

## Autoregressive Integrated Moving Average (ARIMA) Models

### Example

#### MA(1)

$$Y_{1t} = 0.1 + u_{1t} + 0.5 u_{1t-1}$$

#### AR(1)

$$Y_{2t} = 0.1 + 0.5 Y_{2t-1} + u_{2t}$$

#### ARMA(1,1)

$$Y_{3t} = 0.1 + 0.5 Y_{3t-1} + u_{3t} + 0.5 u_{3t-1}$$

#### ARIMA(1,1,1)

$$\Delta Y_{4t} = 0.1 + 0.5 \Delta Y_{4t-1} + u_{4t} + 0.5 u_{4t-1}$$

```
. set obs 500
obs was 0, now 500

. g time=_n

. tsset time
    time variable:  time, 1 to 500
                delta: 1 unit

. set seed 5432

. g y1=0 in 1
(499 missing values generated)

. g y2=0 in 1
(499 missing values generated)

. g y3=0 in 1
(499 missing values generated)

. g y4=0 in 1
(499 missing values generated)

. g dy4 = 0 in 1
(499 missing values generated)

. g u1= rnormal(0,1)

. g u2= rnormal(0,1)

. g u3= rnormal(0,1)

. g u4= rnormal(0,1)

. replace y1=0.1+u1+0.5*1.u1 if time>1
(499 real changes made)

. replace y2=0.1+0.5*1.y2+u2 if time>1
(499 real changes made)

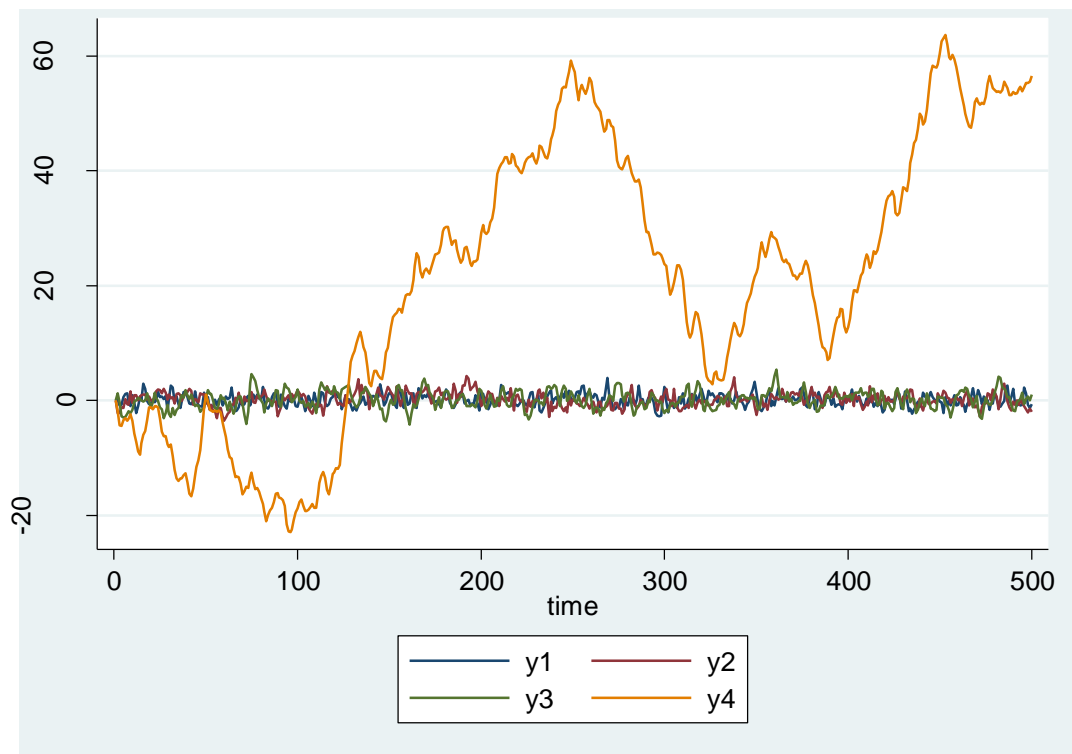
. replace y3=0.1+0.5*1.y3+u3+0.5*1.u3 if time>1
(499 real changes made)

. replace y4=0.1+1.y4+u4+0.5*1.u4 if time>1
(499 real changes made)

. replace dy4=d.y4 if time>1
(499 real changes made)

. replace y4=0.1+1.y4+0.5*1.dy4+u4+0.5*1.u4 if time>1
(498 real changes made)
```

```
. line y1 y2 y3 y4 time
```



### **Example: Finding ARIMA process — Box-Jenkins Methodology**

**Identification and Estimation:** The first step is to identify the order of ARIMA(p,d,q).

To find the value of p, d, and q, we should firstly identify value of d by performing unit root test to find which order the series is integrated.

#### **Unit Root Test**

Find integrated order of the series using unit root test.

```
. tsset time
      time variable: time, 1 to 357
                delta: 1 unit
```

```
. dfuller y, trend lags(1) regress
```

Augmented Dickey-Fuller test for unit root                      Number of obs =                      355

| Test Statistic | ----- Interpolated Dickey-Fuller ----- |                   |                    |        |
|----------------|--|-------------------|--------------------|--------|
|                | 1% Critical Value                      | 5% Critical Value | 10% Critical Value |        |
| z(t)           | -2.152                                 | -3.986            | -3.426             | -3.130 |

Mackinnon approximate p-value for z(t) = 0.5167

| D.y    | Coef.     | Std. Err. | t     | P> t  | [95% Conf. Interval] |           |
|--------|-----------|-----------|-------|-------|----------------------|-----------|
| y      |           |           |       |       |                      |           |
| L1.    | -.0237778 | .0110475  | -2.15 | 0.032 | -.0455054            | -.0020502 |
| LD.    | .0625276  | .0531988  | 1.18  | 0.241 | -.0421009            | .167156   |
| _trend | .0143903  | .0082343  | 1.75  | 0.081 | -.0018044            | .030585   |
| _cons  | 23.00759  | 11.00167  | 2.09  | 0.037 | 1.370103             | 44.64508  |

```
. dfuller y, lags(1) regress
```

```
Augmented Dickey-Fuller test for unit root          Number of obs   =       355
```

|      | Test<br>Statistic | -----<br>1% Critical<br>Value | Interpolated Dickey-Fuller<br>5% Critical<br>Value | -----<br>10% Critical<br>Value |
|------|-------------------|-------------------------------|--|--------------------------------|
| Z(t) | -1.414            | -3.452                        | -2.876   | -2.570                         |

Mackinnon approximate p-value for Z(t) = 0.5757

| D.y   | Coef.     | Std. Err. | t     | P> t  | [95% Conf. Interval] |
|-------|-----------|-----------|-------|-------|----------------------|
| y     |           |           |       |       |                      |
| L1.   | -.0129893 | .0091884  | -1.41 | 0.158 | -.0310604 .0050818   |
| LD.   | .059847   | .0533316  | 1.12  | 0.263 | -.0450416 .1647357   |
| _cons | 14.12272  | 9.784939  | 1.44  | 0.150 | -5.121575 33.36702   |

```
. dfuller d.y, trend lags(1) regress
```

```
Augmented Dickey-Fuller test for unit root          Number of obs   =       354
```

|      | Test<br>Statistic | -----<br>1% Critical<br>Value | Interpolated Dickey-Fuller<br>5% Critical<br>Value | -----<br>10% Critical<br>Value |
|------|-------------------|-------------------------------|--|--------------------------------|
| Z(t) | -12.054           | -3.986                        | -3.426   | -3.130                         |

Mackinnon approximate p-value for Z(t) = 0.0000

| D2.y   | Coef.     | Std. Err. | t      | P> t  | [95% Conf. Interval] |
|--------|-----------|-----------|--------|-------|----------------------|
| D.y    |           |           |        |       |                      |
| L1.    | -.8856237 | .0734696  | -12.05 | 0.000 | -1.030121 -.7411262  |
| LD.    | -.0668939 | .0533317  | -1.25  | 0.211 | -.1717849 .0379971   |
| _trend | .0042522  | .0069007  | 0.62   | 0.538 | -.0093198 .0178243   |
| _cons  | -.4529247 | 1.418181  | -0.32  | 0.750 | -3.242154 2.336305   |

```
. arima y, arima(1,1,1) nolog
```

```
ARIMA regression
```

```
Sample: 2 - 357
```

```
Log likelihood = -1423.146
```

```
Number of obs   =       356  
wald chi2(2)    =         3.13  
Prob > chi2     =       0.2089
```

| D.y    | Coef.     | OPG<br>Std. Err. | z     | P> z  | [95% Conf. Interval] |
|--------|-----------|------------------|-------|-------|----------------------|
| y      |           |                  |       |       |                      |
| _cons  | .3700177  | .8218557         | 0.45  | 0.653 | -1.24079 1.980825    |
| ARMA   |           |                  |       |       |                      |
| ar     |           |                  |       |       |                      |
| L1.    | .2948697  | .6818072         | 0.43  | 0.665 | -1.041448 1.631187   |
| ma     |           |                  |       |       |                      |
| L1.    | -.2368839 | .7055342         | -0.34 | 0.737 | -1.619705 1.145938   |
| /sigma | 13.17852  | .3681099         | 35.80 | 0.000 | 12.45704 13.9        |

```
. est store arima111
```

```
. qui arima y, arima(1,1,2) nolog
```

```
. est store arima112
```

```
. qui arima y, arima(1,1,3) nolog
```

```

. est store arima113
. qui arima y, arima(2,1,1) nolog
. est store arima211
. qui arima y, arima(2,1,2) nolog
. est store arima212
. qui arima y, arima(2,1,3) nolog
. est store arima213
. qui arima y, arima(3,1,1) nolog
. est store arima311
. qui arima y, arima(3,1,2) nolog
. est store arima312
. qui arima y, arima(3,1,3) nolog
. est store arima313
. est table arima11*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

```

| Variable   | arima111     | arima112     | arima113     |
|------------|--------------|--------------|--------------|
| y          |              |              |              |
| _cons      | .37001774    | .37418642    | .3987889     |
| ARMA       |              |              |              |
| ar         |              |              |              |
| L1.        | .29486971    | -.31289111   | .97476406*** |
| ma         |              |              |              |
| L1.        | -.23688394   | .36887941    | -.92682332   |
| L2.        |              | .11337558**  | .03029933    |
| L3.        |              |              | -.10347864   |
| sigma      |              |              |              |
| _cons      | 13.178521*** | 13.127234*** | 13.055651    |
| Statistics |              |              |              |
| N          | 356          | 356          | 356          |
| ll         | -1423.1463   | -1421.7567   | -1420.6373   |
| chi2       | 3.1319865    | 6.7574223    | 9222.4937    |
| aic        | 2854.2926    | 2853.5135    | 2853.2745    |
| bic        | 2869.7923    | 2872.8881    | 2876.5241    |

Legend: \* p<.1; \*\* p<.05; \*\*\* p<.01

```

. est table arima21*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

```

| Variable   | arima211     | arima212     | arima213     |
|------------|--------------|--------------|--------------|
| y          |              |              |              |
| _cons      | .37274811    | .37283121    | .37413313    |
| ARMA       |              |              |              |
| ar         |              |              |              |
| L1.        | -.28216023   | -.66642562** | .80124027**  |
| L2.        | .09539459**  | -.39627716   | -.3650645    |
| ma         |              |              |              |
| L1.        | .33242601    | .71235711*** | -.75250981** |
| L2.        |              | .50413388    | .39755506    |
| L3.        |              |              | -.10013858*  |
| sigma      |              |              |              |
| _cons      | 13.139922*** | 13.112555*** | 13.079159*** |
| Statistics |              |              |              |

|      | N          | 356        | 356       | 356 |
|------|------------|------------|-----------|-----|
| ll   | -1422.0873 | -1421.3508 | -1420.453 |     |
| chi2 | 4.7048993  | 17.316358  | 18.662816 |     |
| aic  | 2854.1745  | 2854.7016  | 2854.9059 |     |
| bic  | 2873.5492  | 2877.9512  | 2882.0304 |     |

Legend: \* p<.1; \*\* p<.05; \*\*\* p<.01

. est table arima31\*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

| Variable   | arima311      | arima312     | arima313     |
|------------|---------------|--------------|--------------|
| y          |               |              |              |
| _cons      | .37269062     | .3751536     | .37488954    |
| ARMA       |               |              |              |
| ar         |               |              |              |
| L1.        | .51072432     | .67284808*   | .63119128    |
| L2.        | .04992461     | -.20028258   | -.15281824   |
| L3.        | -.12395064*** | -.11233502** | -.16502621   |
| ma         |               |              |              |
| L1.        | -.46077982    | -.62568637*  | -.58388774   |
| L2.        |               | .2450268     | .19971418    |
| L3.        |               |              | .05258667    |
| sigma      |               |              |              |
| _cons      | 13.082025***  | 13.075786*** | 13.075313*** |
| Statistics |               |              |              |
| N          | 356           | 356          | 356          |
| ll         | -1420.5393    | -1420.3504   | -1420.3353   |
| chi2       | 11.701303     | 16.675786    | 16.501777    |
| aic        | 2853.0786     | 2854.7008    | 2856.6706    |
| bic        | 2876.3282     | 2881.8253    | 2887.67      |

Legend: \* p<.1; \*\* p<.05; \*\*\* p<.01

## Forecast

### Command: predict, xb

. arima y, arima(1,1,1) nolog

ARIMA regression

Sample: 2 - 357

Log likelihood = -1423.146

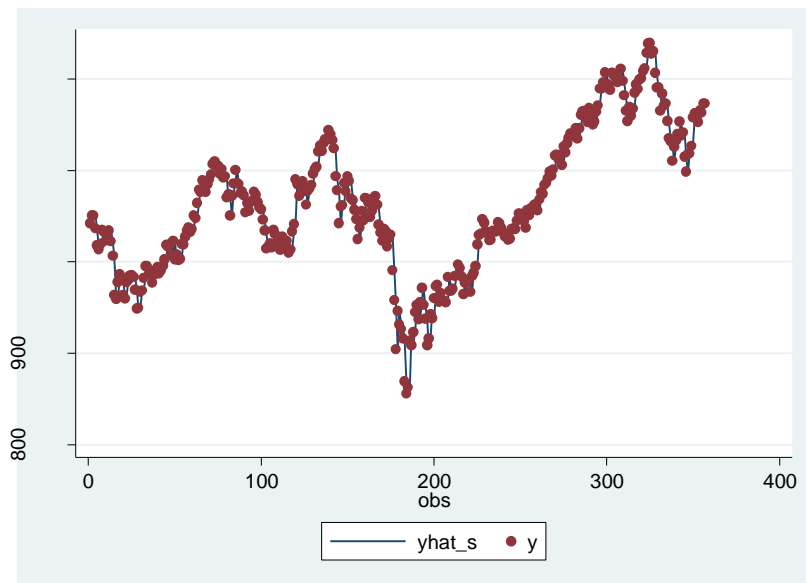
Number of obs = 356  
 Wald chi2(2) = 3.13  
 Prob > chi2 = 0.2089

| D.y    | Coef.     | OPG<br>Std. Err. | z     | P> z  | [95% Conf. Interval] |
|--------|-----------|------------------|-------|-------|----------------------|
| y      |           |                  |       |       |                      |
| _cons  | .3700177  | .8218557         | 0.45  | 0.653 | -1.24079 1.980825    |
| ARMA   |           |                  |       |       |                      |
| ar     |           |                  |       |       |                      |
| L1.    | .2948697  | .6818072         | 0.43  | 0.665 | -1.041448 1.631187   |
| ma     |           |                  |       |       |                      |
| L1.    | -.2368839 | .7055342         | -0.34 | 0.737 | -1.619705 1.145938   |
| /sigma | 13.17852  | .3681099         | 35.80 | 0.000 | 12.45704 13.9        |

. predict dyhat, xb

. g yhat\_s=l.y+dyhat if time>1  
 (3 missing values generated)

```
. twoway (line yhat_s time, sort) (scatter y time, sort)
```



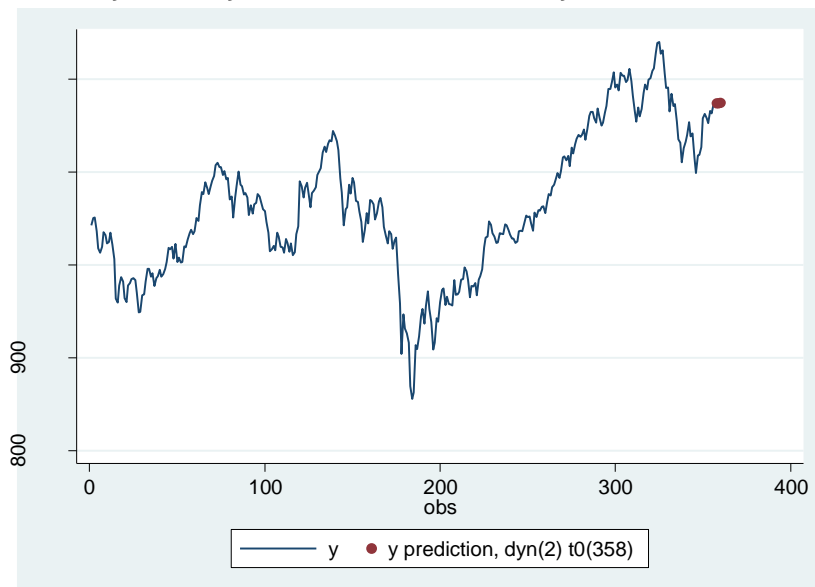
### **Command: predict, y**

#### **Dynamic Forecast**

```
. predict yhat, y dynamic(.) t0(357)
Note: beginning dynamic predictions in period 2
(356 missing values generated)
```

```
. predict yhat2, y dynamic(.) t0(358)
Note: beginning dynamic predictions in period 2
(357 missing values generated)
```

```
. twoway (line y time, sort) (scatter yhat2 time, sort)
```

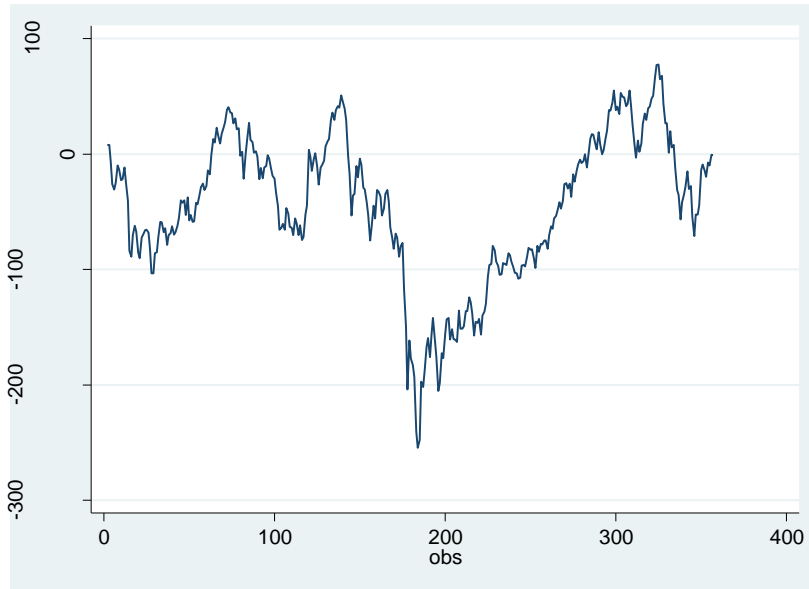


#### **Forecasting Error**

```
. predict yhat_d, y dynamic(.)
Note: beginning dynamic predictions in period 2
(1 missing value generated)
```

```
. g ferror=y-yhat_d if time<358
(4 missing values generated)
```

```
. twoway (line ferror time, sort)
```



### SARIMA

```
. arima y, arima(1,1,1) sarima(1,1,1,13) nolog
```

ARIMA regression

Sample: 15 - 357

Log likelihood = -1390.897

Number of obs = 343  
 Wald chi2(4) = 190.55  
 Prob > chi2 = 0.0000

| DS13.y |           | Coef.     | OPG<br>Std. Err. | z      | P> z  | [95% Conf. Interval] |           |
|--------|-----------|-----------|------------------|--------|-------|----------------------|-----------|
| y      | _cons     | .055653   | .0949183         | 0.59   | 0.558 | -.1303835            | .2416896  |
| ARMA   |           |           |                  |        |       |                      |           |
|        | ar<br>L1. | -.6610302 | 1.885136         | -0.35  | 0.726 | -4.355829            | 3.033769  |
|        | ma<br>L1. | .6497243  | 1.909893         | 0.34   | 0.734 | -3.093598            | 4.393047  |
| ARMA13 |           |           |                  |        |       |                      |           |
|        | ar<br>L1. | -.0235579 | .057453          | -0.41  | 0.682 | -.1361638            | .089048   |
|        | ma<br>L1. | -.9555681 | .0798977         | -11.96 | 0.000 | -1.112165            | -.7989715 |
|        | /sigma    | 13.33896  | .5078398         | 26.27  | 0.000 | 12.34361             | 14.33431  |

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.

### ARMAX

```
. arima y d.usd, arima(1,1,1) nolog
```

ARIMA regression

Sample: 3 - 357

Log likelihood = -1405.373

Number of obs = 355  
 Wald chi2(3) = 41.61  
 Prob > chi2 = 0.0000

| D.y |  | Coef. | OPG<br>Std. Err. | z | P> z | [95% Conf. Interval] |  |
|-----|--|-------|------------------|---|------|----------------------|--|
|-----|--|-------|------------------|---|------|----------------------|--|

---

|       |        |           |          |       |       |           |          |
|-------|--------|-----------|----------|-------|-------|-----------|----------|
| y     | usd    |           |          |       |       |           |          |
|       | D2.    | -25.89824 | 4.143719 | -6.25 | 0.000 | -34.01978 | -17.7767 |
|       | _cons  | .3479213  | .8288965 | 0.42  | 0.675 | -1.276686 | 1.972529 |
| ----- |        |           |          |       |       |           |          |
| ARMA  | ar     |           |          |       |       |           |          |
|       | L1.    | .2162354  | .3554955 | 0.61  | 0.543 | -.4805229 | .9129937 |
|       | ma     |           |          |       |       |           |          |
|       | L1.    | -.0959384 | .3756771 | -0.26 | 0.798 | -.8322521 | .6403752 |
| ----- |        |           |          |       |       |           |          |
|       | /sigma | 12.67782  | .3774993 | 33.58 | 0.000 | 11.93794  | 13.41771 |
| ----- |        |           |          |       |       |           |          |

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.

## Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models

### Example Simulated Data

```

set obs 501
g t=_n
tsset t
g s2=1 in 1
g u=rnormal(0,s2) in 1
g u2=u^2 in 1
forvalue i=2(1)501 {
  replace s2=0.3+0.3*1.u2+0.3*1.s2 in `i'
  replace u=rnormal(0,s2) in `i'
  replace u2=u^2 in `i'
}
g x=rnormal(1,10)
g y=0.5+0.7*x+u
reg y x if t>1
estat archlm
arch y x if t>1, arch(1) garch(1) nolog
predict s2hat, v
twayway (line s2hat t) (scatter s2 t)

```

```

. set obs 501
number of observations (_N) was 0, now 501

. g t=_n

. tsset t
      time variable:  t, 1 to 501
              delta:  1 unit

. g s2=1 in 1
(500 missing values generated)

. g u=rnormal(0,s2) in 1
(500 missing values generated)

. g u2=u^2 in 1
(500 missing values generated)

. forvalue i=2(1)501 {
2.  replace s2=0.3+0.3*1.u2+0.3*1.s2 in `i'
3.  replace u=rnormal(0,s2) in `i'
4.  replace u2=u^2 in `i'
5. }

. g x=rnormal(1,10)

. g y=0.5+0.7*x+u

. reg y x if t>1

```

| Source   | SS         | df  | MS         | Number of obs | = | 500      |
|----------|------------|-----|------------|---------------|---|----------|
| Model    | 23398.9523 | 1   | 23398.9523 | F(1, 498)     | = | 63539.73 |
| Residual | 183.391992 | 498 | .368257012 | Prob > F      | = | 0.0000   |
| Total    | 23582.3443 | 499 | 47.259207  | R-squared     | = | 0.9922   |
|          |            |     |            | Adj R-squared | = | 0.9922   |
|          |            |     |            | Root MSE      | = | .60684   |

|  | y     | Coef.    | Std. Err. | t      | P> t  | [95% Conf. Interval] |
|--|-------|----------|-----------|--------|-------|----------------------|
|  | x     | .7013338 | .0027823  | 252.07 | 0.000 | .6958673 .7068002    |
|  | _cons | .4743123 | .0273053  | 17.37  | 0.000 | .4206645 .5279601    |

```

. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
-----+-----
lags(p) |          chi2          df          Prob > chi2
-----+-----
      1 |          35.821          1          0.0000
-----+-----
          H0: no ARCH effects          vs.          H1: ARCH(p) disturbance

. arch y x if t>1, arch(1) garch(1) nolog

ARCH family regression

Sample: 2 - 501          Number of obs =          500
Distribution: Gaussian          Wald chi2(1) =          84227.65
Log likelihood = -426.751          Prob > chi2 =          0.0000

-----+-----
          y |          Coef.          OPG          z          P>|z|          [95% Conf. Interval]
-----+-----
y
  x |          .7028459          .0024218          290.22          0.000          .6980993          .7075925
  _cons |          .4758044          .023265          20.45          0.000          .4302058          .5214029
-----+-----
ARCH
  arch |
  L1. |          .3970325          .093721          4.24          0.000          .2133428          .5807223
  |
  garch |
  L1. |          .3355659          .112419          2.98          0.003          .1152287          .5559032
  |
  _cons |          .105968          .0289994          3.65          0.000          .0491302          .1628059
-----+-----

. predict s2hat, v

. predict yhat, xb

. mat beta=e(b)

. mat list beta

beta[1,5]
          y:          y:          ARCH:          ARCH:          ARCH:
          x          _cons          L.          L.
y1 .70284588 .47580435 .39703251 .33556595 .10596802

. sca b0=e1(beta,1,2)

. sca b1=e1(beta,1,1)

. sca a0=e1(beta,1,5)

. sca d1=e1(beta,1,4)

. sca a1=e1(beta,1,3)

. g yhat_m=b0+b1*x

. g ehat_m=y-yhat_m

. predict uhat, r

. g uhat2=uhat^2

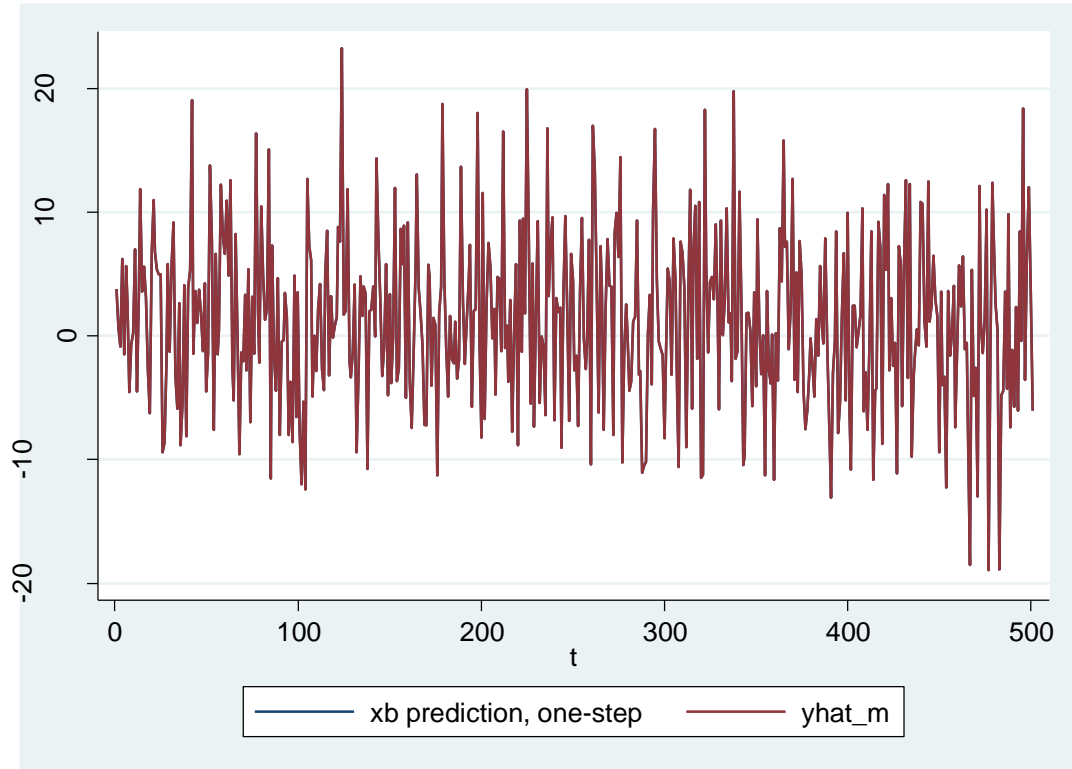
. g s2hat_m=a0+d1*1.uhat2+a1*1.uhat2 in 2

```

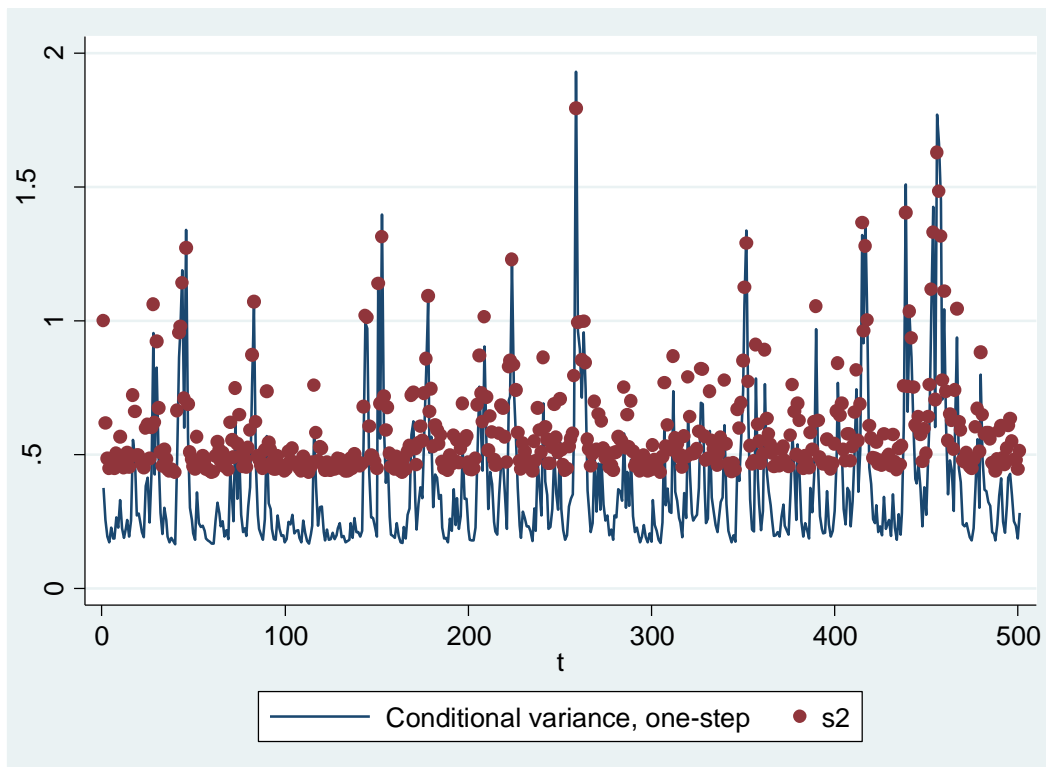
(500 missing values generated)

```
. replace s2hat_m=a0+d1*1.s2hat_m+a1*1.uhat2 if t>2
(499 real changes made)
```

```
. line yhat yhat_m t
```



```
. twoway (line s2hat t) (scatter s2 t)
```



**Example**

$$SET_t = \beta_0 + \beta_1 IBR_t + \beta_2 GOLD_t + \beta_3 USD_t + \varepsilon_t$$

where:  $SET_t$  = Return on Stock Exchange of Thailand (SET).

$IBR_t$  = Inter-bank rate.

$GOLD_t$  = Gold price.

$USD_t$  = Exchange rate (\$US/฿Baht).

$\varepsilon_t$  = Residual which has GARCH(p,q) process:

$$\sigma_t^2 = \alpha_0 + \delta_1 \sigma_{t-1}^2 + \delta_2 \sigma_{t-2}^2 + \dots + \delta_p \sigma_{t-p}^2 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_q \varepsilon_{t-q}^2$$

**Testing GARCH Effect**

Firstly, estimate the model using OLS without ARCH(p) process.

```
. reg rset ibr rgold rUSD
```

| Source   | SS         | df  | MS         | Number of obs = | 356    |
|----------|------------|-----|------------|-----------------|--------|
| Model    | .013629179 | 3   | .00454306  | F( 3, 352) =    | 34.95  |
| Residual | .045752082 | 352 | .000129978 | Prob > F =      | 0.0000 |
|          |            |     |            | R-squared =     | 0.2295 |
|          |            |     |            | Adj R-squared = | 0.2230 |
| Total    | .059381261 | 355 | .000167271 | Root MSE =      | .0114  |

| rset  | Coef.     | Std. Err. | t     | P> t  | [95% Conf. Interval] |           |
|-------|-----------|-----------|-------|-------|----------------------|-----------|
| ibr   | -.0148909 | .0208497  | -0.71 | 0.476 | -.0558966            | .0261148  |
| rgold | -.0855076 | .0551807  | -1.55 | 0.122 | -.194033             | .0230178  |
| rUSD  | -2.060204 | .214308   | -9.61 | 0.000 | -2.481689            | -1.638718 |
| _cons | .0012448  | .0009258  | 1.34  | 0.180 | -.0005761            | .0030656  |

```
. est store ols
```

```
. estat archlm
```

LM test for autoregressive conditional heteroskedasticity (ARCH)

| lags(p) | chi2  | df | Prob > chi2 |
|---------|-------|----|-------------|
| 1       | 8.225 | 1  | 0.0041      |

H0: no ARCH effects vs. H1: ARCH(p) disturbance

In this case, since p-value of the ARCH effect test (F-statistic or Chi-Square (Obs\*R-squared = 8.225) the p-value is less than level of significance 0.05, thus, null hypothesis that there is no ARCH effect is rejected, thus, there exists significant ARCH effect in this model with 0.05 significant level.

**Identify Order (p,q) and Estimation**

The next step is to identify order of GARCH(p,q) by estimating GARCH models in several orders and choose the model with the lowest AIC or SIC.

**Estimate ARCH(q) using MLE**

. arch rset ibr rgold rurd, arch(1) nolog

ARCH family regression

Sample: 2 - 357  
 Distribution: Gaussian  
 Log likelihood = 1094.88  
 Number of obs = 356  
 Wald chi2(3) = 114.64  
 Prob > chi2 = 0.0000

|      |       | Coef.     | OPG<br>Std. Err. | z      | P> z  | [95% Conf. Interval] |           |
|------|-------|-----------|------------------|--------|-------|----------------------|-----------|
| rset |       |           |                  |        |       |                      |           |
|      | ibr   | -.0224536 | .0227516         | -0.99  | 0.324 | -.0670459            | .0221386  |
|      | rgold | -.1352603 | .0481352         | -2.81  | 0.005 | -.2296037            | -.040917  |
|      | rurd  | -1.990394 | .1956511         | -10.17 | 0.000 | -2.373863            | -1.606925 |
|      | _cons | .0019338  | .0009473         | 2.04   | 0.041 | .0000772             | .0037904  |
| ARCH |       |           |                  |        |       |                      |           |
|      | arch  |           |                  |        |       |                      |           |
|      | L1.   | .1410099  | .0526486         | 2.68   | 0.007 | .0378206             | .2441992  |
|      | _cons | .0001097  | 7.65e-06         | 14.34  | 0.000 | .0000947             | .0001246  |

. est store arch1

. qui arch rset ibr rgold rurd, arch(1/2) nolog

. est store arch2

. qui arch rset ibr rgold rurd, arch(1/3) nolog

. est store arch3

. qui arch rset ibr rgold rurd, arch(1/4) nolog

. est store arch4

. est table ols arch\*, star(0.1 0.05 0.01) stat(N rss ll F chi2 aic bic)

| Variable   | ols           | arch1         | arch2         | arch3         | arch4         |
|------------|---------------|---------------|---------------|---------------|---------------|
| ibr        | -.01489091    |               |               |               |               |
| rgold      | -.08550759    |               |               |               |               |
| rurd       | -2.0602036*** |               |               |               |               |
| _cons      | .00124476     |               |               |               |               |
| rset       |               |               |               |               |               |
|            |               | -.02245364    | -.03228511    | -.02934212    | -.02443571    |
|            |               | -.13526031*** | -.15316854*** | -.14704494*** | -.12657214**  |
|            |               | -1.990394***  | -1.8927342*** | -1.8023097*** | -1.7960216*** |
|            |               | .00193379**   | .00262446***  | .00210875**   | .00172192**   |
| ARCH       |               |               |               |               |               |
|            | arch          |               |               |               |               |
|            | L1.           | .14100992***  | .15429368**   | .13569541**   | .11527677*    |
|            | L2.           |               | .22850896***  | .14715617*    | .15313291*    |
|            | L3.           |               |               | .17200035**   | .16646817**   |
|            | L4.           |               |               |               | .12103532***  |
|            | _cons         | .00010965***  | .0000819***   | .00007027***  | .00005809***  |
| Statistics |               |               |               |               |               |
| N          | 356           | 356           | 356           | 356           | 356           |
| rss        | .04575208     |               |               |               |               |
| ll         | 1089.6398     | 1094.8804     | 1101.4192     | 1107.1823     | 1111.1621     |
| F          | 34.952661     |               |               |               |               |
| chi2       |               | 114.64432     | 113.73733     | 111.95123     | 108.03864     |
| aic        | -2171.2795    | -2177.7608    | -2188.8384    | -2198.3645    | -2204.3242    |
| bic        | -2155.7798    | -2154.5112    | -2161.7138    | -2167.3651    | -2169.4499    |

Legend: \* p<.1; \*\* p<.05; \*\*\* p<.01

**Estimate GARCH(p,q) using MLE**

```

. qui arch rset ibr rgold rUSD, arch(1) garch(1) nolog
. est store garch11
. qui arch rset ibr rgold rUSD, arch(1) garch(1/2) nolog
. est store garch12
. qui arch rset ibr rgold rUSD, arch(1/2) garch(1) nolog
. est store garch21
. qui arch rset ibr rgold rUSD, arch(1/2) garch(1/2) nolog
. est store garch22
. est table garch*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

```

| Variable          | garch11       | garch12       | garch21       | garch22       |
|-------------------|---------------|---------------|---------------|---------------|
| <b>rset</b>       |               |               |               |               |
| ibr               | -.02671053    | -.02670456    | -.02669459    | -.02681346    |
| rgold             | -.12290548*** | -.12283479**  | -.12267326**  | -.09998894**  |
| rUSD              | -1.7803476*** | -1.7804212*** | -1.7802643*** | -1.7176602*** |
| _cons             | .00205671***  | .00205576***  | .00205456***  | .00201858***  |
| <b>ARCH</b>       |               |               |               |               |
| arch              |               |               |               |               |
| L1.               | .12274144***  | .12226525**   | .12140524**   | .14859642***  |
| L2.               |               |               | .00283013     | .07798055*    |
| garch             |               |               |               |               |
| L1.               | .82872934***  | .8361347      | .82647574***  | -.09243062    |
| L2.               |               | -.00680923    |               | .79361191***  |
| _cons             | 6.190e-06*    | 6.175e-06     | 6.290e-06*    | 9.638e-06     |
| <b>Statistics</b> |               |               |               |               |
| N                 | 356           | 356           | 356           | 356           |
| ll                | 1116.3772     | 1116.3774     | 1116.3778     | 1117.2825     |
| chi2              | 132.26126     | 131.01417     | 130.93732     | 135.01264     |
| aic               | -2218.7545    | -2216.7548    | -2216.7555    | -2216.565     |
| bic               | -2191.6299    | -2185.7554    | -2185.7561    | -2181.6906    |

Legend: \* p<.1; \*\* p<.05; \*\*\* p<.01

**Forecast using ARCH(1)**

```
. arch rset ibr rgold rUSD, arch(1/2) garch(1/2) nolog
```

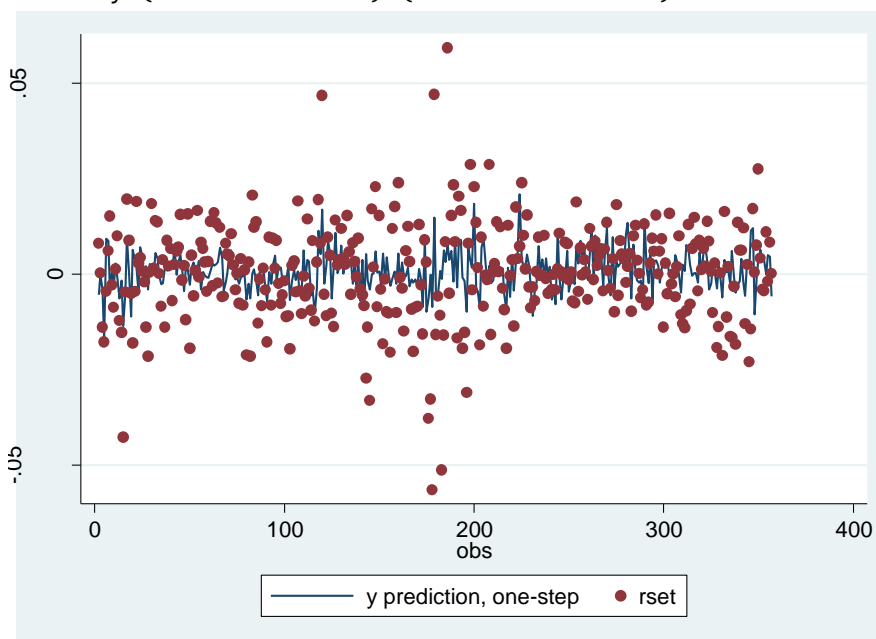
ARCH family regression

```
Sample: 2 - 357
Distribution: Gaussian
Log likelihood = 1117.283
Number of obs = 356
Wald chi2(3) = 135.01
Prob > chi2 = 0.0000
```

|      |       | Coef.     | OPG<br>Std. Err. | z      | P> z  | [95% Conf. Interval] |           |
|------|-------|-----------|------------------|--------|-------|----------------------|-----------|
| rset |       |           |                  |        |       |                      |           |
|      | ibr   | -.0268135 | .0163996         | -1.64  | 0.102 | -.0589561            | .0053292  |
|      | rgold | -.0999889 | .047902          | -2.09  | 0.037 | -.193875             | -.0061028 |
|      | rUSD  | -1.71766  | .164051          | -10.47 | 0.000 | -2.039194            | -1.396126 |
|      | _cons | .0020186  | .000773          | 2.61   | 0.009 | .0005035             | .0035337  |
| ARCH |       |           |                  |        |       |                      |           |
|      | arch  |           |                  |        |       |                      |           |
|      | L1.   | .1485964  | .0449342         | 3.31   | 0.001 | .0605269             | .2366659  |
|      | L2.   | .0779806  | .0458139         | 1.70   | 0.089 | -.0118131            | .1677742  |
|      | garch |           |                  |        |       |                      |           |
|      | L1.   | -.0924306 | .0664156         | -1.39  | 0.164 | -.2226027            | .0377415  |
|      | L2.   | .7936119  | .0744962         | 10.65  | 0.000 | .647602              | .9396218  |
|      | _cons | 9.64e-06  | 5.99e-06         | 1.61   | 0.107 | -2.09e-06            | .0000214  |

```
. predict rsetf, y
(4 missing values generated)
```

```
. twoway (line rsetf time) (scatter rset time)
```

**Estimate Variance (Volatility) of Dependent Variable from Estimated Model**

Variance or volatility of dependent variable (in this case, return on SET) can be estimated from the estimated result.

```
. predict sigma2, variance
(11 missing values generated)
```

The estimated variance series is then estimated and keep as sigma2.

```
. line sigma2 t
```

