Risk and Return

FN 201: Business Finance

Bachelor of Economics

Definition

- Gross return

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

- Net return

$$r_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1$$

where

P_t : price today

P_{t+1} : price tomorrow

D_{t+1} : dividend tomorrow

Net return

$$r_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1$$

$$= \frac{D_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$$= \text{income yield} + \text{capital gain/loss}$$

- Income yield : cash payout
- Capital gain/loss : change in security price

Expected versus Realized return

- At the start of the period, some variables are not known, so we can calculate only expected return

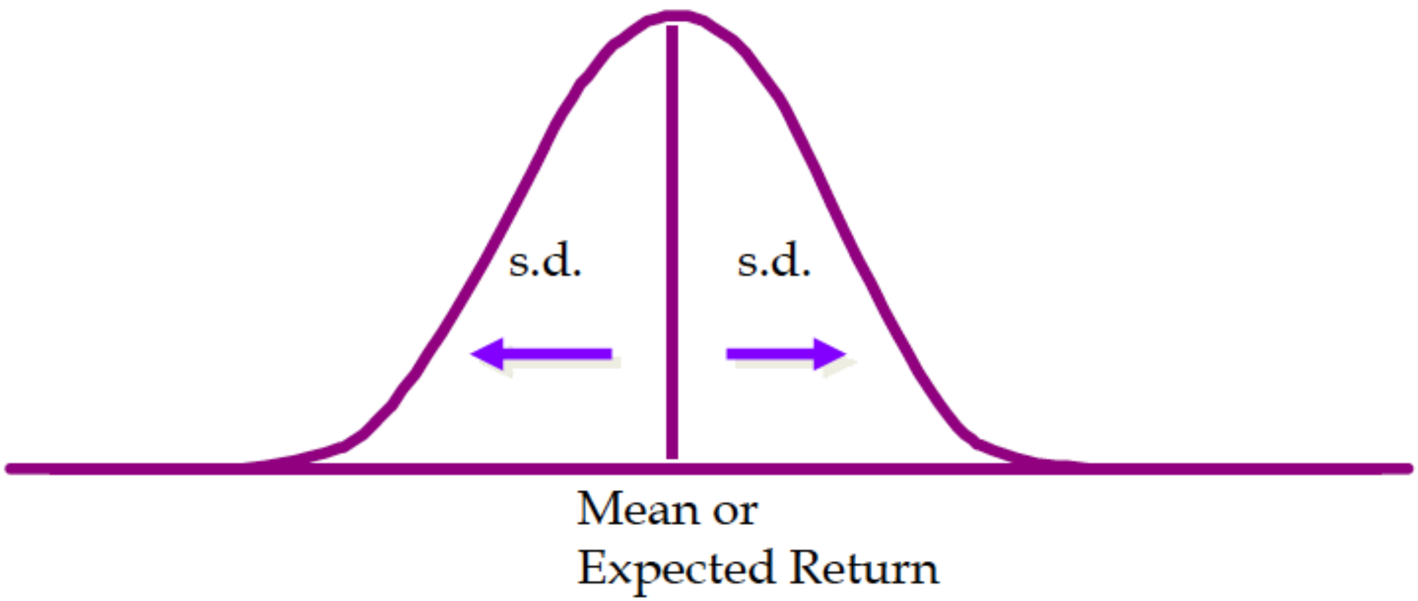
$$\text{Expected Return; } \mathrm{E}_t\left[r_{t+1}\right] = \frac{\mathrm{E}_t\left[P_{t+1}\right] + \mathrm{E}_t\left[D_{t+1}\right]}{P_t} - 1$$

$$\text{Realized Return; } \quad r_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1$$

- ‘Expected’ and ‘Realized’ returns are very different!!!

Descriptive statistics

- Mean : Expected return
- Variance (standard deviation) : dispersion
- Normal Distribution



The Normal Distribution

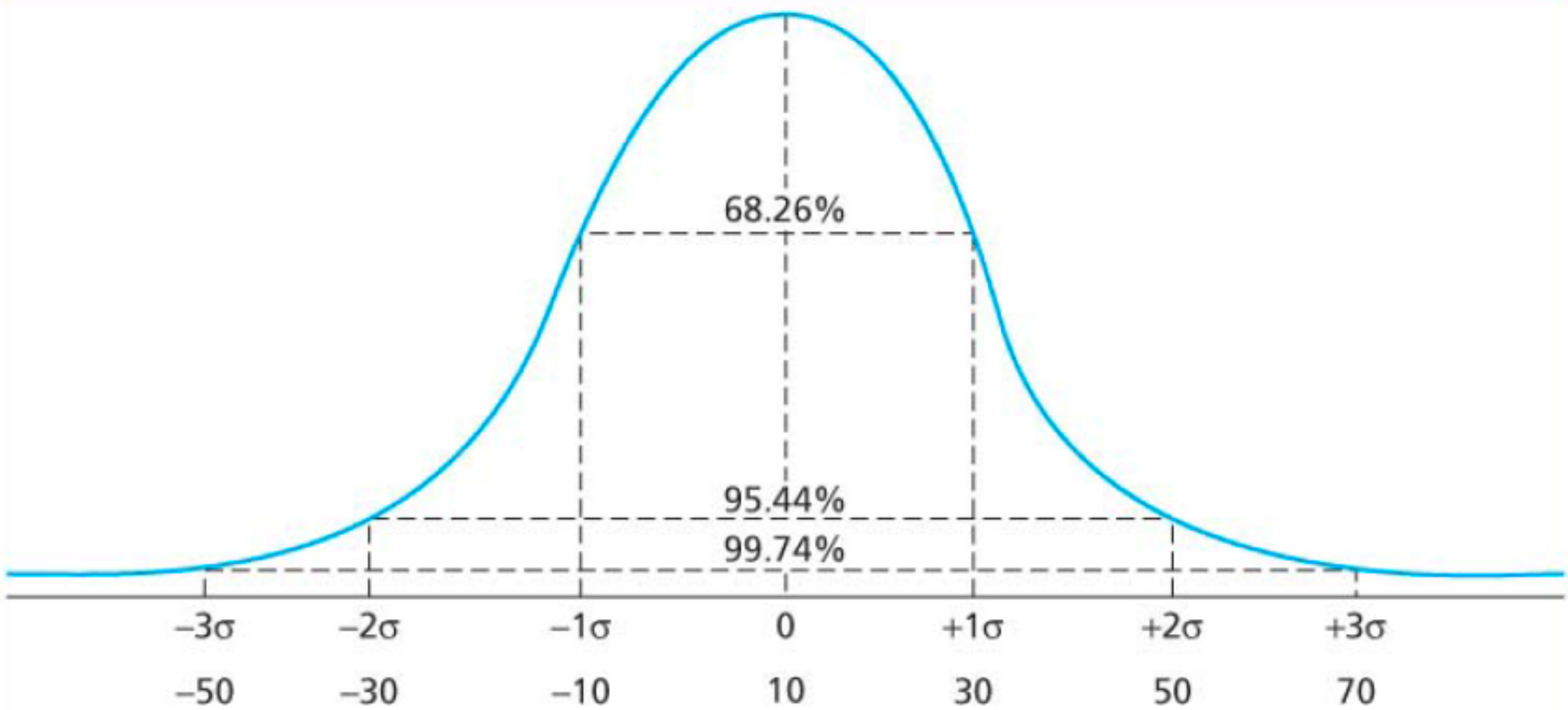
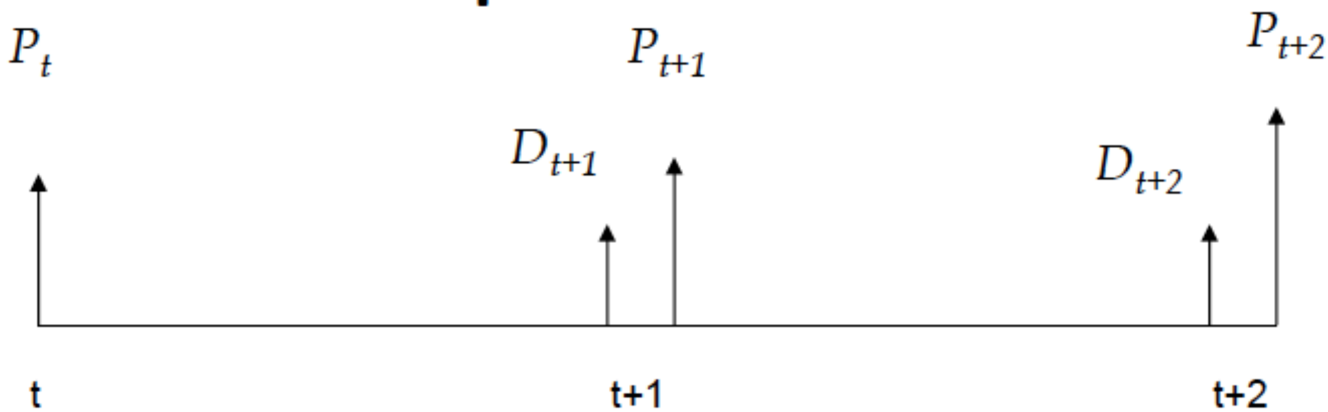


Figure 5.4 The normal distribution with mean 10% and standard deviation 20%.

Multi-period returns



- r_{t+1} : return from time t to t+1
- r_{t+2} : return from time t+1 to t+2
- $r_{t+2}(2)$: return from time t to t+2

$$(1+r_{t+2}(2)) = (1+r_{t+1})*(1+r_{t+2})$$

Compounding returns

- Example: you earn 10% in year 1 and 20% in year 2; your 2-year return is:

$$(1+\quad)(1+\quad)-1=1.\quad -1=\quad=\quad\%$$

  - Calculation assumes that we immediately reinvest the dividends
  - The 2-year return is *not the sum of* annual returns (30%)

Compounding returns (contd..)

- In the previous example, what is the *average annual* return? Put in another way, what is the return per annum that would give us the same amount in 2 years?
- We want a number such that

  $(1+r)*(1+r) = (1+r_{t+1})*(1+r_{t+2}) = 1.32$

  $r = \sqrt{[(1+r_{t+1})*(1+r_{t+2})]} - 1$

  $r = \sqrt{1.1*1.2} - 1 = 0.149 = 14.9\%$

  - Note $.32 = (1+.149)^2 - 1$
  - It is *not the arithmetic average* annual return [15%=(10%+20%)/2]
  - It is the *geometric average* of 10% and 20%

Arithmetic vs. Geometric Mean Return

Year	Re *turn*
1	0.30
2	−0.20
3	0.20
4	0.50

Arithmetic mean return

$$r_a = \frac{1}{T}\sum_{t=1}^{T} r_t = \frac{1}{4} \qquad = 0. \quad \Rightarrow \quad \%$$

Geometric mean return

$$(1+r_g)^T = (1+r_1)(1+r_2)\bullet\bullet\bullet\bullet(1+r_T) = (\quad)(\quad)(\quad)(\quad) = 1.87$$

$$1 + r_g = 1.87^{0.25} \Rightarrow r_g = 0.17 \Rightarrow 17\%$$

Arithmetic vs. Geometric Mean Return

S_0	S_1	S_2
100	200	100

r_1=100% r_2=-50%

r_a =   /2 =   =   %

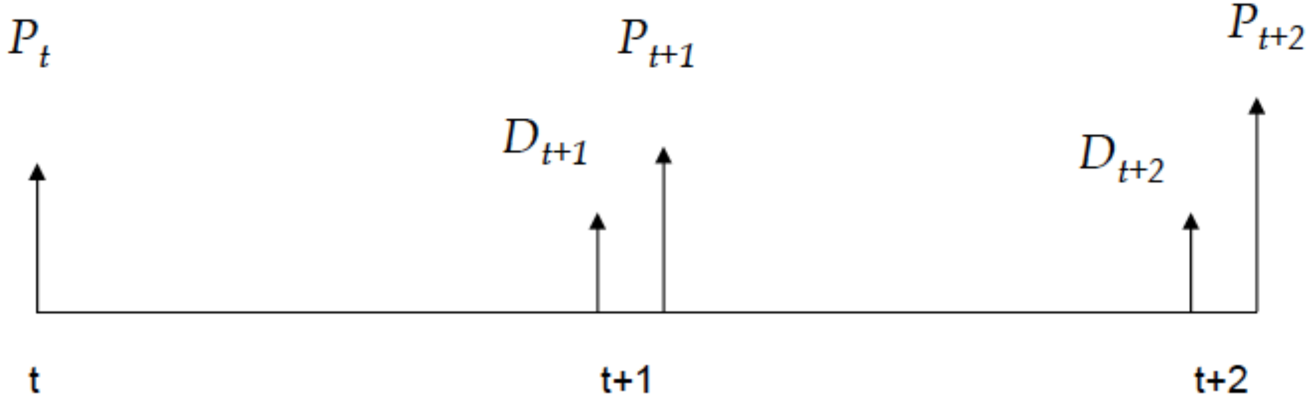
r_g = [(1+   )(1-   )]$^{0.5}$ – 1 =   – 1 =   %

Which one makes more sense?

Arithmetic vs. Geometric average

- Geometric average is always lower than the arithmetic average
  - Difference depends on the horizon and the variability of returns
- Which one is better?
  - Geometric average is an excellent measure of *past* performance
  - Arithmetic average is an unbiased estimate of the *expected future* return in one period

Continuously compounded returns



$$e^{r_{t+1}^c} = \frac{P_{t+1} + D_{t+1}}{P_t} \qquad e^{r_{t+2}^c} = \frac{P_{t+2} + D_{t+2}}{P_{t+1}}$$

$$r_{t+1}^c = \ln\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \qquad r_{t+2}^c = \ln\left(\frac{P_{t+2} + D_{t+2}}{P_{t+1}}\right)$$

Continuous compounding

- Continuously compounded returns make our lives easier
  - Two-period returns are just the sum of one period returns

$$e^{r_{t+2}^{c}(2)} = e^{r_{t+1}^{c}} e^{r_{t+2}^{c}} = e^{r_{t+1}^{c} + r_{t+2}^{c}}$$

$$r_{t+2}^{c}(2) = r_{t+1}^{c} + r_{t+2}^{c}$$

  - Average return obtained on a period

$$e^{r^{c} \times 2} = e^{r_{t+1}^{c}} e^{r_{t+2}^{c}} = e^{r_{t+1}^{c} + r_{t+2}^{c}}$$

$$r^{c} = \frac{r_{t+1}^{c} + r_{t+2}^{c}}{2}$$

Past and future

- We want to find the best possible portfolio to invest in.
- For that, we need to forecast returns in the future and evaluate the risk of the investments.
- The best guide for the future performance of assets is their past performance
  - But we need to be very careful extrapolating past performance to the future!!!

Historical averages

- We look at the historical returns of 5 portfolios
  - Treasury Bills: as safe as an investment can get
  - Long-term government bonds: interest risk
  - Long-term corporate bonds: interest risk + default risk
  - S&P Composite Index (S&P 500): 500 stocks of over 7000, but they account for over 70% of market
  - Small Stocks: smallest 20% of NYSE
- Nominal and Real Averages
- We take averages over long periods of time (83 y)
- Returns on these portfolios coincide with our intuitive risk ranking

Historical Averages

AVERAGE RATES OF RETURN (1926 - 2008 : 83 YEARS OF DATA)

PORTFOLIO	AVERAGE ANNUAL RATE OF RETURN NOMINAL	REAL	AVERAGE RISK PREMIUM (EXTRA RETURN VS. TBILLS)	
Treasury bills	3.8%	0.7%	0.0%	
Long-Term government bonds	6.1%	3.0%	2.3%	
Long-Term corporate bonds	6.2%	3.1%	2.4%	
Large company stocks	11.7%	8.6%	7.9%	← Equity risk premium
Small company stocks	16.4%	13.3%	12.6%	

Inflation = 3.1%

$r = r_f$ + risk premium

Interest Rates and Inflation, 1926-2009

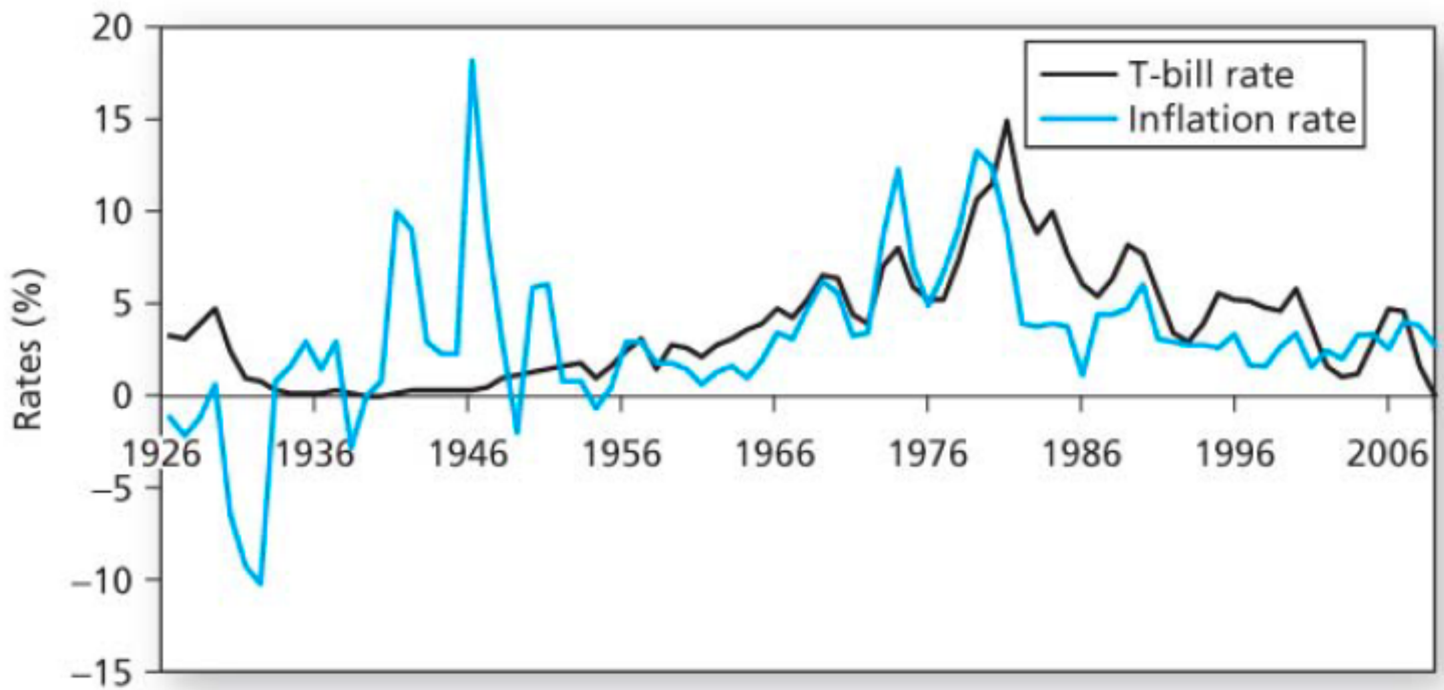
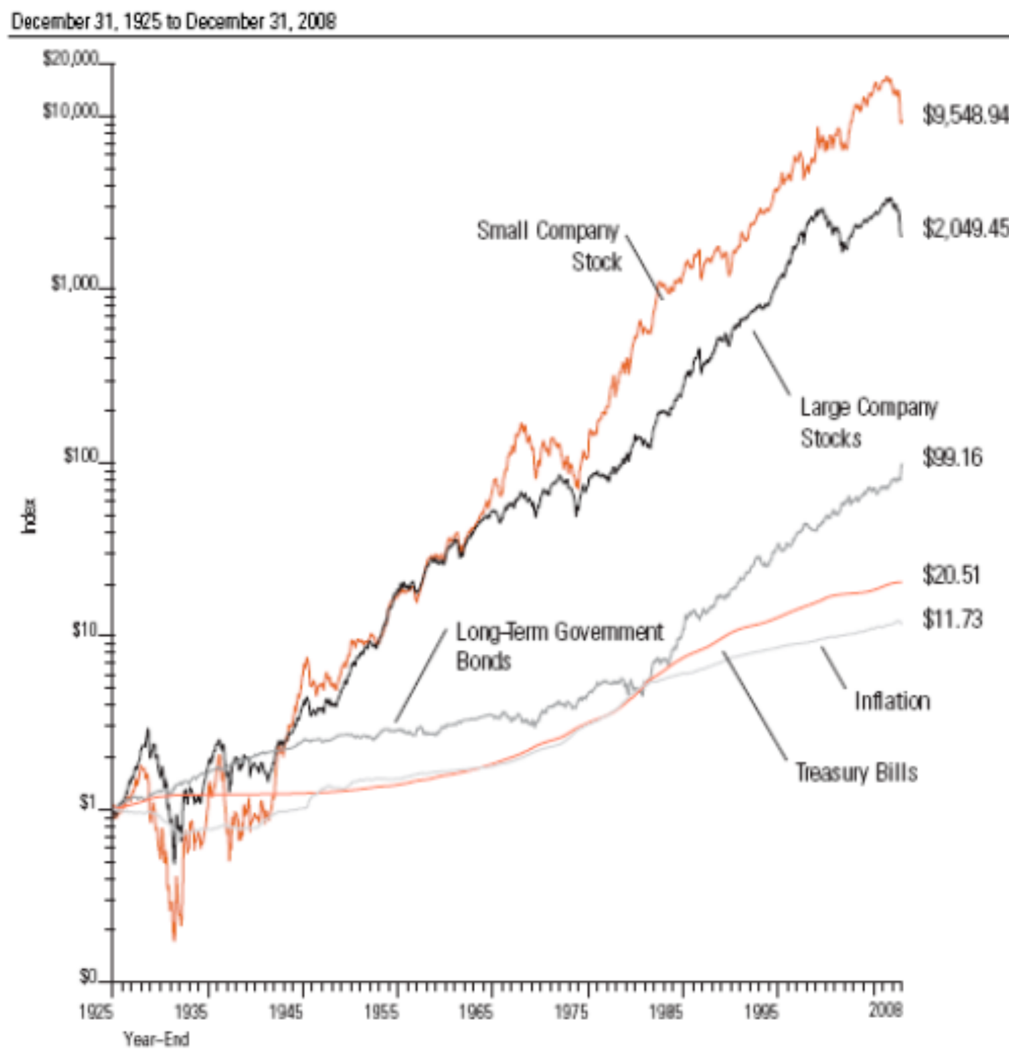


Figure 5.3 Interest and inflation rates, 1926–2009

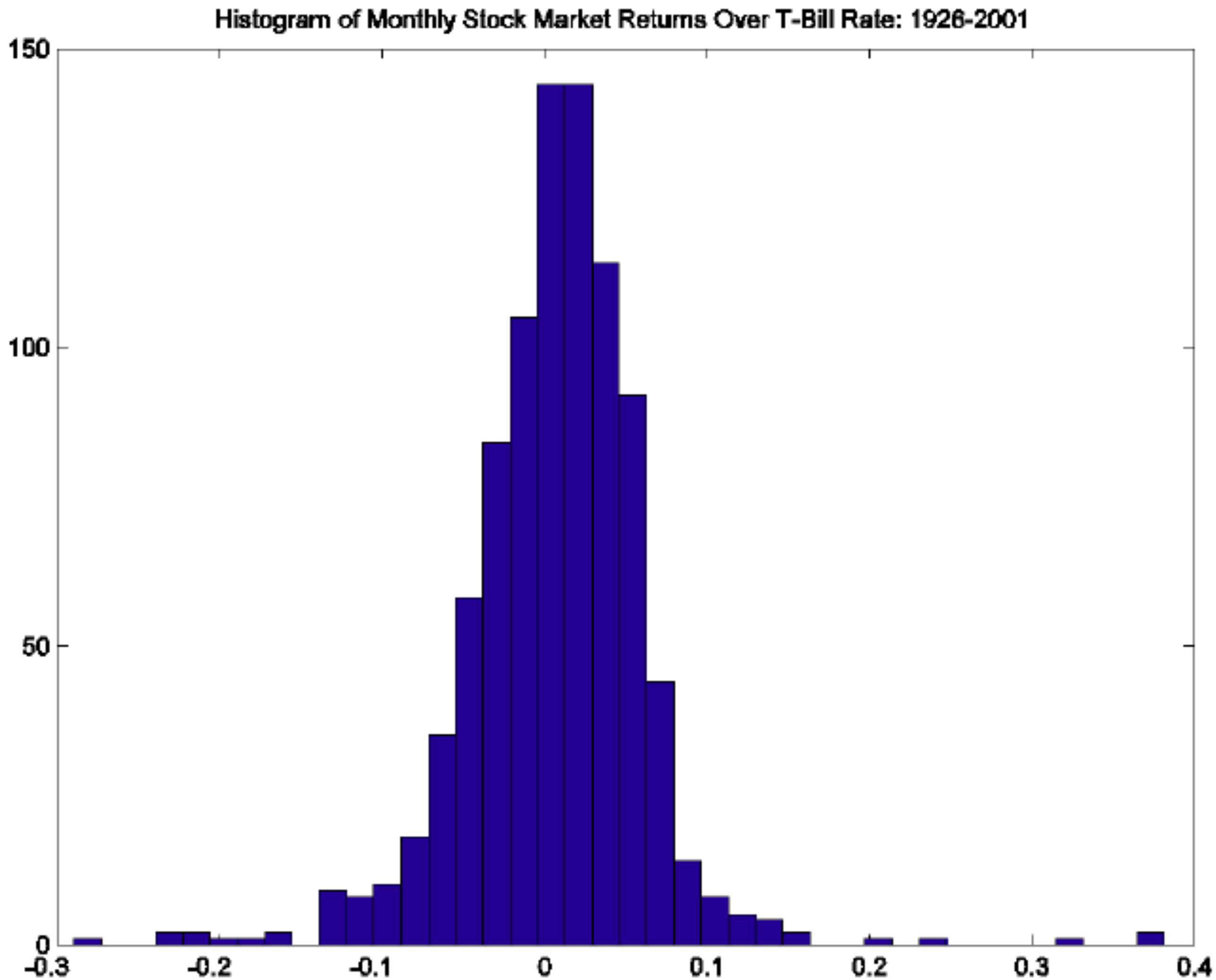
Cumulative returns – Ibbotson data



Equity premium

- Excess of stock returns over risk-free (T-bills) return
  - 7.9% over 1926-2008
  - Note that there is no need to adjust excess returns for inflation
- Given the superior performance of stocks over such long period, why does anyone hold bonds?

But Stocks are Riskier!



Risk

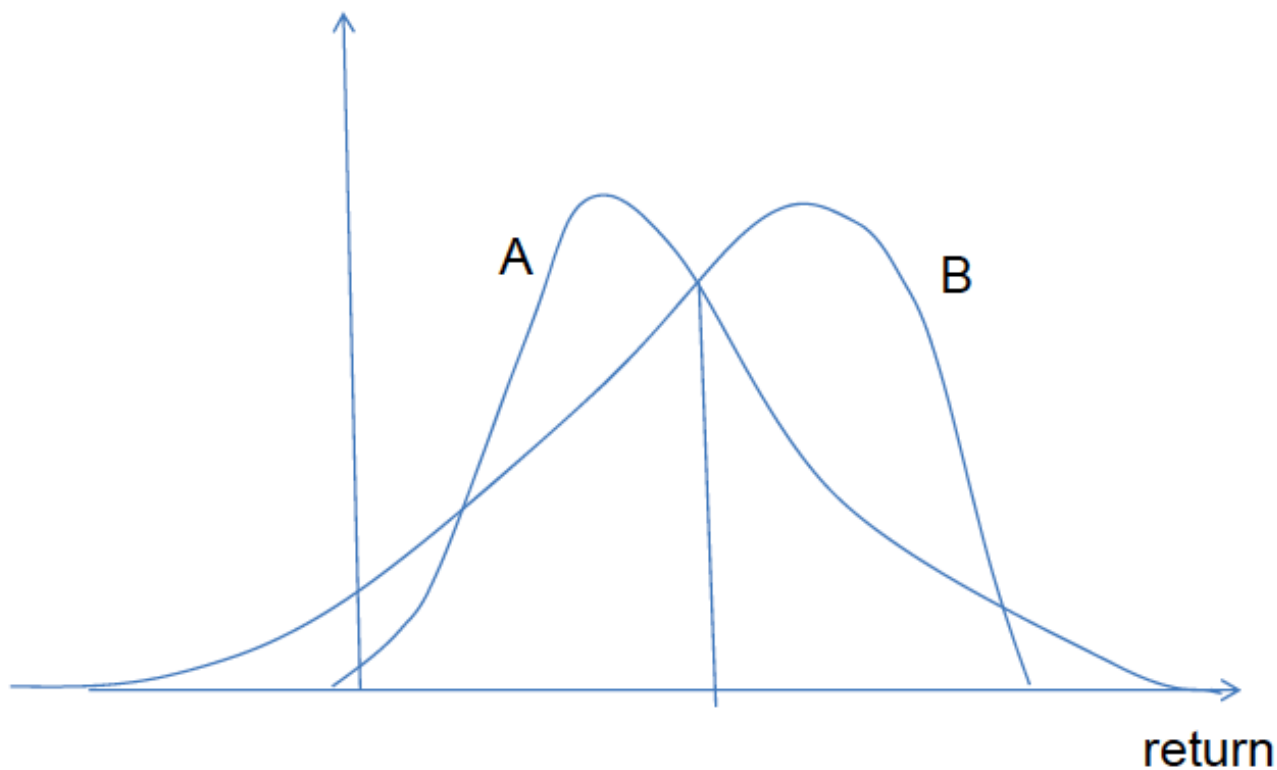
- Risk is uncertainty about the future
  - Probability Distributions
  - While stocks do better on average, investors know that in any one year, stocks may do much worse
- Summarize risk through standard deviation, σ, a measure of dispersion
  - Using historical data
  - Frequency Distributions (Histograms)

Standard Deviations

AVERAGE RATES OF RETURN (1926 - 2008 : 83 YEARS OF DATA)

PORTFOLIO	AVERAGE ANNUAL RATE OF RETURN Arithmetic mean	Geometric mean	Standard Deviation
Treasury bills	3.8%	3.7%	3.1%
Long-Term government bonds	6.1%	5.7%	9.4%
Long-Term corporate bonds	6.2%	5.9%	8.4%
Large company stocks	11.7%	9.6%	20.6%
Small company stocks	16.4%	11.7%	33.0%

If A and B have the same mean and standard deviation.
Which one do you prefer?



Skewness Preference: most people prefer right skeness

Normal and Skewed Distributions

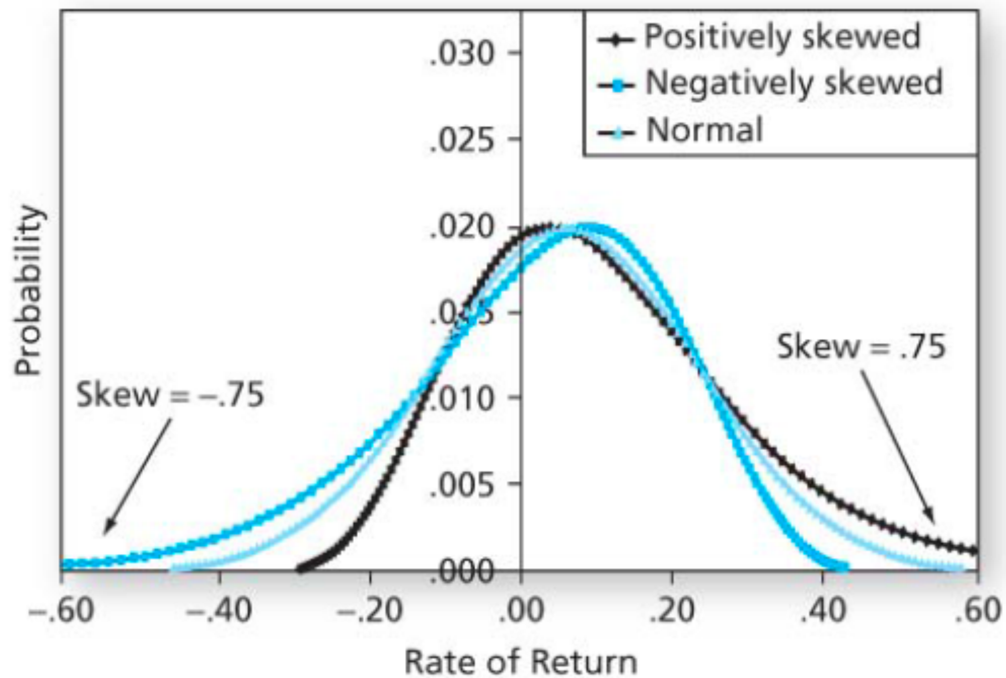


Figure 5.5A Normal and skewed distributions (mean = 6%, SD = 17%)

Normal and Fat-Tailed Distributions

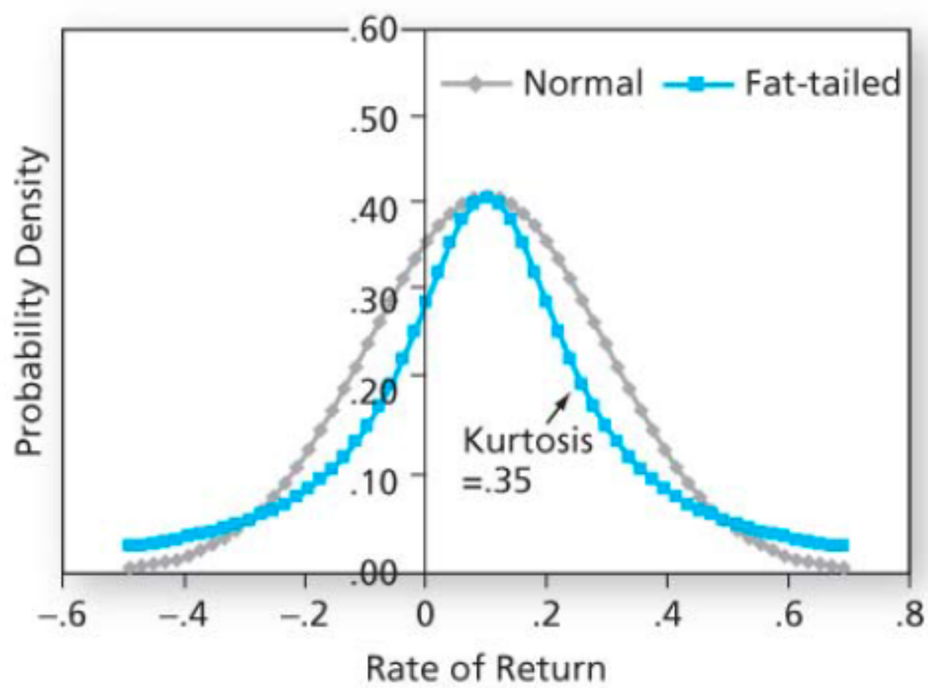


Figure 5.5B Normal and fat-tailed distributions (mean = .1, SD = .2)

Standard deviation

- If r_1, r_2, r_3, …, r_T, are yearly returns, first compute the sample variance of the returns

$$\mathrm{V}[\tilde{r}] = \hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2$$

  – Using the sample mean

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

  – Sample standard deviation is then

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

  – Functions *average*(), *var*(), and *stdev*() in Excel

Estimating means and variances

- We are implicitly assuming that the returns came from the same probability distribution in each year of the sample
- The estimated mean and variance are themselves random variables since there is estimation error that depends on the particular sample of data used (sampling error)
  - We can calculate the standard error of our estimates and figure out a confidence interval for them
  - This contrasts with the true (but unknown) mean and variance which are fixed numbers, not random variables

Computing historic standard deviation

Year	Return	
1	10%	$\bar{r} = 8\%$
2	30%	$\sigma^2 = 370$
3	-20%	$\sigma = 19.2\%$
4	0%	
5	20%	

$\sigma^2 = [(\quad)^2+(\quad)^2+(\quad)^2+(\quad)^2+(\quad)^2]/4$

$= [4+484+784+64+144]/4$

$= [1480]/4$

In general it is better to do the calculation in decimals (.10) instead of in percentages (10%)

Risk and standard deviation

- We use historic estimates as approximations for true *ex ante* means and variances, since we cannot see into the future
  - Is using past data to forecast the future the best method?
  - For variances is not bad, but for means…..
- Is risk just standard deviation?
  - What about skewness, fat tails?

Risk aversion

- People are *risk averse*
  - They prefer a *sure* outcome of \$100 versus a *lottery* of \$200 (heads) and \$0 (tails)
  - Demand compensation for risk
- Stocks offer higher average returns than bonds because:
  - To be willing to hold risky security, investors must receive higher expected return as reward
- However, studies have shown that we would need unreasonably high risk aversion to justify an equity premium of 6% to 8%, given the risk of stocks

Equity premium puzzle

- The equity premium is one of the most important numbers in your life!!!
  - At what age can you retire? How much money will you have?
  - How much should you save now?
- The consequences of an equity premium of 6% to 8% are huge!
  - Borrow and invest in equities
  - Only invest in projects with high expected returns (the opportunity cost is high)
- However, historical (realized) returns are not necessarily an indication of future (expected) returns

Some pause for consideration

- We are very uncertain about the size of the premium
  - Standard error on mean estimate is 2.2%
  - Two-standard error confidence interval is [3.5%,12.3%]
- Survivorship bias
  - Only examined U.S. market, which is perhaps the best performing and surviving market over the last century
  - When examining the global market, including emerging equities, the premium is smaller
- Other risks besides historical volatility
  - Catastrophe risk

Nominal and Real Equity Returns Around the World, 1900-2000

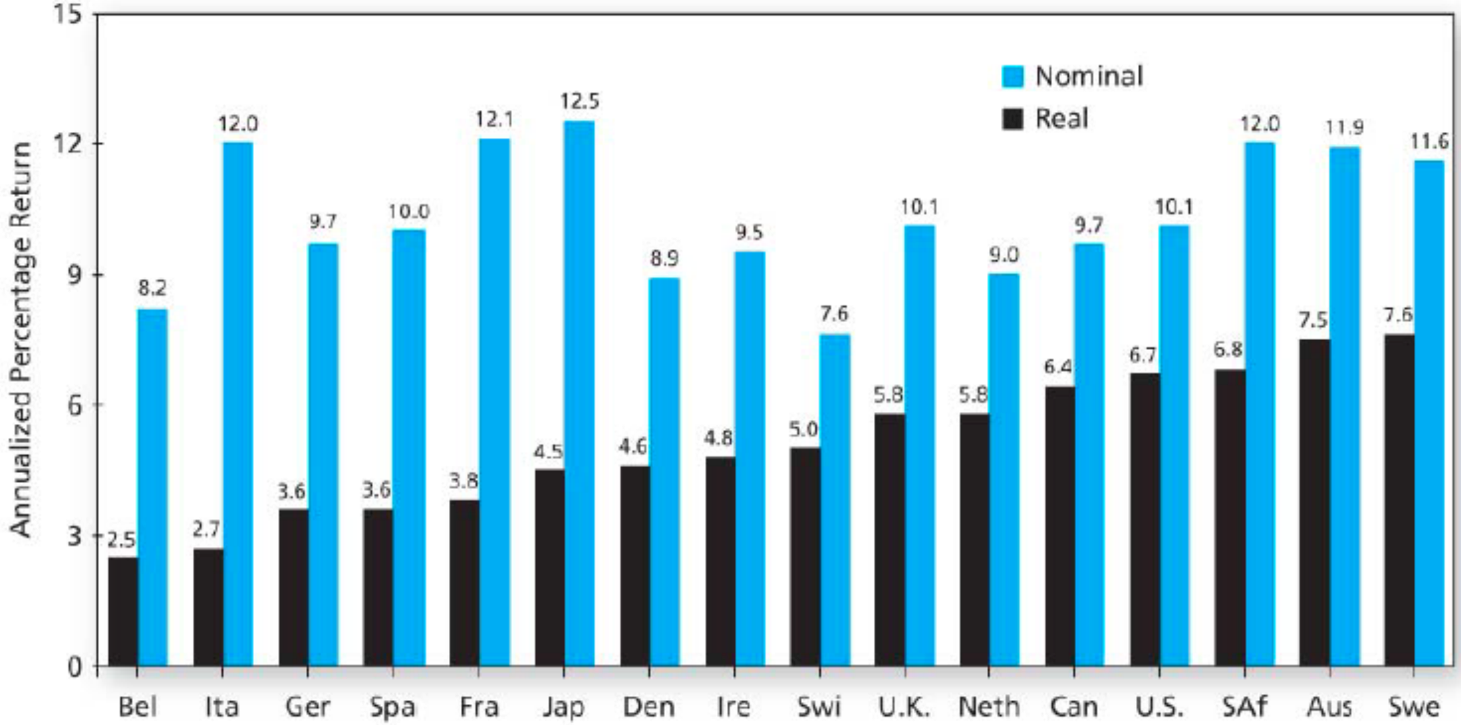


Figure 5.7 Nominal and real equity returns around the world, 1900–2000

Source: Elroy Dimson, Paul Marsh, and Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton: Princeton University Press, 2002), p. 50. Reprinted by permission of the Princeton University Press.

Standard Deviations of Real Equity and Bond Returns Around the World, 1900-2000

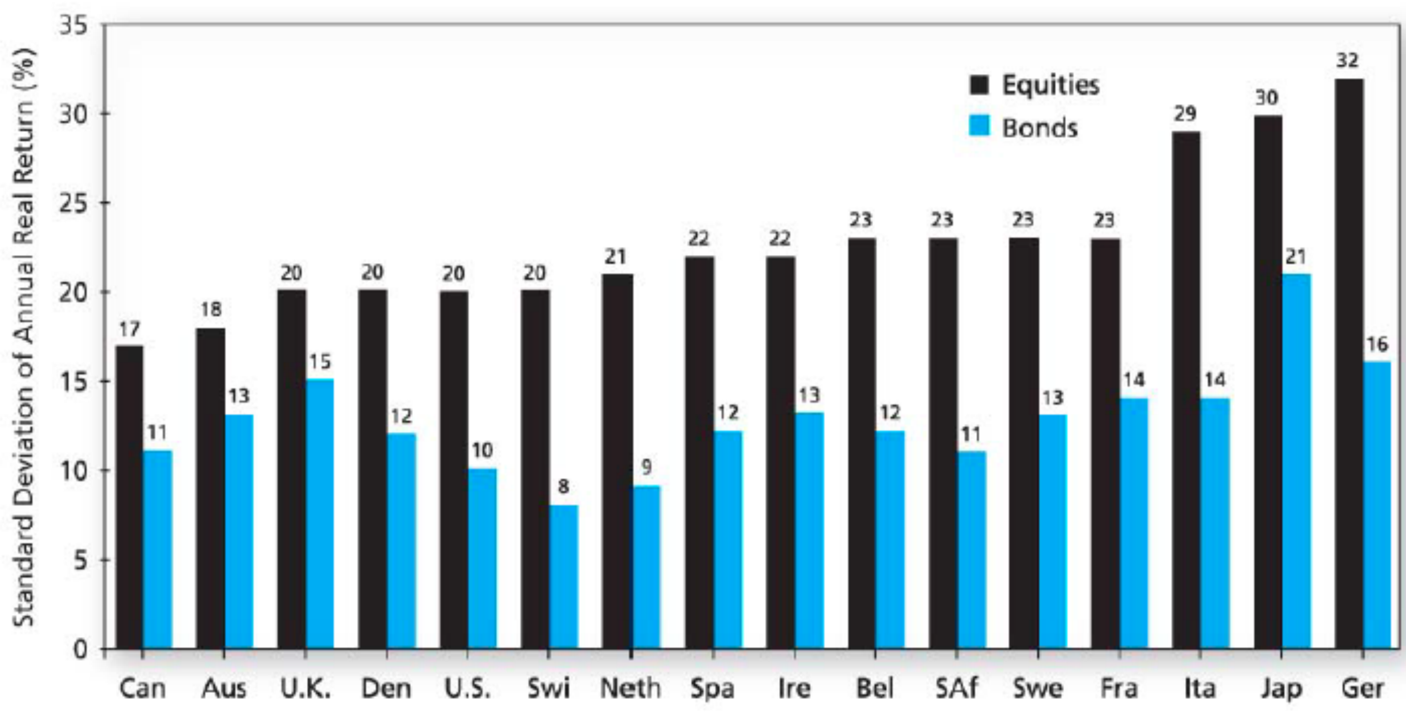


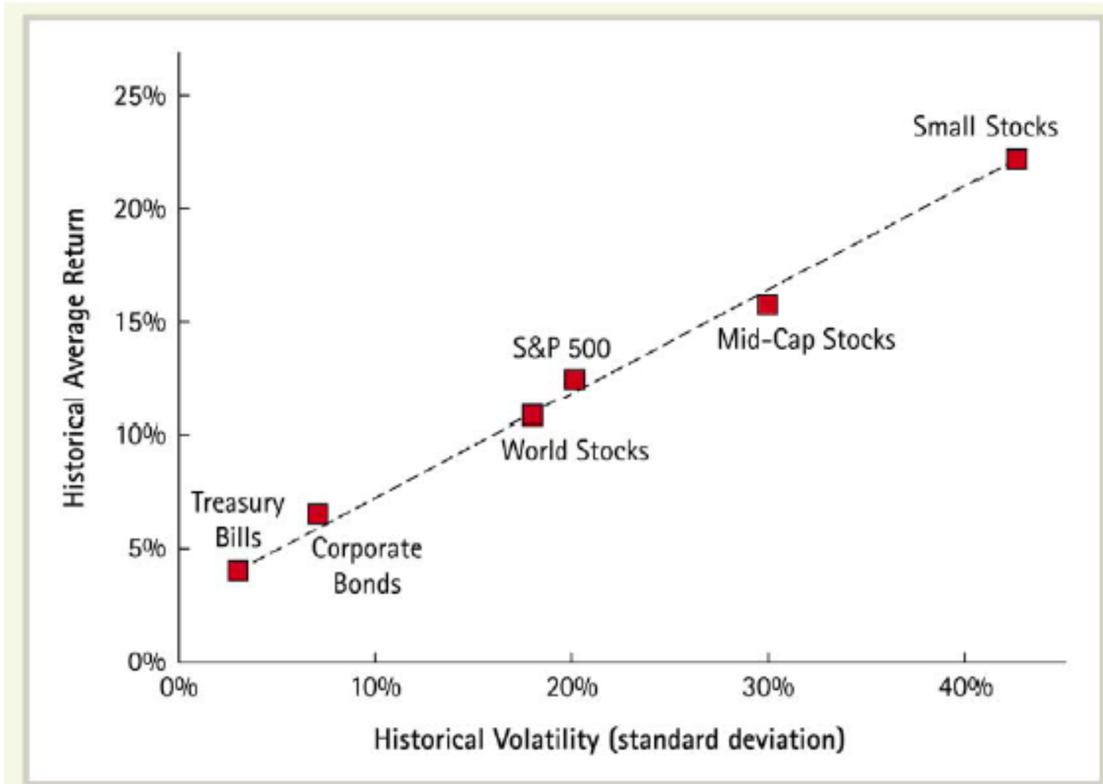
Figure 5.8 Standard deviations of real equity and bond returns around the world, 1900–2000

Source: Elroy Dimson, Paul Marsh, and Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton: Princeton University Press, 2002), p. 61. Reprinted by permission of the Princeton University Press.

Points to remember

- How to compound returns?
- Difference between
  - Income yield and capital gain/loss
  - Expected vs. realized returns
  - Geometric vs. arithmetic averages
- Historical behavior of returns
  - The equity premium puzzle
- Risk is uncertainty about future
  - Usually measured by standard deviation

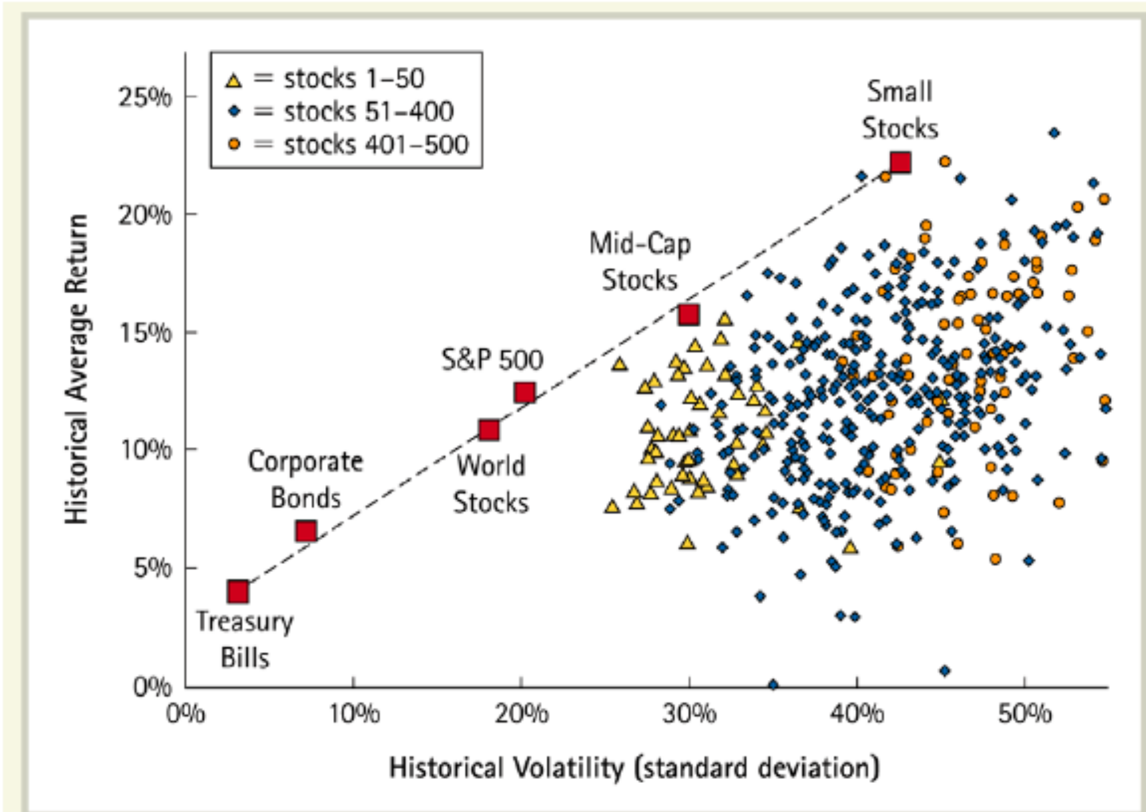
Figure 10.5 The Historical Tradeoff Between Risk and Return in Large Portfolios, 1926–2004



Also included are a mid-cap portfolio composed of the 10% of U.S. stocks whose size is just below the median of all U.S. stocks, and a world portfolio of large stocks from North America, Europe, and Asia. Note the general increasing relationship between historical volatility and average return for these large portfolios.

Source: CRSP, Morgan Stanley Capital International and Global Financial Data.

Figure 10.6 Historical Volatility and Return for 500 Individual Stocks, by Size, Updated Quarterly, 1926–2004



Unlike the case for large portfolios, there is no precise relationship between volatility and average return for individual stocks. Individual stocks have higher volatility and lower average returns than the relationship shown for large portfolios.

Alternative Example

- **Problem:**
  - What were the realized annual returns for NRG stock in 2008 and in 2012?

Alternative Example (cont'd)

- **Solution**
  - First, we look up stock price data for NRG at the start and end of the year, as well as dividend dates. From these data, we construct the following table:

Date	Price ($)	Dividend ($)	Return	Date	Price ($)	Dividend ($)	Return
12/31/2007	58.69			12/31/2011	6.73	0	
1/31/2008	61.44	0.26	5.13%	3/31/2012	5.72	0	-15.01%
4/30/2008	63.94	0.26	4.49%	6/30/2012	4.81	0	-15.91%
7/31/2008	48.5	0.26	-23.74%	9/30/2012	5.2	0	8.11%
10/31/2008	54.88	0.29	13.75%	12/21/2012	2.29	0	-55.96%
12/31/2008	53.31		-2.86%				

Alternative Example (cont'd)

- **Solution**
  - We compute each period's return using Equation 10.4. For example, the return from December 31, 2007 to January 31, 2008 is: $\frac{61.44+0.26}{58.69}-1=5.13\%$
  - We then determine annual returns using Eq. 10.5:

$$R_{2008}=(1.0513)(1.0449)(0.7626)(1.1375)(0.9714)-1=-7.43\%$$

$$R_{2012}=(0.8499)(0.8409)(1.0811)(0.440)-1=-66.0\%$$

Alternative Example (cont’d)

- **Solution**
  - Note that, since NRG did not pay dividends during 2012, the return can also be computed as:

$$\frac{2.29}{6.73} - 1 = -66.0\%$$

References

- Berk and DeMarzo, Corporate Finance, 3th edition, 2013.