Answer Keys Chp2

1. A) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 0 \\ 7 & 13 & 6 \end{bmatrix}$

   b) $\begin{bmatrix} -9 & 1 & 1 \\ 2 & -2 & 6 \\ -7 & -11 & -2 \end{bmatrix}$

   c) $\begin{bmatrix} 9 & -1 & -1 \\ -2 & 2 & -6 \\ 7 & 11 & 2 \end{bmatrix}$

   d) $\begin{bmatrix} -13 & 16 & 4 \\ 31 & 33 & 15 \\ 14 & 25 & 5 \end{bmatrix}$

   e) $\begin{bmatrix} -13 & 31 & 14 \\ 16 & 33 & 25 \\ 4 & 15 & 5 \end{bmatrix}$

   f) $\begin{bmatrix} 1 & 2 & 7 \\ -1 & 0 & 13 \\ 1 & 0 & 6 \end{bmatrix}$

   g) $\begin{bmatrix} -2 & 6 \\ -13 & -10 \\ 44 & 7 \end{bmatrix}$

   h) $\begin{bmatrix} 4 & 3 & 16 \\ -6 & 4 & 4 \end{bmatrix}$

2. det A= -8 , det B= -8

   Choose Row 1; detC = $2\begin{vmatrix} 2 & 5 & 1 \\ 2 & 1 & 4 \\ 4 & 3 & 2 \end{vmatrix} - 4\begin{vmatrix} 3 & 5 & 1 \\ 1 & 1 & 4 \\ 3 & 3 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} - 5\begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & 1 \\ 3 & 4 & 3 \end{vmatrix}$

   =2

3. $A^T = \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}$, $B^T = \begin{bmatrix} 2 & 3 & 1 \\ 3 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$, $C^T = \begin{bmatrix} -1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, $D^T = \begin{bmatrix} 2 & 1 & 9 \end{bmatrix}$

4. $A^{-1} = \frac{1}{8}\begin{bmatrix} 3 & -5 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 3/8 & -5/8 \\ -1/4 & 3/4 \end{bmatrix}$

$$B^{-1}= \begin{bmatrix} -7 & 2 & -6 \\ 12 & -12 & 6 \\ -1 & -4 & 12 \end{bmatrix}$$

5. $$AB = \begin{bmatrix} 4 & -2 & 0 \\ 1 & 7 & -1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 \\ 3 & 5 & -5 \\ 1 & 5 & 1 \end{bmatrix} = \begin{bmatrix} -10 & -10 & 22 \\ 19 & 30 & -33 \\ 7 & 15 & -9 \end{bmatrix}$$

From det A= 28+8+2=38

$$\text{adj } A = \begin{bmatrix} \begin{vmatrix} 7 & -1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 7 \\ 0 & 2 \end{vmatrix} \\ -\begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & -2 \\ 0 & 2 \end{vmatrix} \\ \begin{vmatrix} -2 & 0 \\ 7 & -1 \end{vmatrix} & -\begin{vmatrix} 4 & 0 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 4 & -2 \\ 1 & 7 \end{vmatrix} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 9 & -1 & 2 \\ 2 & 4 & -8 \\ 2 & 4 & 30 \end{bmatrix}^{T} = \begin{bmatrix} 9 & 2 & 2 \\ -1 & 4 & 4 \\ 2 & -8 & 30 \end{bmatrix}$$

Then, $$A^{-1} = \frac{1}{\det A}(\text{adj}A) = \begin{bmatrix} 9/38 & 2/38 & 2/38 \\ -1/38 & 4/38 & 4/38 \\ 2/38 & -8/38 & 30/38 \end{bmatrix}$$

$$[A^{-1}]^{T} = \begin{bmatrix} 9/38 & -1/38 & 2/38 \\ 2/38 & 4/38 & -8/38 \\ 2/38 & 4/38 & 30/38 \end{bmatrix} \text{---1)}$$

And $$A^{T} = \begin{bmatrix} 4 & 1 & 0 \\ -2 & 7 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$, det A^{T} = 28+8+2 = 38

$$\text{Adj } A^{T} = \begin{bmatrix} \begin{vmatrix} 7 & 2 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -2 & 7 \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ 7 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 0 \\ -2 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ -2 & 7 \end{vmatrix} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 9 & 2 & 2 \\ -1 & 4 & 4 \\ 2 & -8 & 30 \end{bmatrix}^{T} = \begin{bmatrix} 9 & -1 & 2 \\ 2 & 4 & -8 \\ 2 & 4 & 30 \end{bmatrix}$$

Then, $[A^T]^{-1} = \begin{bmatrix} 9/38 & -1/38 & 2/38 \\ 2/38 & 4/38 & -8/38 \\ 2/38 & 4/38 & 30/38 \end{bmatrix}$---2)

It shows that 1) = 2), so $[A^{-1}]^T = [A^T]^{-1}$

6. $$\begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ \frac{3}{4} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The solution is X= -1 and Y=1.

7. a) $\begin{bmatrix} 4 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 1/8 & 1/4 \\ -1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The Solution is X=2 and Y=1

b) $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 1 & 1 \\ 3 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 1 & 1 \\ 3 & -2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

The Solution is X=2, Y=3 and Z=-1.

8. 4X+5Y=2, 11X+Y+2Z=3, X+5Y+2Z=1

Let $A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}$, det A= - 132

From Cramer's rule

$$X=\frac{\begin{vmatrix}2 & 5 & 0\\ 3 & 1 & 2\\ 1 & 5 & 2\end{vmatrix}}{-132}=\frac{-36}{-132}=\frac{3}{11}$$

$$Y=\frac{\begin{vmatrix}4 & 2 & 0\\ 11 & 3 & 2\\ 1 & 1 & 2\end{vmatrix}}{-132}=\frac{-24}{-132}=\frac{2}{11}$$

$$Z=\frac{\begin{vmatrix}4 & 5 & 2\\ 11 & 1 & 3\\ 1 & 5 & 1\end{vmatrix}}{-132}=\frac{12}{-132}=\frac{-1}{11}$$

9. Q_d=220-5P and Q_s=-20+3P

$$\begin{bmatrix}1 & 5\\ 1 & -3\end{bmatrix}\begin{bmatrix}Q\\ P\end{bmatrix}=\begin{bmatrix}220\\ -20\end{bmatrix}$$

$$\begin{bmatrix}Q\\ P\end{bmatrix}=\begin{bmatrix}1 & 5\\ 1 & -3\end{bmatrix}^{-1}\begin{bmatrix}220\\ -20\end{bmatrix}$$

$$=\begin{bmatrix}3/8 & 5/8\\ 1/8 & -1/8\end{bmatrix}\begin{bmatrix}220\\ -20\end{bmatrix}$$

$$=\begin{bmatrix}70\\ 30\end{bmatrix}$$

Then, equilibrium quantity=70 and equilibrium price=30.

10. $18P_1 - P_2$=87 and $-2P_1+36P_2$=98

$$\begin{bmatrix}18 & -1\\ -2 & 36\end{bmatrix}\begin{bmatrix}P1\\ P2\end{bmatrix}=\begin{bmatrix}87\\ 98\end{bmatrix}$$

$$\begin{bmatrix}P1\\ P2\end{bmatrix}=\begin{bmatrix}18 & -1\\ -2 & 36\end{bmatrix}^{-1}\begin{bmatrix}87\\ 98\end{bmatrix}$$

$$=\begin{bmatrix}\frac{18}{323} & \frac{1}{646}\\ \frac{1}{323} & \frac{9}{323}\end{bmatrix}\begin{bmatrix}87\\ 98\end{bmatrix}$$

$$=\begin{bmatrix}5\\ 3\end{bmatrix}$$

Equilibrium price1=5 and equilibrium price2 =3.

When there is a change,

$$\begin{bmatrix} 18 & -1 \\ -2 & 36 \end{bmatrix} \begin{bmatrix} P1' \\ P2' \end{bmatrix} = \begin{bmatrix} 174 \\ 196 \end{bmatrix}$$

$$\begin{bmatrix} P1' \\ P2' \end{bmatrix} = \begin{bmatrix} \frac{18}{323} & \frac{1}{646} \\ \frac{1}{323} & \frac{9}{323} \end{bmatrix} \begin{bmatrix} 174 \\ 196 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

New Equilibrium price1=10 and new equilibrium price2 =6.

11. $Q_{dp}=82-3P_p+P_c$, $Q_{sp}=-5+15P_p$

$Q_{dc}=92+2P_p-4P_c$, $Q_{sc}=-6+32P_c$

$Q_{dp}=Q_{sp}=Q_p$ and $Q_{dc}=Q_{sc}=Q_c$

$$\begin{bmatrix} 1 & 0 & 3 & -1 \\ 1 & 0 & 15 & 0 \\ 0 & 1 & -2 & 4 \\ 0 & 1 & 0 & -32 \end{bmatrix} \begin{bmatrix} Qp \\ Qc \\ Pp \\ Pc \end{bmatrix} = \begin{bmatrix} 82 \\ -5 \\ 92 \\ -6 \end{bmatrix}$$

Then, equilibrium price of pork =5 and equilibrium price of chicken =3.

Consider $82-3P_p+P_c$

$\frac{dQ_{dp}}{dP_c}$ = 1>0 , so it shows that pork and chicken are substitute goods.

12.

$$Q_d - Q_s = 0$$

$$Q_d + 5P = 220$$

$Q_s - 3(P-1) = -20$, so $Q_s - 3P = -23$

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 5 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} Q_{d*} \\ Q_{s*} \\ P* \end{bmatrix} = \begin{bmatrix} 0 \\ 220 \\ -23 \end{bmatrix}$$

$Q_* = 68.125$ $P* = 30.375$

13.

$$A=\begin{bmatrix} 4 & 0 & 1 \\ 19 & 1 & 3 \\ 5 & 4 & 7 \end{bmatrix}$$ det A= 28+76 – 5 – 48 = 51

So, A^{-1} can be calculated.

$$B=\begin{bmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 13 & -3 & -4 \end{bmatrix}$$ det B= -28 – 52 +84 – 4 =0

So, A^{-1} can't be calculated.

14. 0.3Y+100R-252=0 and 0.25Y-200R-176=0

$$\begin{bmatrix} 0.3 & 100 \\ 0.25 & -200 \end{bmatrix}\begin{bmatrix} Y \\ R \end{bmatrix} = \begin{bmatrix} 252 \\ 176 \end{bmatrix}$$

$$\begin{bmatrix} Y \\ R \end{bmatrix} = \begin{bmatrix} 0.3 & 100 \\ 0.25 & -200 \end{bmatrix}^{-1}\begin{bmatrix} 252 \\ 176 \end{bmatrix}$$

$$= \frac{-1}{85}\begin{bmatrix} -200 & -100 \\ -0.25 & 0.3 \end{bmatrix}\begin{bmatrix} 252 \\ 176 \end{bmatrix}$$

$$= \begin{bmatrix} 800 \\ 0.12 \end{bmatrix}$$

Equilibrium income (Y*) is 800 and equilibrium interest rate (R*) is 0.12.

15. C=85+0.9Y

I_0=55

G_0=20

X=100

M=10+0.3Y

$$\begin{bmatrix} 1 & -1 & 1 \\ -0.9 & 1 & 0 \\ -0.3 & 0 & 1 \end{bmatrix}\begin{bmatrix} Y \\ C \\ M \end{bmatrix}=\begin{bmatrix} 175 \\ 85 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} Y \\ C \\ M \end{bmatrix}=\begin{bmatrix} 625 \\ 647.5 \\ 197.5 \end{bmatrix}$$

Equilibrium income is 625, equilibrium consumption expenditure is 647.5 and equilibrium import is 197.5.

16. 3P+Q-71=0 and 6P-Q-10=0

a) $$\begin{bmatrix} 3 & 1 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} 71 \\ 10 \end{bmatrix}$$

Det $\begin{bmatrix} 3 & 1 \\ 6 & -1 \end{bmatrix}$ = -9

From Cramer's rule, P*= $\frac{\begin{vmatrix} 71 & 1 \\ 10 & -1 \end{vmatrix}}{-9} = \frac{81}{9} = 9$

Q*= $\frac{\begin{vmatrix} 3 & 71 \\ 6 & 10 \end{vmatrix}}{-9} = \frac{396}{9} = 44$

b) Impose tax on seller; 6(P – 2) –Q – 10 =0

6P – Q =22

$$\begin{bmatrix} 3 & 1 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} 71 \\ 22 \end{bmatrix}$$

From Cramer's rule, P*'= $\frac{\begin{vmatrix} 71 & 1 \\ 22 & -1 \end{vmatrix}}{-9} = \frac{93}{9} = 10.33$

Q*'= $\frac{\begin{vmatrix} 3 & 71 \\ 6 & 22 \end{vmatrix}}{-9} = \frac{360}{9} = 40$

17. IS: Y= C+I+G =100+0.75(Y- 20 - 0.2Y)+1200-30R+ 935

0.4Y+30R=2220

LM: M_d= M_s >> 1375+0.25Y-25R =2500

0.25Y – 25R= 1125

$$\begin{bmatrix} 0.4 & 30 \\ 0.25 & -25 \end{bmatrix}\begin{bmatrix} Y \\ R \end{bmatrix} = \begin{bmatrix} 2220 \\ 1125 \end{bmatrix}$$

$$\det\begin{bmatrix} 0.4 & 30 \\ 0.25 & -25 \end{bmatrix} = -17.5$$

From Cramer's rule; Y*= $\frac{\begin{vmatrix} 2220 & 30 \\ 1125 & -25 \end{vmatrix}}{-17.5}$ =5100

$$R^* = \frac{\begin{vmatrix} 0.4 & 2220 \\ 0.25 & 1125 \end{vmatrix}}{-17.5} = 6$$