Exercise 1 (Part 2)

1. Determine whether the statement forms are logically equivalent. In each case, construct a truth table to justify your answer.

   (a) $(p \to q) \to (q \to p)$ and $(p \vee q) \to (p \wedge q)$

   (b) $\sim p \to (q \to r)$ and $q \to (p \vee r)$

   (c) $(p \vee q) \wedge \sim (p \wedge q)$ and $p \leftrightarrow q$

   (d) $(p \leftrightarrow q) \leftrightarrow r$ and $p \leftrightarrow (q \leftrightarrow r)$

2. Determine whether or not the statement $p \wedge q \to (p \to q)$ is a tautology or a contradiction.

3. Let p, q and r be statements such that $(p \leftrightarrow q) \wedge (q \leftrightarrow r)$ is **true**. Determine the truth value of $(p \vee q \vee r) \wedge (\sim p \vee \sim q \vee \sim r)$.

4. Consider the following statement.

   If I eat spicy food and I drink beer, then I feel sick or I have a bad dream.

   (a) Write the **negation** of the above statement.

   (b) Write the **contrapositive**, **inverse**, and **converse** of the above statement.

5. Use truth tables to determine whether the argument forms are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer.

   (a)
   $$\begin{array}{ll} & p \to \sim q \\ & \sim q \to p \\ \therefore & p \vee \sim q \end{array}$$

   (b)
   $$\begin{array}{ll} & r \vee \sim q \\ & p \to q \\ & \sim r \\ \therefore & \sim p \end{array}$$

   (c)
   $$\begin{array}{ll} & \text{If I answer the quiz correctly, then I understand the class material.} \\ & \text{I understand the class material.} \\ \therefore & \text{I answer the quiz correctly.} \end{array}$$

6. Consider the following premises.

   (i) It is not sunny this afternoon and it is colder than yesterday.

   (ii) We will go swimming only if it is sunny.

   (iii) If we do not go swimming, then we will take a canoe trip.

   (iv) If we take a canoe trip, then we will be home by sunset.

   From the above premises (i)-(iv), does the conclusion that *we go home by sunset* make a valid argument? Explain your answer by using rules of inferences.