Question 0: (required for all)

0.1) Given that $Z = \frac{x^3-y^3}{x^2y^2}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -Z$

0.2) Given that $Z = \frac{x-y}{x+y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

0.3) If $z = 2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

0.5) Given that $ln\,(x + y + z) + xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

Members

Purin	6204640954
Wanrada	6204641200
Gulanut	6204641416
Hyeseong	6204641606

0.1) $\frac{\partial z}{\partial x} = \frac{(x^2y^2)(3x^2) - (x^3-y^3)(2xy^2)}{(x^2y^2)^2}$

$= \frac{3x^4y^2 - 2x^4y^2 + 2xy^5}{(x^2y^2)^2}$

$= \frac{x^4y^2 + 2xy^5}{(x^2y^2)^2}$

$\frac{\partial z}{\partial y} = \frac{x^2y^2(-3y^2) - [(x^3-y^3)2x^2y]}{(x^2y^2)^2}$

$= \frac{-3x^2y^4 - 2x^5y + 2x^2y^4}{(x^2y^2)^2}$

$= \frac{-x^2y^4 - 2x^5y}{(x^2y^2)^2}$

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -z$

$x\left[\frac{x^4y^2 + 2xy^5}{(x^2y^2)^2}\right] + y\left[\frac{-x^2y^4 - 2x^5y}{(x^2y^2)^2}\right]$

$= \frac{x^5y^2 + 2x^2y^5 - x^2y^5 - 2x^5y^2}{(x^2y^2)^2}$

$= \frac{x^2y^5 - x^5y^2}{(x^2y^2)^2}$

$= \frac{x^2y^2(y^3-x^3)}{(x^2y^2)^2}$

$\therefore -z = \frac{y^3-x^3}{(x^2y^2)}$

0.2) $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$= \frac{(x+y) - (x-y)}{(x+y)^2}dx + \frac{(x+y)(-2) - (x-y)}{(x+y)^2}dy$

$= \frac{2y}{(x+y)^2}dx - \frac{2x}{(x+y)^2}dy$

$dx = 2 \quad dy = -2 \quad x = 1 \quad y = 1$

$\left(\frac{2(1)}{(2)^2} \cdot 2\right) - \left(\frac{2(1)}{(2)^2} \cdot -2\right) = 1 + 1 = 2$

∴ when x increased by 2 units, while y decreased by 2 units. However, z will increared by 2 units.

0.3a) $\frac{\partial z}{\partial s} = \left[\frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s}\right] + \left[\frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}\right]$

$= (4xy + 3y)(2r) + (2x^2 + 3x + 2y)(-4)$

$\frac{\partial z}{\partial r} = \left[\frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r}\right] + \left[\frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}\right]$

$= (4xy + 3y)(2r + 2s) + (2x^2 + 3x + 2y)(2)$

0.3b) $r = 1,\ s = 0$

$x = (1)^2 + 2(1)(0) = 1 \qquad y = 2(1) - 4(0) = 2$

$\frac{\partial z}{\partial s} = (8+6)(2) + (2+3+4)(-4)$

$= 28 - 36$

$= -8$ $\quad\therefore$ when r increase by 1 unit, z will decrease by 8 units

$\frac{\partial z}{\partial r} = (8+6)(2+0) + (2+3+4)(2)$

$= 28 + 18$

$= 46$ $\quad\therefore$ when r increase by 1 unit, z will increase by 46 units.

0.4) $2x^2 + 3y^2 + 2z^2 = 16$

$z^2 = \frac{16}{2} - \frac{2x^2}{2} - \frac{3y^2}{2}$

$z = \pm\sqrt{8 - x^2 - \frac{3}{2}y^2}$

$\frac{\partial z}{\partial y} = \frac{1}{2}(8 - x^2 - \frac{3}{2}y^2)^{-\frac{1}{2}}(-3y)$

$= \frac{1}{2}\left[\frac{1}{\pm\sqrt{8 - x^2 - \frac{3}{2}y^2}}\right](-3y)$

$= \frac{1}{2} \cdot \frac{1}{z} \cdot (-3y)$

$= -\frac{3y}{2z}$

$\left.\frac{\partial z}{\partial y}\right|_{x=1, y=2, z=-1} = \frac{-3(2)}{2(-1)} = 3$

$\therefore$ when y increase by 1 unit, z will increase by 3 units

0.5) $F(x,z) = 0 = \ln(x+y+z) + xyz - ze^{x+y+z}$

$$= -\frac{F_x}{F_z} = \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{\frac{1}{x+y+z} + yz - ze^{x+y+z}}{\frac{1}{x+y+z} + xy - (ze^{x+y+z} + e^{x+y+z})}$$

$$= -\frac{\frac{1}{1} + 0 - 0}{\frac{1}{1} + 0 - e^{1}} = \boxed{\frac{1}{e}}$$

1.1) • $\frac{\partial Q_x}{\partial P_x} = -4$

• $\frac{\partial Q_x}{\partial P_y} = -25 P_y^{-3/2} = \frac{25}{P_y^{3/2}}$

$\therefore$ if P_x increase 1 unit quantity of x will decrease by -4
However, if P_y increase 1 unit quantity of x will increase by $\frac{25}{P_y^{3/2}}$.
So, good x and good y are substitute product.

1.2) $\frac{\partial Q_x}{\partial I} = I$

If income increase by 1 unit, quantity of x will increase by I unit.
So, quantity of x is normal goods.

1.3) $Q_x = 100 - 4P_x - 50(P_y)^{-1/2} + 0.5I^2$

$= 100 - 4(10) - 50(25)^{-1/2} + 0.5(10)^2$

$= 100 - 40 - 10 + 50$

$= 100$ units

1.4) $= \frac{\partial Q_x}{\partial P_x} \cdot \frac{P_x}{Q_x}$

$= -4 \cdot \frac{10}{100} = -0.4$

1.5) $= \frac{\partial Q_x}{\partial P_y} \cdot \frac{P_y}{Q_x}$

$= \frac{25}{P_y^{3/2}} \cdot \frac{25}{100}$

$= \frac{25}{4P_y^{3/2}} = \frac{1}{4 \cdot 5} = \frac{1}{20}$

1.6) $= \frac{\partial Q_x}{\partial I} \cdot \frac{I}{Q_x}$

$= I \cdot \frac{I}{Q_x}$

$= 10 \cdot \frac{10}{100} = 1$

$\therefore$ Goods whose income elasticity of demand is between 0 and 1 are typically referred to as necessary goods, which are products and services that consumer will buy regardless of change income levels.

3.1) $\boxed{\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}}$: MU of good x

$\boxed{\frac{dy}{dx} = \frac{1}{2}y^{-\frac{1}{2}}}$: MU of good y

3.2) $\left.\begin{aligned} \frac{d^2y}{dx^2} &= -\frac{1}{4}x^{-\frac{3}{2}} \\ \frac{d^2y}{dy^2} &= -\frac{1}{4}y^{-\frac{3}{2}} \end{aligned}\right\} < 0$: law of diminishing

3.3) when consumers consume more units of good y, the curve of MU of good x has no effect.

3.4) $u = x^{\frac{1}{2}} + y^{\frac{1}{2}}$

$= 1^{\frac{1}{2}} + 2^{\frac{1}{2}}$

$= 1 + \sqrt{2} \approx 2.414$

3.5) $du = \frac{dy}{dx} \cdot dx + \frac{dy}{dy} \cdot dy$

$= \frac{1}{2}x^{-\frac{1}{2}} \cdot dx + \frac{1}{2}y^{-\frac{1}{2}} \cdot dy$

$= \frac{1}{2}(1)^{-\frac{1}{2}} \cdot 3 + \frac{1}{2}(2)^{-\frac{1}{2}} \cdot -1$

$= \frac{3}{2} - \frac{\sqrt{2}}{4}$

$= \frac{6 - \sqrt{2}}{4} \approx 1.1464$

3.6) $MRS = \frac{MUx}{MUy} = \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{y}}} = \frac{2\sqrt{y}}{2\sqrt{x}} = \frac{\sqrt{y}}{\sqrt{x}}$

$\therefore$ when x increase by 1 unit, MRS will decrease by $\frac{\sqrt{y}}{\sqrt{x}}$ unit.