

$$\boxed{\int f(x)dx = F(x) + C} \quad \text{with} \quad \frac{dF(x)}{dx} = f(x)$$

Basic Rule of Integration

The power rule

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c \quad (n \neq -1)$$

The exponential rule

$$\int e^x dx = e^x + c$$

The logarithmic rule

$$\int \frac{1}{x} dx = \ln x + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c \quad [f(x) > 0]$$

Rule of operation

Integral of a sum

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

Integral of a multiple

$$\int kf(x) dx = k \int f(x) dx$$

Rule Involving Substitution

the substitution rule

$$\int g(x) dx = \int f(u) \frac{du}{dx} dx = \int f(u) du = F(u) + c \quad ; g(x) = f(u) \frac{du}{dx}$$

Integration by parts

$$\int v du = uv - \int u dv$$

Definite Integrals

From Indefinite Integral:

$$\int f(x)dx = F(x) + C$$

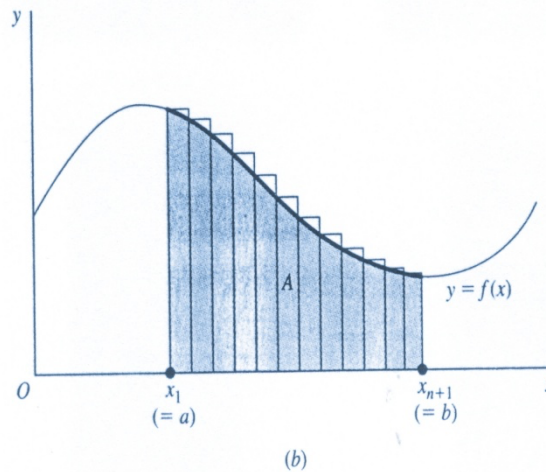
If we choose a and b , where $a < b$, in Domain of x and find the difference:

$$[F(b) + c] - [F(a) + c] = F(b) - F(a)$$

We call this “definite integral of $f(x)$ from a to b ”, with a be lower limit of integration and b the upper limit of integration.

$$\int_a^b f(x)dx = F(x) \Big|_a^b = F(b) - F(a)$$

A definite Integral can be thought of as an area under the curve between a and b .



Some Properties of Definite Integral

Property I: The interchange of the limits of integration changes the sign of the definite integral:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

Property II: A definite integral has a value of zero when the two limits of integration are identical

$$\int_a^a f(x)dx = F(a) - F(a) = 0$$

Applications of Integration in Economics



Recovering Total function from Marginal function

Integrate MC to get TC

Let marginal cost function be $MC(Q)$. We can find the underlying total cost as:

$$TC(Q) = \int MC(Q)dQ$$

Example: $MC(Q) = 3 + 2Q$

$TC(Q) =$

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Note: Q: What is the constant in a total cost function?

A: It is fixed costs, so using a marginal cost function we are unable to recover the level of fixed costs that a firm faces. To figure out the total fixed cost, we need initial condition or boundary condition.

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Integrate MR to get TR

$$TR(Q) = \int MR(Q)dQ$$

Integrate MP_L to get Production function

$$TP_L(L) = \int MP_L(L)dL$$

Try: Find TP_L when $MP_L = \alpha$

Integrate MPS to get S

Let Mr. Yahoo’s marginal propensity to save be $S'(Y) = 0.3 - 0.1Y^{-\frac{1}{2}}$. If Mr. Yahoo has US\$ 81, he will not have any saving. What is Mr. Yahoo’s saving function?

$$S(Y) = \int MPS(Y) dY$$

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Integrate MPC to get C

$$C(Y) = \int MPC(Y) dY$$

Try: Find $C(Y)$ when $MPC(Y) = b$

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Integrate Marginal Profit function to get Total Profit function

$$\text{Marginal Profit}(MP(Q)) = \frac{d\pi(Q)}{dQ} = \frac{d(TR(Q) - TC(Q))}{dQ} = MR(Q) - MC(Q)$$

$$\pi(Q) = \int MP(Q) dQ$$

If $MR = 50 - 2Q$ and $MC = 10 + Q$ and a firm produces at $\hat{q} = 10$, how much is the profit of this firm?

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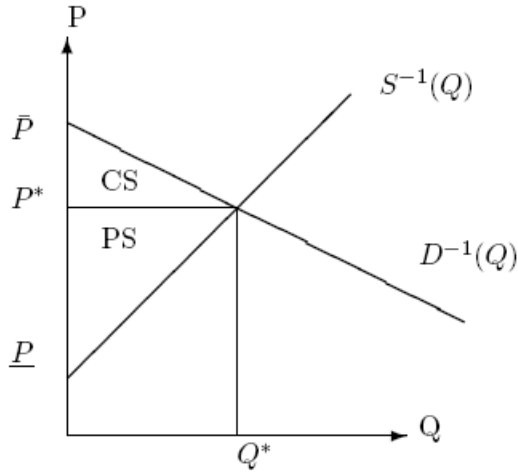
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Consumer and Producer Surplus



CS =

$$CS = \int_{P^*}^{\bar{P}} D(P) dP$$

PS =

$$PS = \int_{\underline{P}}^{P^*} S(P) dP$$

TSW(Total Social Welfare) =

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Example:

Let demand function be $Q^D = D(P) = \frac{25}{2} - \frac{1}{2}P \quad \therefore P^D = D^{-1}(Q) = 25 - 2Q$

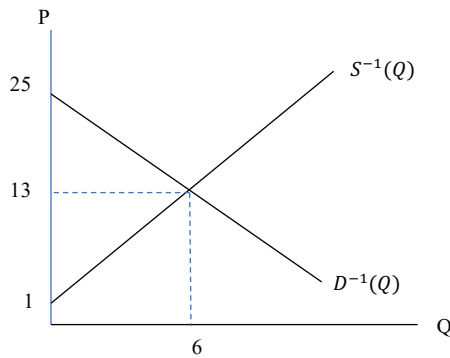
Let supply function be $Q^S = S(P) = -\frac{1}{2} + \frac{1}{2}P \quad \therefore P^S = S^{-1}(Q) = 1 + 2Q$

Find consumer surplus and producer surplus

Competitive market

At equilibrium,

$$\begin{aligned} Q^D &= Q^S \\ D(P) &= S(P) \\ \frac{25}{2} - \frac{1}{2}P &= -\frac{1}{2} + \frac{1}{2}P \\ P^* &= 13, Q^* = 6 \end{aligned}$$



$$CS = \int_0^6 (25 - 2Q)dQ - 13(6)$$

$$PS = 13(6) - \int_0^6 (1 + 2Q)dQ$$

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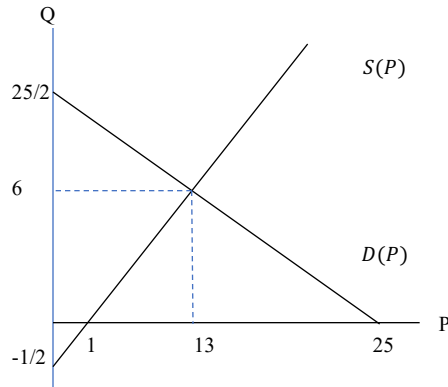
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$$CS = \int_{13}^{25} \left(\frac{25}{2} - \frac{1}{2}P \right) dP$$

$$PS = \int_1^{13} \left(-\frac{1}{2} + \frac{1}{2}P \right) dP$$

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$$TSW = CS + PS = \int_0^6 [(25 - 2Q) - (1 + 2Q)] dQ$$

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🔔 Monopoly market: Find dead weight loss

