6204644564
Nanthasri K.

Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$$\hat{\beta}_2 = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} = \frac{-174.2}{1098.8} = -0.1585$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 21.03 - (-0.1585)(12.2)$$
$$= 22.9637$$

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_i$$
$$\hat{Y} = 22.9637 - 0.1585 X_i$$

Interpretation

$\hat{\beta}_1$; if $X = 0$, y will be equal to 22.9637

$\hat{\beta}_2$; if X increased by 1 unit , y will decreased by 0.1585 unit

b) Find r^2 and explain its meaning.

$$r^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\Sigma \hat{u}_i^2}{\Sigma y_i^2}$$

$$= 1 - \frac{873.14}{882.97} = 0.0111$$

Meaning that Y can be explained by regression model at 1.11%.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

$$E(Y | X = 5)$$
$$\hat{Y} = 22.9637 - 0.1585(5) = 22.1712$$

Meaning that if $X_i = 5$, expected $\hat{Y}$ will be equal to 22.1712

d) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

$$Var(u_i) = \frac{\Sigma \hat{u}^2}{n-K} = \frac{873.14}{30-2} = 31.1836 = \sigma^2$$

$$var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \cdot \Sigma(x_i - \bar{X})} \cdot \sigma^2 = \frac{5564}{30(1098.8)} \cdot 31.1836$$

$$= 5.2635$$

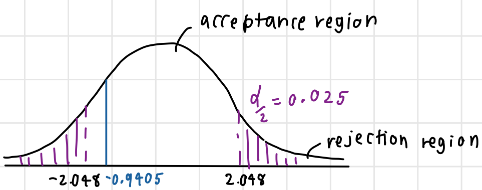
$$var(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma(X_i - \bar{X})^2} = \frac{31.1836}{1098.8} = 0.0284$$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$\alpha = 0.05$



$$t_{28, 0.025} = \pm 2.048$$

$$t = \frac{\hat{\beta}_2 - \beta_2}{Se\,\hat{\beta}_2} = \frac{-0.1585 - 0}{\sqrt{0.0284}} = |-0.9405|$$

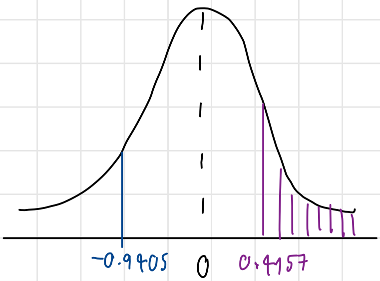
$\therefore$ as $0.9405 < 2.048$, t fall in the acceptance region
hence, coefficients are not different from zero ($\hat{\beta}_2 = 0$)
At 95% confidence level, x has no effect on y

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

$H_0 : \beta_2 \leq 0$

$H_1 : \beta_2 > 0$

$\alpha = 0.01$



$t = \frac{\hat{\beta}_2 - \beta_2}{Se\ \hat{\beta}_2} = \frac{\hat{\beta}_2 - \beta_2}{\sqrt{Var\ \hat{\beta}_2}} = \frac{-0.1585 - 0}{\sqrt{0.0284}} = -0.9405$

The upper found $= \beta_2 + t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{\beta}_2} = 0 + 2.467(\sqrt{0.028470})$

$= 0.4157$

t_{cal} lies within acceptance region, meaning that H_0 is cannot be denied and we can say for sure that β_2 is less than 0 99 out of 100 times.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

Yes, as the function has a negative slope it means that x and y has a negative relationship, which making sense because as the year of usage increase, the price of car should be decreasing as the durable good must be depreciation over ages.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

$$\hat{Y}_0 - (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{Y}_0}) \leq Y_0 \leq \hat{Y}_0 + (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{Y}_0})$$

If $X_i = 5$, $\hat{Y}_0 = 7{,}836 - 502.4(5) = 5{,}324$

$$Var(\hat{Y}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\Sigma (X_i - \bar{X})^2} \right]$$

$$Var(\hat{Y}_0) = 212{,}877 \left[\frac{1}{11} + \frac{(5 - 7.45)^2}{98.73} \right]$$

$$= 35{,}582.5345$$

$$\sigma_{\hat{Y}_0} = \sqrt{35{,}582.5345} = 188.6333$$

$t_{0.025} = 2.262$

upper bound $= \hat{Y}_0 + (t_{\alpha/2} \cdot \sigma_{\hat{Y}_0}) = 5{,}324 + (2.262 \times 188.6333)$

$= 5{,}750.6885$

lower bound $= \hat{Y}_0 - (t_{\alpha/2} \cdot \sigma_{\hat{Y}_0}) = 5{,}324 - (2.262 \times 188.6333)$

$= 4{,}897.3115$

$$4{,}897.3115 \leq Y_0 \leq 5{,}750.6885$$

$\therefore$ If the car has been used for 5 years, the estimated average price will be between 4,897.3115 and 5,750.6885.

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

$$\text{SRF } \times (10) = \hat{y} = \underset{(52)}{7836} - \underset{(4118)}{5024}(X)$$

d) Calculate the elasticity of market price when a car is 10 years old.

$$\varepsilon_p = \frac{dP}{dA} \times \frac{A}{P}$$

$$= -502.4 \times \frac{10}{2812}$$

$$= -1.7866$$