Introductory to mathematical economics

EE320

Semester 2/2015

Kittichai Saelee

Agendas

- Logistics about this course
- What is this course all about?
- Structure of economy analysis
- Element of mathematical economics model

Logistics

- Instructor:
  - Kittichai Saelee (Golf)
    - (No, it's not "Chalee" as shown on the website!)
  - Email: kittichai_lee[at]econ.tu.ac.th
  - Office: rm#516. (Tue/Thu: 1.30-2.30 pm, or by appointment)
- Teaching assistant/Tutor:
  - Sahaphon (kay) Vairungroj

Logistics

- Lecture:
  - rm# 303.
  - Class will be starting at 11.10 am and finished at 12.30 pm**. (Don’t come late.)**
  - Teaching will be lecture-based.
  - I don’t check for your class attendances; however, I encourage everyone to attend my class.

Logistics

- Rules
  - Please don't come late. It's so distracting when I see people keep coming after I've started the class.
  - No talk, No Cellphone → penalty is a pop-up quiz.

Logistics

- Evaluations
  - Quiz (10%)
    - 6 times. Dropping the lowest one out. (Each counts 2%)
  - Homework (10%)
    - 6 assignments.
    - You have to attempt all, but grading will be based only one/two question(s) randomly selected.
  - Mid-term (30%): March, 8th (2 hours: 11 am – 1 pm)
  - Final (50%): (3 hours: May, 30th: 9.00 am – 12.00 pm)

Logistics

- We have two sections in this semester.
  - 50% of the question in the two exams (final/midterm) will be shared for both sectors.
  - Grading is based on our class's distribution. (nothing to do with the distribution of the other class.)

Logistics

- Materials:
  - Chiang and Wainwright (henceforth CW)
    - This is very a classic one. Any editions fine! Well-written.
  - My own lecture notes.
  - Lecture slide/note will be uploaded on BE Moodle.
    - Passcode is 1174. Please do not share this code with others who do not enroll in this class.
    - Any announcements thereafter will be made via Moodle as well.

Logistics

- To do well in this course,
  - Study hard "*regularly*".
  - I suggest that you form up your own study group, and help one another.
  - Do the practice problem set/assignment.
- Don't piggy-back your friend. Free-riding doesn't help you in the long-term. It only works to get you 10% for the homework, but the rest of 90% is on your own.

Logistics

- Your TA will be holding his discussion sections **every** Monday between 5 pm and 6 pm, in rm#303.
  - Go over some practice questions in further details/homework problems.
  - Cover some topics that I didn't get into all the details during the lecture.
  - Available to answer you any specific questions related to this course.
  - In some weeks, you have to do the quiz.

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What is this course all about?

- This course is mainly about the **application** of mathematics for economics analysis.
  - That is, we show students how to represent and analyze some of the economic theories that you've learned in ee211/ee212 in the mathematical form.
  - MA216 tools + some new tools.

Why using math in economic analysis

- Sound difficult, so why bother using mathematics in the economic analysis.
  - Limitations of the *narrative and graphical* approach in addressing the complex analysis that requires understanding the relationship of variables in multi-dimensional array.
  - Mathematical model is an important ground for taking the model to test with the data: a foundation for econometric model that links between theory and data.

Why using math in economic analysis

- In terms of economic theory, we are **not** going to introduce any new theoretical concepts: all these should be already seen in 212/213.
- In terms of mathematics, we will focus on three important tools:
  - Function/relation
  - Method of solving system of equations (linear and non-linear)
  - Univariate and multivariate calculus
  - Optimization theory

What is this course all about?

- Function/Relation in MA216:
  - $y = f(x)$

- Function in EE320 is to put “y” and “x” into economic context.
  - Describing relationship between y and x, using equation.

What is this course all about?

- Method of solving system of equations

$$
\begin{aligned}
a_{11}x_1 + \ a_{12}x_2 + \cdots + \ a_{1n}x_n &= b_1 \\
a_{21}x_1 + \ a_{22}x_2 + \cdots + \ a_{2n}x_n &= b_2 \\
\vdots \qquad\quad & \vdots \qquad\qquad\quad \vdots \qquad\quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m .
\end{aligned}
$$

  - Most economic concepts can be formulated as system of equations, which equilibrium outcome of the model can be obtained by solving for the solution of such a system of equations.

What is this course all about?

- Univariate calculus
  - Single variable: $\mathrm{y} = f(x), \frac{dy}{dx} = f'(x),$
    $higher\ order\ f''(x), \ldots . f^n(x)$

- Multivariate calculus
  - Multiple variable: $\mathrm{y} = f(x_1, x_2, \ldots, x_n)$
  - Various concepts of finding the derivative for multivariable case: partial derivative, total differential, etc.

What is this course all about?

- **Optimization theory** is the central toolkit for mathematical economics model.
- Application of calculus to solve for the extreme point problem.
  - Choosing for "x" that maximize or minimize a function.
- Suppose that we interpret this problem in economic context, what can they be?
  - Most economics problems are about the extreme point problem!!
  - The concept of optimization problem matches perfectly well with the idea of economic decision making, in which agent chooses for any actions that best generate the highest returns/benefits, given the constraints facing.
  - New tools given is the optimization under constraints.

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Structure of economic analysis

- How do we study economics?
  - Model-based analysis
- Model is an abstract form of the problem that we are interested in.
  - The problem that we are interested in aims at getting us some insights, helping us understand something about how the world/economy works.
  - Typically, all the results in a certain model hinges on the assumptions used in the analysis.

Structure of economic analysis

- Models developed to answer three broad classes of analytical questions that economists conceptually ask:
  - Determine the equilibrium ➔ static equilibrium analysis
  - If something happens, make predictions about the new equilibrium ➔ comparative static equilibrium
  - How do we move from old to new equilibrium ➔ dynamic analysis

Structure of economic analysis

- Micro: Price determination model
  - Motivation: how is the price determined in the market? At what price and quantity of output, market will trade?
  - To understand this issue, we need to conceptualize behavior of the two parties in trading arrangements: buyers and sellers

Structure of economic analysis

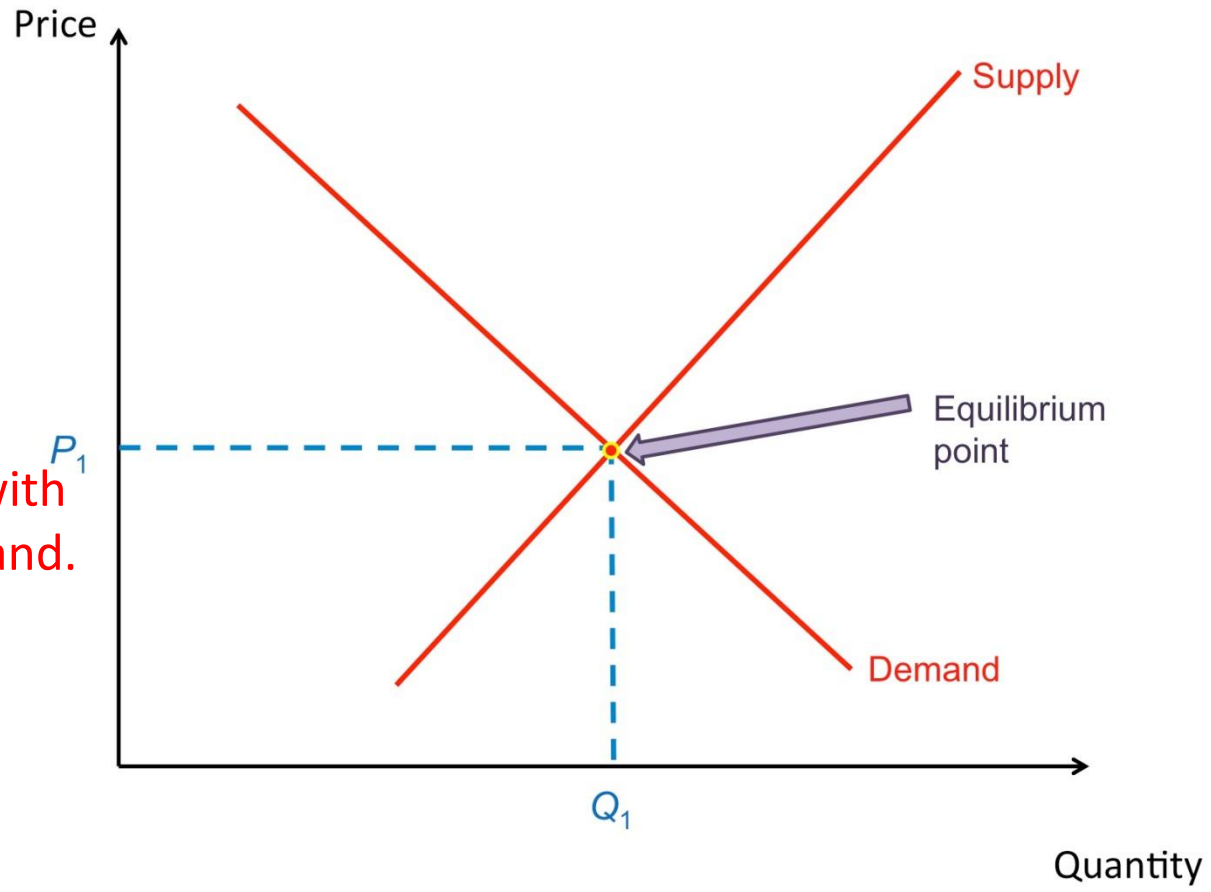
- How do we do that?
  - Conceptual framework for understanding the behavior of these two parties are often summarized in terms of DEMAND and SUPPLY.
  - Conventionally, behavioral assumptions are that
    - Demand: Given all other factors, quantity demanded is negatively related to price
    - Supply: Given all other factors, quantity supplied is positively related to price.

Structure of economic analysis: Static equilibrium analysis

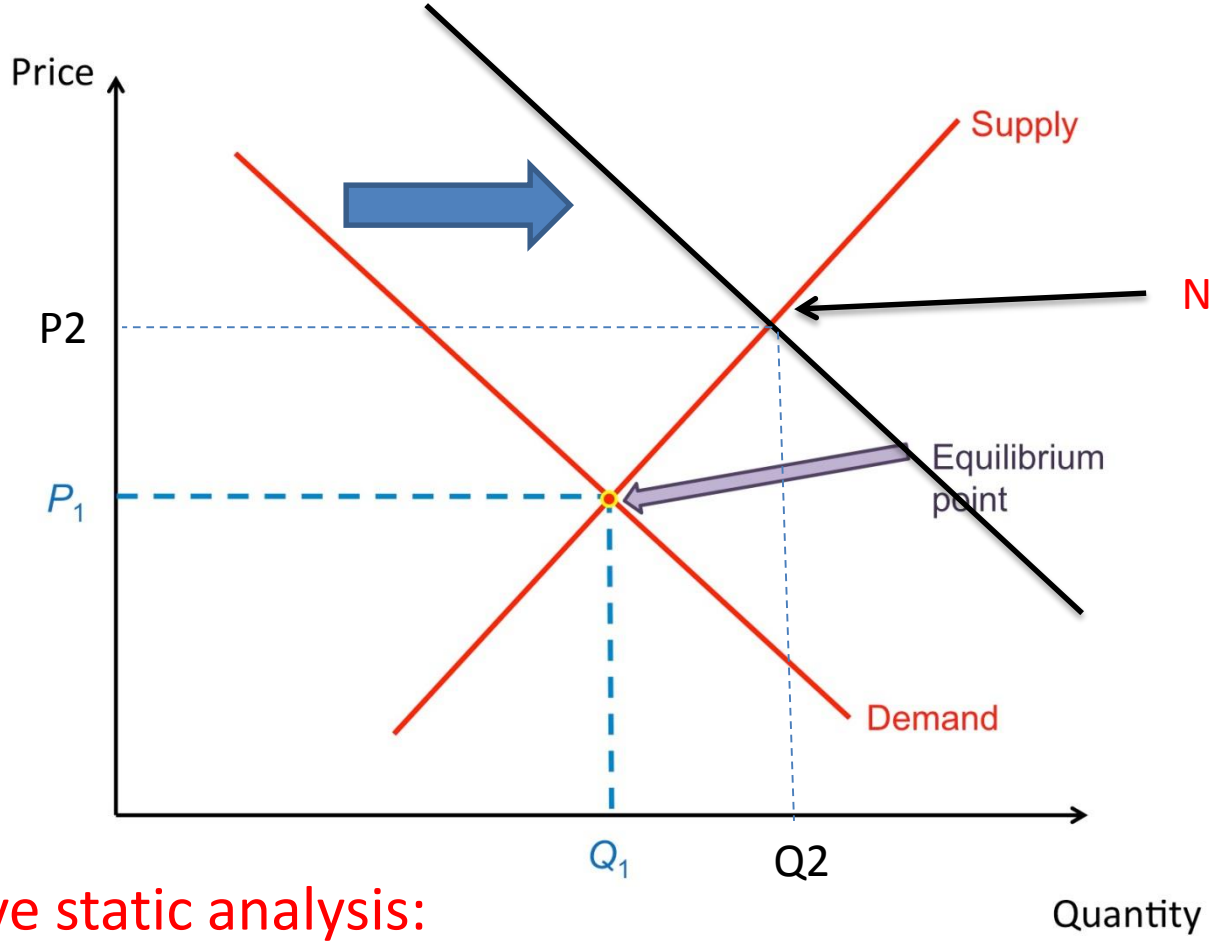
Static analysis:

- Draw demand/supply onto p-q diagram

- Equilibrium occurs when price clears the market. That is, demand matches with supply, aka no excess demand.

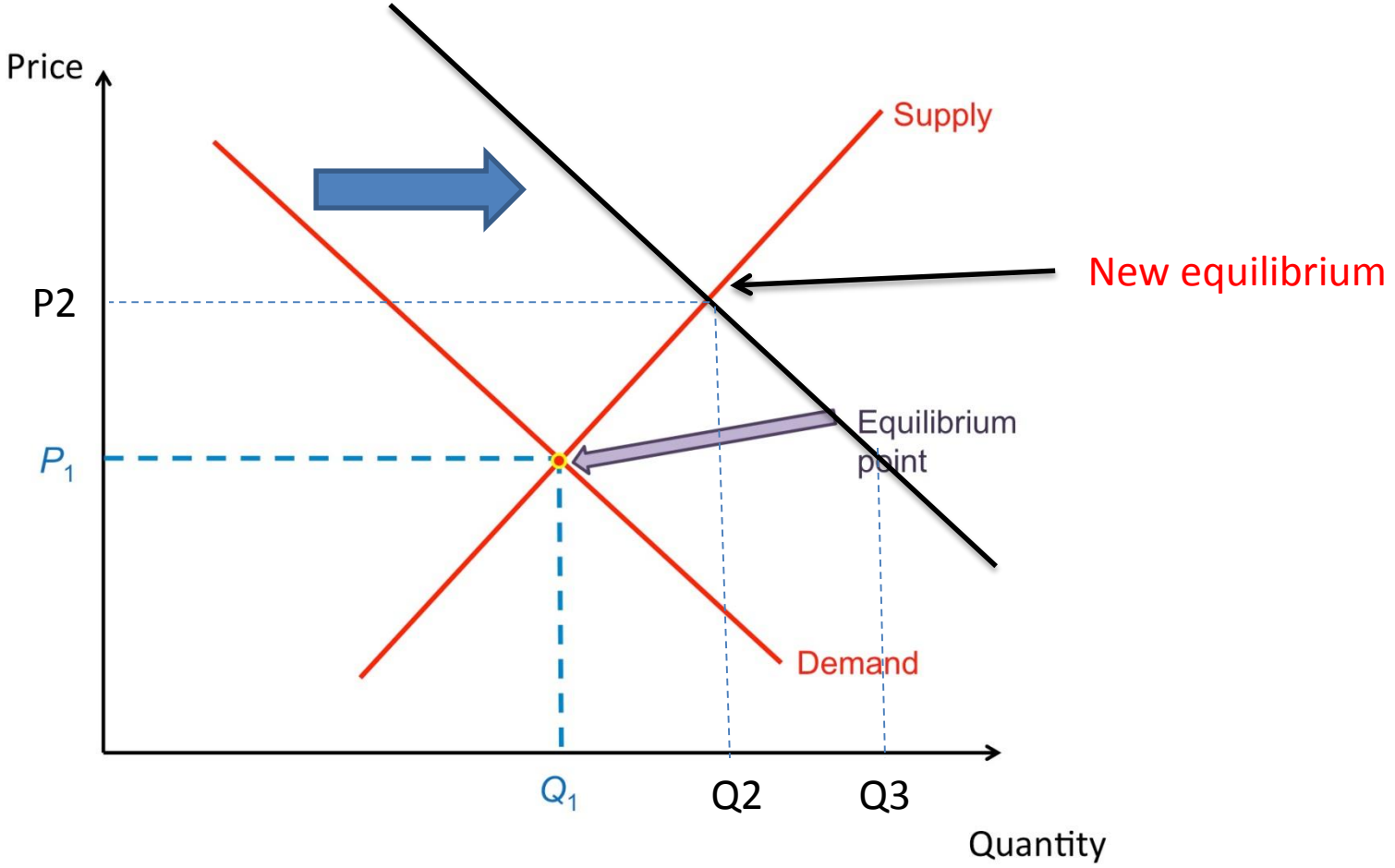


Structure of economic analysis: Comparative static analysis

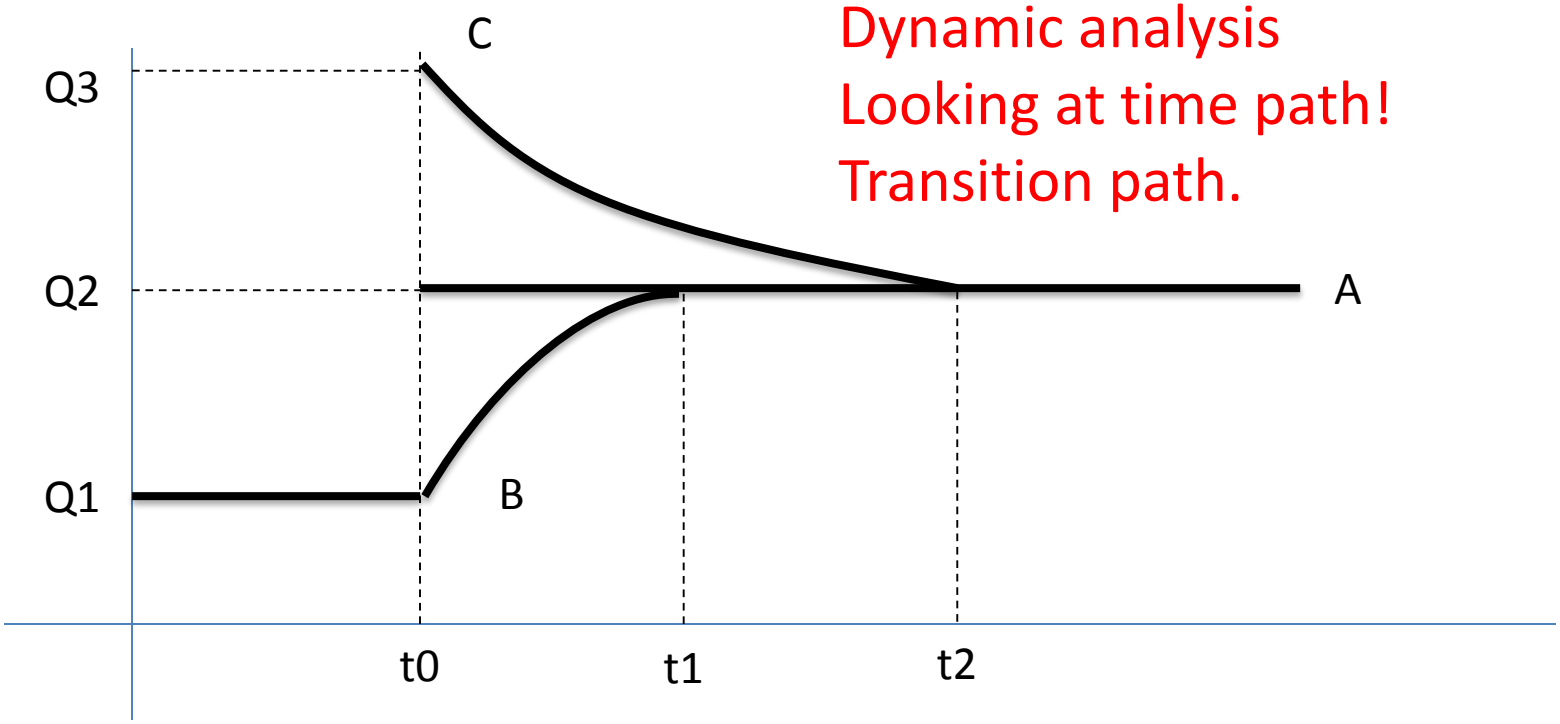


Comparative static analysis:
For example, what if income increases?

Structure of economic analysis:
Dynamic analysis



Structure of economic analysis



Dynamic analysis
Looking at time path!
Transition path.

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Mathematical economics model

- Variable:
  - Quantity or amount of some objects whose value can be changed.
  - For example, Y is a variable that represents “your income in a day”. X is another variable that represents how many hours you have worked in a day.

Mathematical economics model

- Equation:
  - An equation describes the relationship of variables, at the least two.
  - How Y and X can be linked together is summarized by an equation.
  - For example,

    $Y = 9X$ if hourly wage is $9.

Mathematical economics model

- Mathematical economics model typically comprises of sets of equations (more than one), each of which describes the interlink among sets of variables.
  - Type of equations
  - Type of variables
- Consider market equilibrium model as an example.

Mathematical economics model

- Market equilibrium model aims at understanding the determination of “market” price and quantity.
- Price and quantity that emerges as a market equilibrium results from the interaction between buyers and sellers.

Mathematical economics model

Equation 1: $Q_d = a - bP - cY;$

Equation 2: $Q_s = d + eP - fT;$

Equation 3: $Q_d = Q_s$

Notations defined as follows:
Q_d = quantity demanded, Q_s = quantity supplied, P = price,
Y = income, T= temperature ➔ variables

(a, b, c, d, e, f) = coefficients or parameters that are all positive

Mathematical economics model

_____: $Q_d = a - bP - cY;$

_____: $Q_s = d + eP - fT$

Behavioral equations

Treatment of variables 1

Single equation concept:

Dependent : _____

Independent: _____

Mathematical economics model

Demand: $Q_d = a - bP - cY;$

Supply: $Q_s = d + eP - fT$

Behavioral equations

Equilibrium: $Q_d = Q_s$ **Equilibrium condition/equation**

Treatment of variables 1
Single equation concept:
Dependent : _____
Independent: _____

Mathematical economic model

- Type of variables treated/distinguished in the model
  - Exogenous variables = treated as "given" in the model. Their values are given as a known thing, and *will not be solved* within the model.
  - Endogenous variable = set of variables that will be solved for.

Mathematical economics model

Demand: $Q_d = a - bP - cY;$

Supply: $Q_s = d + eP - fT$

Behavioral equations

Equilibrium: $Q_d = Q_s$ **Equilibrium condition/equation**

Treatment of variables 1
Single equation concept:
Dependent : Qd, Qs
Independent: P, Y, T

Treatment of variables 2
Model concept:
ENDO : Qd, Qs, P
EXO: Y, T

Mathematical economic model

- Type of variables treated/distinguished in the model
  - Exogenous variables = treated as "given" in the model. Their values are given as known thing, and *will not be solved* within the model.
  - Endogenous variable = set of variables that will be solved for.
- Typically, the solution will depend on (i) exogenous variables and (ii) parameters.
  - Reduced-form equation or Equilibrium solution

Mathematical economic model

- I'd like you to see a big picture of where we are headed.
- Consider the next example: **the National income model**
  - Known as the Keynesian cross diagram model
  - Idea first appeared in economics field after his treatise work by John Meynard Keynes, and subsequently reinterpreted and put into formal analysis by Sir John Hicks.

Mathematical economic model

A very basic/simple version of Keynesian cross model

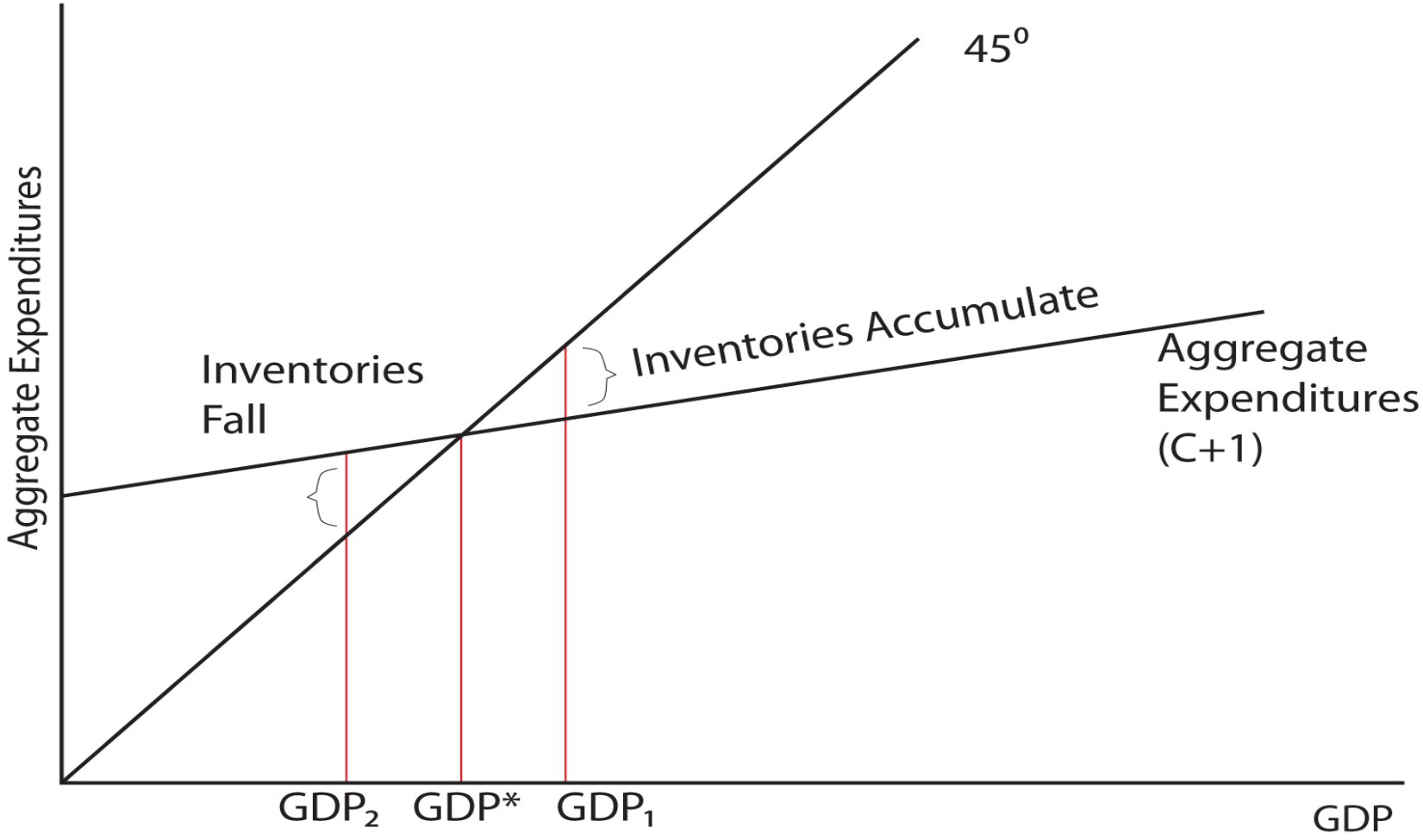
- Consumption function: $C = \mathrm{a} + \mathrm{bY}$
- Investment equation: $I = I_0$
- Government equation: $G = G_0$
- Equilibrium condition: $Y = C + I + G$

Variable Lists:

Y = income, C = consumption

I = investment and G = government expenditure

In EC212… construct AE line



Mathematical economic model

- Static analysis:
  - characterizing the equilibrium solution of the model by solving for a solution of the system of equations

$$Y^* = f(I_0, G_0, a, b)$$
$$C^* = g(I_0, G_0, a, b)$$

- From the example,

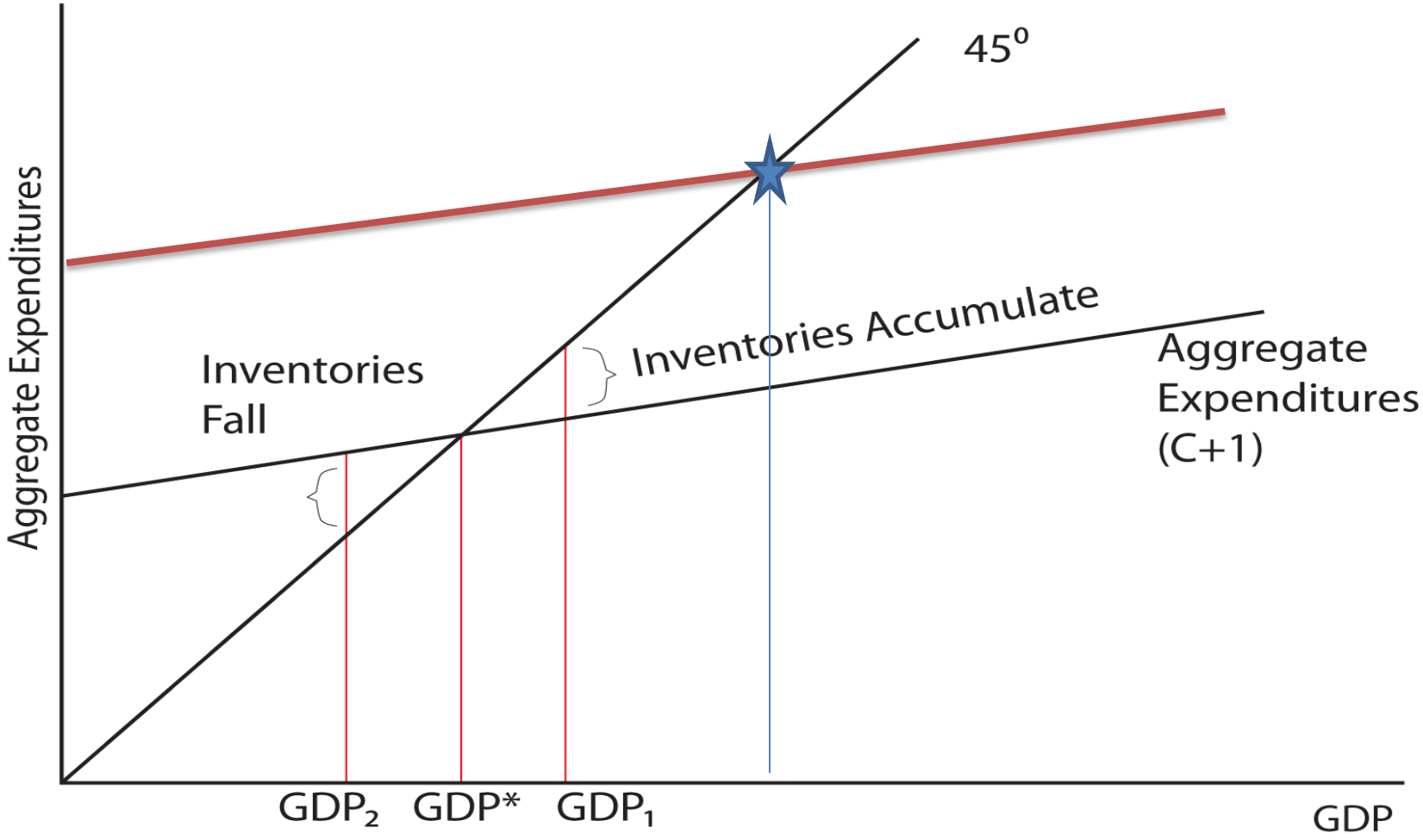
$$Y^* = \frac{a+I_0+G_0}{1-b},\ C^* = \frac{a+b(I_0+G_0)}{1-b}$$

Mathematical economic model

- From the example,
- if a = 1, I_0 = 1, G_0 = 1 and b = 0.5, this yields us,

$$Y^* = \frac{a+I_0+G_0}{1-b} = 6, \quad C^* = \frac{a+b(I_0+G_0)}{1-b} = 4$$

In EC212… construct AE line



Mathematical economic model

- Comparative Static analysis:
- What if G is now 2, how big is the change in Y* and C*?
  - New Y* = 8?
- How much a dollar increase in G has caused a change in Y*?
  - Multiplier of government expenditure is 2?.

Mathematical economic model

- In term of where we are headed, we will learn how to construct a model, and solve for its solution.
- Then, we will apply *derivative method* to perform some comparative static analysis.
  - calculate the multiplier and figure out the impact of changes in exogenous variable/parameters on endogenous variable.
  - Good thing is that we don't need to keep substituting numeric value of variables and parameters, but still we can figure out the size of multiplier.

Mathematical economic model

- Dynamic analysis:
  - Transition path for the change from Y* = 6 to Y* = 8.
  - Fortunately or unfortunately, we won't discuss this issue in this course.
  - You would need to know a more sophisticated technic called differential/difference equations.
  - Welcome to learn this on your own or from EE421/EE422.

Done for today

- Next time, we will cover topic on static equilibrium analysis, in more details. Meaning is: lot of examples and applications.
- We will start from basic mathematical concept for relation, function and system of equations.
- Then, we proceed to a variety of economic model that applies those mathematical tools developed/discussed.
- Read chapter 3 and 4 of CW