Graphs for Economics

A ***point*** in a graph of xy plane is identified by an ordered pair. $A = (2,3), B = (3,2)$

- A point tells the value of x and $y$simultaneously.

A ***line*** is a continuation of points. It shows the relationship between x and y.

Point	x	Y
	0	10
A	1	12
B	2	14
C	3	16

$\Leftrightarrow \quad y = 10 + 2x$

- A line can be thought as a function of x, $y = f(x)$
- A line can be linear or nonlinear.
- A linear line is a straight line with constant slope

Slope = rate of change of y per unit change of x
$=\frac{\Delta Y}{\Delta X}$

For linear line, from A to B

$$\Delta y =$$

$$\Delta x =$$

$$Slope \ = \frac{\Delta y}{\Delta x} =$$

From A to C,

$$\Delta y =$$

$$\Delta x =$$

$$Slope \ = \frac{\Delta y}{\Delta x} =$$

Linear line ⟺ Slope is constant

- x can change by any amount and y will change proportionately that we have same slope
- Slope is the same if we change from B to A.
- Slope > 0 ⟹ when x increases y also increases
  ⟹ x and y have positive relationship

Example when Slope < 0, $y = 10 - 3x$:

Nonlinear Line—Slope is not constant.

Example $y = 10 + x^2$

Point	x	y	slope
	0	10	
A	1	11	
B	2	14	
C	3	19	

- We can find slope by drawing a straight line tangent to the point we want to find the slope. This line is called a ***tangent line***.
- Slope changes with the value of x. Thus, slope can be called instantaneous rate of change because it is the rate of change at the instant at a particular value of x.

We can find the slope by taking derivative of the function:

$$y = f(x) = 10 + x^2$$
$$\frac{dy}{dx} = f'(x) = 2x$$

- Note that in this example, the slope increases as x increases

Approximation of change of y as a result of change of x

- When $x_1 = 2, y_1 = 14$. If $\Delta x = 0.1$, we can approximate

$$\begin{aligned} \Delta y &\approx f'(x_1).\Delta x \\ &= f'(2).0.1 \\ &= 2(2)\cdot 0.1 = 0.4 \end{aligned}$$

- What is the real Δy?

$$y_2 = f(2.1) = 10 + (2.1)^2 = 14.41$$
$$\Delta y = y_2 - y_1 =$$

- We underestimate the real change of y.
- What if $\Delta x = -0.2$? Approximate the change of y.

HW Given $y = 10 + \sqrt{x}$,

a) Find the derivative $f'(x)$.

b) Fill in the table

Point	X	Y	$f'(x)$
	0	10	
A	1	11	
B	2	11.414	
C	3	11.732	

c) Does the slope increase as x increases?

d) Approximate the change in Y when $\Delta x = 0.2$ at $x_1 = 3$. Is the approximation under- or over-estimate?

Note: If the function $f(x)$ is linear, the approximation is exact.

Shift of Graph

1) Linear with positive slope

- Change of intercept

$y = 10 + 2x$
$y = 12 + 2x$

- Change of slope

$y = 10 + 2x$
$y = 10 + 3x$

2) Linear with negative slope

- Change of intercept

$y = 10 - 2x$
$y = 12 - 2x$

- Change of slope

$y = 10 - 2x$
$y = 10 - 3x$

3) Nonlinear with change in the intercept

$f(x)$	x	y	$f'(x)$
$y = 10 + x^2$	2	14	
$y = 14 + x^2$	2	18	

$f(x)$	x	y	$f'(x)$
$y = 10 + \sqrt{x}$	2	14	
$y = 14 + \sqrt{x}$	2	18	

Second-order Derivative: Slope of Slope

$$y = f(x) = 10 + x^2$$
$$f'(x) =$$
$$f''(x) =$$

Point	x	y	$f'(x)$	$f''(x)$
	0	10	0	
A	1	11	2	
B	2	14	4	
C	3	19	6	

- The 2nd -order derivative indicates the curvature of the graph to be convex or concave.

HW Find the 2nd -order derivative of $y = f(x) = 10 + \sqrt{x}$ and fill in the table:

Point	x	y	$f'(x)$	$f''(x)$
	0	10		
A	1	11		
B	2	11.414		
C	3	11.732		

Plot the graph of y and $f'(x)$. Is $f'(x)$ linear?