

EE320 (2/2012)

INTRODUCTORY MATHEMATICAL ECONOMICS

MATRIX ALGEBRA AND ITS APPLICATION

(Part 2 – Matrix Applications)

Topics

- Partial market equilibrium
- General market equilibrium
- Excise tax and market equilibrium
- Simple macroeconomic model
- IS-LM model
- Input-output model

Partial Market Equilibrium (1)

- Given the system of equations

$$Q_d = 24 - 4P \quad \text{-- (1)}$$

$$Q_s = -27 + 13P \quad \text{-- (2)}$$

$$Q_d = Q_s \quad \text{-- (3)}$$

- Write the system in matrix form $AX = d$:

Partial Market Equilibrium (2)

- Using Cramer's rule to find Q^* and P^*

General Market Equilibrium (1)

- Market for good 1

$$Q_{d1} - Q_{s1} = 0 \quad (1)$$

$$Q_{d1} = a_0 + a_1P_1 + a_2P_2 \quad (2)$$

$$Q_{s1} = b_0 + b_1P_1 + b_2P_2 \quad (3)$$

- Market for good 2:

$$Q_{d2} - Q_{s2} = 0 \quad (4)$$

$$Q_{d2} = \alpha_0 + \alpha_1P_1 + \alpha_2P_2 \quad (5)$$

$$Q_{s2} = \beta_0 + \beta_1P_1 + \beta_2P_2 \quad (6)$$

- Write the above system of equations in the matrix form:

General Market Equilibrium (2)

- Find the equilibrium prices and quantities for good 1 and good 2.

Excise Tax and Market Equilibrium: Specific Tax (1)

- Suppose a specific tax t baht per unit is imposed on consumer.

$$Q_D = Q_S \quad (1)$$

$$Q_D = a - b(P + T) \quad (2)$$

$$Q_S = -c + dP \quad (3)$$

- Write the above system of the equations in the matrix form:

Excise Tax and Market Equilibrium: Specific Tax (2)

- Find the new equilibrium prices and quantity.

Excise Tax and Market Equilibrium: Ad Varolem Tax (1)

- Suppose an $t\%$ tax is imposed on consumer.

$$Q_D = Q_S \quad (1)$$

$$Q_D = a - bP^T \quad (2)$$

$$Q_S = -c + dP \quad (3)$$

$$P^T = (1+t)P \quad (4)$$

- Write the above system of the equations in the matrix form:

Excise Tax and Market Equilibrium: Specific Tax (2)

- Find the new equilibrium prices and quantity.

Simple Macroeconomic Model (1)

- Keynesian model:

$$Y = C + I_0 + G_0 + X_0 - M$$

$$C = a + bY_d$$

$$M = mY_d$$

$$Y_d = Y - T$$

$$T = tY$$

- Reduce the above system into:

Simple Macroeconomic Model (2)

- Write the above system in the matrix form, and find the equilibrium national income.

IS-LM Model (1)

- Commodity Market:

$$Y = C + I + G_0$$

$$C = a + bY_d \quad (a > 0, 0 < b < 1)$$

$$I = I_0 - ir \quad (I_0, i > 0)$$

$$Y_d = Y - T \quad \text{where } T = tY \quad (0 < t < 1)$$

$$\rightarrow Y = a + b(1-t)Y + I_0 - ir + G_0 \quad : \text{IS}$$

- Money Market:

$$M^S = M_0$$

$$M^D = kY - hr \quad (k, h > 0)$$

$$\rightarrow M_0 = kY - hr \quad : \text{LM}$$

- Write a matrix form of the above IS-LM equations:

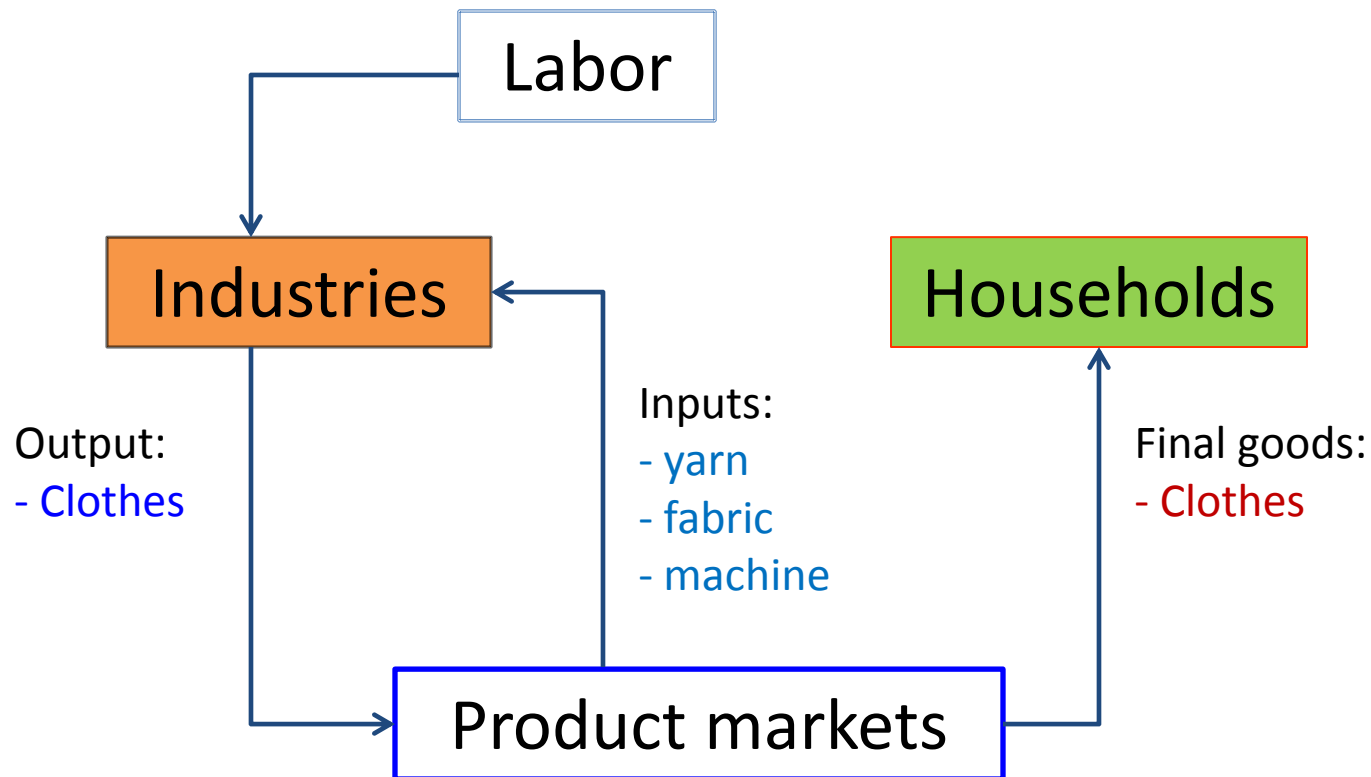
IS-LM Model (2)

- Use Cramer's rule to find the equilibrium national income and interest rate.

Leontif Input-Output Model (1)

- This model characterizes input-output relationships among n interlinked industries in an economy.
- It seeks to determine the output level that each of the n industries should produce to meet the total demand for that product.
- **Application:** Use in production planning, e.g. planning for economic development of a country
- **Assumptions:**
 - Each industry produces only one homogenous commodity.
 - Each industry uses a fixed input ratio for the production of its output.
 - Production in every industry is subject to constant returns to scale.

Leontief Input-Output Model (2)



Leontif Input-Output Model (3)

- Consider an economy with n industries.
 - All the outputs are produced need to meet:
 - (i) the **input demand** of the same i industries
 - (ii) the **final demand** (e.g. consumer demand)
 - 2 types of inputs: (i) **intermediate inputs**
 - (ii) **primary inputs** (e.g. labor)
- Notations:
 - x_i = total number of units of good i that industry i is going to produce
 - a_{ij} = the number of units of good i needed to produce one unit of good j
(a_{ij} = **input coefficient** and $\sum_i a_{ij} < 1$)
 - $a_{ij}x_j$ = the number of units of good i needed to produce x_j unit of good j
 - d_i = final demand for good i

Leontif Input-Output Model (4)

- Equilibrium between supply and demand for each good i :

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n + b_i$$

- The system of equations for n industries:

$$x_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + b_1$$

$$x_2 = a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + b_2 \quad \text{-- } (*)$$

.....

$$x_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n + b_n$$

- Input-coefficient matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Leontif Input-Output Model (4)

- Rearranging the system of equations (*):

$$\begin{aligned}
 (1 - a_{11})x_1 - a_{12}x_2 - \dots - a_{1n}x_n &= b_1 \\
 -a_{21}x_1 + (1 - a_{22})x_2 - \dots - a_{2n}x_n &= b_2 \\
 \dots &\dots \\
 -a_{n1}x_1 - a_{n2}x_2 - \dots + (1 - a_{nn})x_n &= b_n
 \end{aligned}$$

- Leontif model in matrix form:

$$\underbrace{\begin{bmatrix} (1-a_{11}) & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & (1-a_{22}) & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{n1} & -a_{n2} & \cdots & (1-a_{nn}) \end{bmatrix}}_{\text{Leontif Matrix}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} \quad \text{or} \quad (I - A)x = d$$

- Given that $(I - A)^{-1}$ exists, the unique solution of the system is:

Leontif Input-Output Model (5)

- A numerical example:

Industry	Input Demand			Final Demand (\$)
	Agriculture	Manufacture	Service	
Agriculture	0.2	0.3	0.2	10
Manufacture	0.4	0.1	0.2	5
Service	0.1	0.3	0.2	6

- Questions:
 - What level of output should each industry produce?
 - Find the input coefficients of the primary input (labor) for each output, and determine the required amount of the primary input needed in this economy.

Leontif Input-Output Model (6)

- Write the input-coefficient matrix:
- The input-output system in the matrix form $(I-A)x = d$:

Leontif Input-Output Model (7)

- Given that
$$\begin{bmatrix} 0.8 & -0.3 & -0.2 \\ -0.4 & 0.9 & -0.2 \\ -0.1 & -0.3 & 0.8 \end{bmatrix}^{-1} = \frac{1}{0.38} \begin{bmatrix} 0.66 & 0.3 & 0.24 \\ 0.34 & 0.62 & 0.24 \\ 0.21 & 0.27 & 0.6 \end{bmatrix}$$

- The solutions for the output levels to be produced are:

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_2^* \end{bmatrix} = \frac{1}{0.38} \begin{bmatrix} 0.66 & 0.3 & 0.24 \\ 0.34 & 0.62 & 0.24 \\ 0.21 & 0.27 & 0.6 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 6 \end{bmatrix} =$$

- The level of primary input (labor) required for this economy: