

CHAPTER 6

Optimization without Constraints:

One Independent Variable Case

Topics:

- ➔ Maximize profits $\text{Max } \pi$
 - Competitive market case
 - Monopoly case
- ➔ Effects of taxes $\text{Max } \pi_{AT}$
 - Lump-sum tax t
 - Profit tax
 - Excise tax
- ➔ Maximization of tax revenue $\text{Max } t$ Tax Revenue

Our attention will be turned to the study of **goal equilibrium**, in which the equilibrium state is defined as the optimum position for a given economic unit(e.g. a household, a business firm) and in which the said economic unit will be deliberately striving for attainment of that equilibrium.

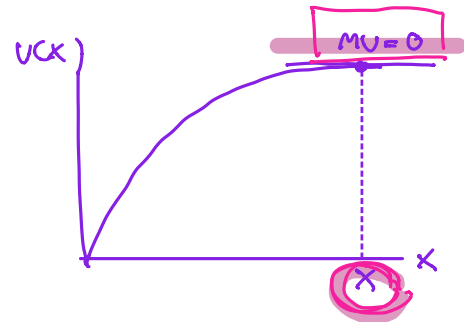
We will use the optimization to study how consumers, producers, or public policy makers choose their best alternatives given their limited resources, as the essence of the optimization problem is to **choose the BEST alternative available**.

Utility Maximization:

FOC: $MU(x) = 0$

→ solve for x^*

$$\max_x U(x)$$



Profit Maximization:

$$\max_Q \pi(Q)$$

FOC: $MR(Q) = MC(Q)$

The optimal point is Q_1^* .

Cost Minimization:

$$\min_Q AC(Q)$$

FOC: $\frac{dAC(Q)}{dQ} = 0$

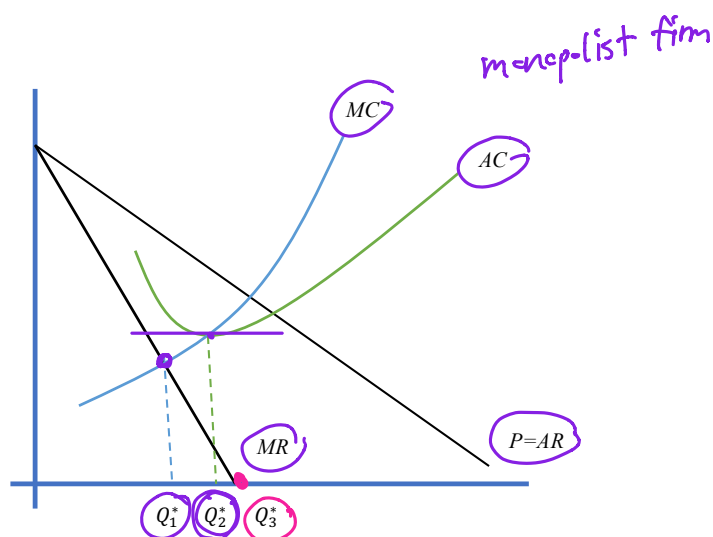
The optimal point is Q_2^* .

Total Revenue Maximization:

$$\max_Q TR(Q)$$

FOC: $MR(Q) = 0$

The optimal point is Q_3^* .



Tax Revenue Maximization:

$$\max_t \text{Tax Revenue}(t)$$

H.W. Find critical values and the maximum/minimum of the following functions:

$$\text{✂ } y = f(x) = x^3 - 12x^2 + 36x + 8$$

$$\text{✂ } y = f(x) = \frac{1}{3}x^3 - x^2 + x + 10$$

$$\text{✂ } y = f(x) = (2-x)^4$$

$$\text{✂ } \text{Let } TC = f(Q) = Q^3 - 4Q^2 + 8Q. \text{ Find } Q \text{ that minimizes AC}$$



Optimization problem and producers
 Get the correct objective fn

Objective:

$$\pi(q) = TR(q) - TC(q) \quad : \text{Profit fn}$$

Environment:

TR	TC
A competitive firm A monopolist An oligopolist A monopolistic competition	Production function Structure of market of factors of productions

Firm's Profit Maximization:

$$\max_q \pi(q)$$

$$\text{FOC: } \frac{d\pi(q)}{dq} = 0$$

$$\frac{d(TR(q) - TC(q))}{dq} = 0$$

$$\boxed{MR = MC} \Rightarrow \text{find critical value of } q, q^*$$

➤ Producers in Competitive Market

Recap: Characteristics of competitive market

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| <ol style="list-style-type: none">1. Many Buyers, many sellers2. Homogeneous product3. Perfect information among buyers and Sellers4. Free entry and perfect mobility |
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Firm's Profit Maximization:

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Example: Suppose Peter decides to open a company selling shirts in a competitive market. He has total cost $C = 100 + Q^2$, where Q is his level production. If market price for this shirt is 60 baht per shirt, how many shirt should he produce?

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If $Q = 2L$, how many employee should Peter hire so that he can maximize his profit?

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➤ **A monopolist**

Recap: Characteristics of monopoly market
1. One producer
2. Inelastic market demand
3. Economy of scale in production
4. Barrier to entry

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Let market demand be $Q = a - bP$.

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Total Revenue: TR

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Total Cost: TC

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Objective:

$\pi =$

Find Q that maximizes profit

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FONC:

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SOSC:

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Rule of Thumb for Pricing:

$$MC = MR = p \left(1 + \frac{1}{E_p} \right)$$



Measuring Monopoly Power

$$L = \frac{(P - MC)}{P} = -\frac{1}{E_p}$$

L is called “Lerner index” measuring Lerner’s Degree of Monopoly Power.

The Lerner Index is between 0 and 1 since the range of demand that a monopolist chooses to produce has $1 < |E_p| < \infty$

For producers in a competitive market, Lerner Index is 0, since $P=MC$.

The higher the Lerner index is, the higher the monopoly power is.



The effect of tax on profit maximization

Before tax:

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After tax:

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Three types of tax:

Lumsum tax

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Profit tax:

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Specific tax:

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Let market demand be

$$P = a - bQ$$

For a monopolist,

$$TR =$$

$$\frac{d(aQ - bQ^2 - cQ - f - \bar{T})}{dQ} = 0$$

$$a - 2bQ - c = 0$$

$$Q_{AT}^* = \frac{a - c}{2b}$$

$$P_{AT}^* = a - b \left(\frac{a - c}{2b} \right) = \frac{a}{2} + \frac{c}{2}$$

$$\pi_{AT}(Q_{AT}^*) = aQ_{AT}^* - bQ_{AT}^{*2} - cQ_{AT}^* - f - \bar{T}$$

$$\pi_{AT}(Q_{AT}^*) = a \left(\frac{a - c}{2b} \right) - b \left(\frac{a - c}{2b} \right)^2 - c \left(\frac{a - c}{2b} \right) - f - \bar{T}$$

SOC:

$$\frac{d^2\pi_{AT}}{dQ^2} < 0$$

$$\frac{d(a - 2bQ - c)}{dQ} < 0$$

$$-2b < 0, \text{ since } b > 0$$

From FONC, at the level of output at maximizing profit

$$Q_{AT}^* = Q_{BT}^*$$

The profit-maximizing price is

$$P_{AT}^* = P_{BT}^*$$

Profit after tax is

$$\pi_{AT}^* = \pi_{BT}^* - \bar{T}$$

Therefore, if lumpsum tax is collected, the optimal quantity and price will be as same as before tax, but the level of profit will be reduced by the tax amount \bar{T} . Lumpsum tax can be called neutral tax.

Case 2 :Profit Tax $T = t\pi_{BT}$

Suppose tax rate t % of the profit is collected.

Total Profit after Tax:

$$\begin{aligned}\max_Q \pi_{AT}(Q) &= (1-t)\pi_{BT} = (1-t)(TR(Q) - TC(Q)) \\ \max_Q \pi_{AT}(Q) &= (1-t)\pi_{BT} = (1-t)(aQ - bQ^2 - cQ - f)\end{aligned}$$

Find the level of output that maximizes profit:

FOC:

$$\frac{d\pi_{AT}}{dQ} = 0$$

$$\frac{d((1-t)(TR(Q) - TC(Q)))}{dQ} = 0$$

$$\frac{d((1-t)(aQ - bQ^2 - cQ - f))}{dQ} = 0$$

$$(1-t)(a - 2bQ - c) = 0$$

$$Q_{AT}^* = \frac{a-c}{2b}$$

$$P_{AT}^* = a - b\left(\frac{a-c}{2b}\right) = \frac{a}{2} + \frac{c}{2}$$

$$\begin{aligned}\pi_{AT}(Q) &= (1-t)(aQ_{AT}^* - bQ_{AT}^{*2} - cQ_{AT}^* - f) \\ \pi_{AT}(Q) &= (1-t)\left(a\left(\frac{a-c}{2b}\right) - b\left(\frac{a-c}{2b}\right)^2 - c\left(\frac{a-c}{2b}\right) - f\right)\end{aligned}$$

SOC:

$$\frac{d^2\pi_{AT}}{dQ^2} < 0$$

$$\frac{d(1-t)(a - 2bQ - c)}{dQ} < 0$$

$$-2b(1-t) < 0, \text{ since } b > 0, 0 < t < 1$$

From FONC, at the level of output at maximizing profit

$$Q_{AT}^* = Q_{BT}^*$$

From FONC, at the level of output at maximizing profit

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The profit-maximizing price is

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Profit after tax is

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If unit tax is collected, producer will reduce production and increase price. The level of profit will be reduced. Unit tax is distortion tax.

H.W.: Comparative Static analysis can be done to see the effect of increase in tax rate by figuring out $\frac{\partial Q^*}{\partial t}$, $\frac{\partial P^*}{\partial t}$ and $\frac{\partial \pi^*}{\partial t}$

