**Comparison of exponential function and logarithmic function**

<table>
<tr><th>Exponential Function</th><th>Logarithmic Function</th></tr>
<tr><td>$y = f(x) = b^x$</td><td>$y = \log_b x \iff x = b^y$<br>So please <u>accept</u> $y = \log_b x$ as another way to say $x = b^y$</td></tr>
<tr><td colspan="2">$b > 0, b \neq 1$<br>b is called base</td></tr>
<tr><td>e.g. $y = 2^x$<br>$\therefore x = 1, y = 2$</td><td>e.g. $y = \log_2 x \iff x = 2^y$<br>$\therefore x = 1, y = 0$</td></tr>
<tr><td colspan="2">Note The two functions give different values of y from the same input, x. This is simply because they are two different functions.</td></tr>
<tr><td>Consider again $y = 2^x$<br>$\therefore x = 1 \Rightarrow y = 2$</td><td>Consider $y = \log_2 x \iff x = 2^y$<br>$\therefore x = 2 \Rightarrow y = 1$</td></tr>
<tr><td colspan="2">Note: The input x of exponential function is equal to y of logarithmic function when y of exponential function is equal to x of logarithmic function. (See also the table below)<br>We said the two functions are inverse functions of each other.</td></tr>
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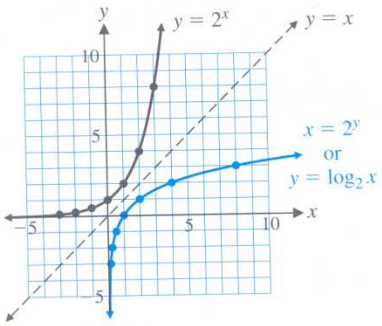


FIGURE 2

| Exponential Function | | Logarithmic Function | |
|---|---|---|---|
| $x$ | $y = 2^x$ | $x = 2^y$ | $y$ |
| $-3$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $-3$ |
| $-2$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $-2$ |
| $-1$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-1$ |
| 0 | 1 | 1 | 0 |
| 1 | 2 | 2 | 1 |
| 2 | 4 | 4 | 2 |
| 3 | 8 | 8 | 3 |

Ordered pairs reversed

Consider the graphs, the graph of $y = \log_2 x$ is a mirror image of the graph of $y = 2^x$ using the line *y* = *x* as an axis, i.e., trying to fold the graph paper on the line *y* = *x*, then the two graphs lie on top of each other.

**Comparison of exponential function and logarithmic function (Cont.)**

| Exponential Function | Logarithmic Function |
|---|---|
| Domain of exponential function is a set of real numbers. | Domain of logarithmic function is x > 0.<br>Because $x = b^y$ can only be positive numbers as b>0 and b is not equal to 1. |
| Range is *y* > 0 because when b>0 and b is not equal to 1, $b^x$ can only be positive numbers. | Range is a set of real numbers as when we consider $x = b^y$, *y* can be any real numbers. |
| **Note:**<br>Domain of exponential function = Range of ogarithmic function<br>Range of exponential function = Domain of logarithmic function<br>Another sign to show that the two functions are inverse functions of each other. | |
| When base, *b*, is equal to *e* (=2.718 281 828 459 …)<br>The exponential function is called natural exponential function. | When base, *b*, is equal to *e* (=2.718 281 828 459 …)<br>The logarithmic function is called natural logarithmic function or $y = \log_e x = \ln x$ |

**How to solve an equation involving exponential functions**

1. Learn exponential properties given in 5.1.3 and logarithmic properties given in 5.2.2.
2. Make bases match.

   $b^x = b^z \Leftrightarrow x = z$

   Explanation

   Consider the function $y = b^x$, one value of *x* only gives one value of *y*. Hence, for $b^x = b^z$, then *x* must be equal to *z*.

   For similar reason, $\log_b x = \log_b z \Leftrightarrow x = z$
3. No matter what, still try to make bases match. This is when exponential and logarithmic properties become useful!!!!

   Ex. 1 $4^y = 2^z$, we can rewrite $4 = 2^2$

   $\therefore 2^{2x} = 2^z \rightarrow 2x = z \rightarrow x = \dfrac{z}{2}$

Ex. 2 $\log_2 x = \log_4 y$

Using logarithmic property given in 5.2.2 that $\log_b x = \dfrac{\ln x}{\ln b}$

$$\log_2 x = \frac{\ln y}{\ln 2^2}$$

Using logarithmic property given in 5.2.2 that $\log_b u^r = r\log_b u$ and $\log_b x = \dfrac{\ln x}{\ln b}$

$$\begin{aligned}\log_2 x &= \frac{\ln y}{2\ln 2} \\ &= \frac{1}{2} \times \frac{\ln y}{\ln 2} \\ &= \frac{1}{2}\log_2 y \\ &= \log_2 y^{1/2}\end{aligned}$$

Hence, $x = y^{1/2}$