

EE 415 / 418 Game Theory (2/2011)

Lecture 8. Repeated Games

- Finite-Stage Repeated Game
- Read topic 2.3A

motivation

- We analyze whether threats and promises about future behavior (payoff or rewards) can influence current behavior in repeated interaction.
- Yes if it is credible. When you have to punish, you will not back off.
- How we create such strategy
- How we define subgame perfect NE for repeated games.

Finite-Stage Repeated Games

- Consider the Prisoners' Dilemma given in normal form representation in Figure 1.

		Player 2	
		L_2	R_2
Player 1	L_1	1, 1	5, 0
	R_1	0, 5	4, 4

Figure 1

Finite-Stage Repeated Games

- Two players play this simultaneous-move game twice, observing the outcome of the first round then play the second round.
- The payoff for the entire game is the sum of the payoff from each round (with no discounting).
- Call this game the Two-Stage Prisoners' Dilemma.

Finite-Stage Repeated Games

- In this Two-Stage Prisoners' Dilemma there exist a unique Nash equilibrium for the second-stage of the game which is independent of the first-stage outcome.
- Because the payoffs of the entire game is the sum of payoffs from each round, one can simply add the second-stage equilibrium payoff into each cell of the first stage payoff matrix.

Finite-Stage Repeated Games

- The entire game can then be restated by normal representation in Figure 2.

	L_2	R_2
L_1	2, 2	6, 1
R_1	1, 6	5, 5

- The subgame-perfect equilibrium outcome is then (L_1, L_2) for both rounds.
- Cooperation, (R_1, R_2) , cannot be achieved in either stage.

Finite-Stage Repeated Games

- The previous argument holds more generally.
- Let $G = \{A_1, \dots, A_n; u_1, \dots, u_n\}$ denote a static game of complete information in which player 1 to player n simultaneously choose his/her action a_1 through a_n respectively, and the payoffs are $u_1(a_1, \dots, a_n)$ through $u_n(a_1, \dots, a_n)$.
- G is called the stage game of the repeated game.

Finite-Stage Repeated Games

Definition

Given a stage game G , let $G(T)$ denote finitely repeated game in which G is played T times, with the outcome of all preceding plays observed before the next play begins. The payoffs for $G(T)$ are simply the sum of the payoffs from the T stage game.

Finite-Stage Repeated Games

Proposition

If the stage game G has a unique Nash equilibrium then, for any finite T , the repeated game $G(T)$ has a unique subgame-perfect equilibrium: the Nash equilibrium of G is played in every stage.

Finite-Stage Repeated Games

- What if for each stage game, G , there exist multiple (pure) Nash equilibria?
- Consider a two-stage repeated game in which normal form representation of the stage game is given by Figure 3.

	L_2	M_2	R_2
L_1	1, 1	5, 0	0, 0
M_1	0, 5	4, 4	0, 0
R_1	0, 0	0, 0	3, 3

- We add R_1 and R_2 . Now, this game has two NE's: (L_1, L_2) and (R_1, R_2) . This allows us not to stick with (L_1, L_2) in the second stage.

- You can think of (M_1, M_2) as cooperation in Prisoner's dilemma game.

- you want to support (M_1, M_2) and (R_1, R_2) as SPNE.
- You can think (L_1, L_2) as a punishment if not cooperating.
- Possible strategy (trickle strategy):
 - Cooperate with M, then give reward R
 - Not cooperate (not play M), then punish with L

- if (M_1, M_2) is played in the first stage then (R_1, R_2) will be played in the second stage, otherwise (L_1, L_2) will be played in the second stage.
- Then one can add the anticipated equilibrium payoffs to the first stage payoff matrix give by Figure 4.

	L_2	M_2	R_2
L_1	2, 2	6, 1	1, 1
M_1	1, 6	7, 7	1, 1
R_1	1, 1	1, 1	4, 4

- If (M_1, M_2) is played, then the payoff is $(7, 7)$, otherwise $(1, 1)$ resulted from playing (L_1, L_2) is added to each cells.

Finite-Stage Repeated Games

- There are three pure strategy Nash equilibria associated with the subgame depicted by Figure 4; (L_1, L_2) , (M_1, M_2) , and (R_1, R_2) .
- (L_1, L_2) : $((L_1, L_2), (L_1, L_2))$ in the entire game and
- (R_1, R_2) : $((R_1, R_2), (L_1, L_2))$.
- (M_1, M_2) : $((M_1, M_2), (R_1, R_2))$,
- Note that in the first-two equilibria, Nash equilibrium is played in each stage game
- In the last one (M_1, M_2) itself is never a Nash equilibrium in the stage game.

Finite-Stage Repeated Games

- $((M_1, M_2), (R_1, R_2))$ can be sustained because players anticipated that the "better" equilibrium in the second stage can be achieved if they cooperate in the first stage.
- Therefore, if the stage game is associated with multiple equilibria, cooperation can be archived.
- Generally, if G is a static game of complete information with multiple equilibria then there may be subgame-perfect outcome of repeated game $G(T)$ in which, for any $t < T$, the outcome in stage t is not a Nash equilibrium of G .



There are two points to extract from this example

- Credible treats or promises about future behavior can influence current behavior. (via trickle strategy)
- Subgame-perfection may not embody a strong credibility. That is, threat to punish with (L_1, L_2) , the “bad equilibrium” if players do not cooperate in the first stage is not credible. If the players reason that let the bygone be bygone, then they might unanimously preferred stage game equilibrium (R_1, R_2) to be played in the second stage instead.
- When it is your time to punish, you are reluctant to hurt yourself too. Or you want to renegotiate (occurs between the first and second stages).

- To suggest a solution to this renegotiation problem, consider a stage game in Figure 5

	L_2	M_2	R_2	P_2	Q_2
L_1	1, 1	5, 0	0, 0	0, 0	0, 0
M_1	0, 5	4, 4	0, 0	0, 0	0, 0
R_1	0, 0	0, 0	3, 3	0, 0	0, 0
P_1	0, 0	0, 0	0, 0	$4, \frac{1}{2}$	0, 0
Q_1	0, 0	0, 0	0, 0	0, 0	$\frac{1}{2}, 4$

- Think of (P_1, P_2) as a stick for player 1 to punish player 2 (hurt 2 more than hurt 1)
- Similarly, (Q_1, Q_2) as a stick for player 2 to punish player 1.

- 4 NE's for this stage game: (L_1, L_2) , (R_1, R_2) , (P_1, P_2) , and (Q_1, Q_2) .

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- Suppose player 2, fails to cooperate in the first stage, how would player 1 want to punish player 2?
 - Among the four Nash equilibria, player 1 gets best payoff if (P_1, P_2) is played (compare 1,3, 4, and 0.5)
 - Thus punishment now is credible (not Pareto dominated as (L_1, L_2) in the previous example).
 - Thus it is best for player 1 to punish player 2 by playing P_1 in the second stage if player 2 do not cooperate.
 - The same logic applies to player 2 analogously.

Finite-Stage Repeated Games

- Check if it is good to stick with cooperation (get $4+3=7$), payoff from deviation then being punished ($5 + 0.5 = 5.5$), payoff from not cooperate at all ($1+1=2$).
- Thus, we can support the cooperation with the following strategy:
 - (R_1, R_2) will be played in the second stage if (M_1, M_2) is played in the first period. If (M_1, x) , is played in the first period (P_1, P_2) will be played in the second period. If (y, M_2) , is played in the first period (Q_1, Q_2) will be played in the second period.
- In this game, $((M_1, M_2), (R_1, R_2))$ is a SPNE.

Finite-Stage Repeated Games

- The main difference between the two game is that in the game based on Figure 4, players can punish one another for deviating in the first stage only by playing Pareto dominated equilibrium in the second stage therefore after first stage is completed, both players have incentive to “renegotiate”.
- In the repeated game based on Figure 5, there are three equilibria on the Pareto frontier, one reward both players if they cooperate, and other two to be used not only to punish a player who deviate but also reward the punisher.
- If punishment is called for in the second stage, there will be only one equilibrium in which the punisher would prefer so that the punisher cannot be persuaded to renegotiate.