Church & Ware Chapter 10 Problem 1

(i) Monopoly output & profit

$$\Pi = TR - TC$$
$$= PQ - Q^2$$
$$= (20 - Q)Q - Q^2$$

FOC: $\frac{\partial \Pi}{\partial Q} = 0 = 20 - 4Q \Rightarrow Q = 5\#,\ P = 20 - 5 = 15\#$

$$\Pi = PQ - Q^2 = (15 \times 5) - 5^2 = 50\#$$

b) If 2 firms were colluding to maximize industry profit

$$\Pi^{industry} = \Pi^{firm1} + \Pi^{firm2} = \overbrace{(TR - TC)}^{firm1} + \overbrace{(TR - TC)}^{firm2}$$

By symmetry $= (TR - TC) \times 2$

$$= (Pq - q^2) \times 2$$
$$= ([20 - 2q]q - q^2) \times 2$$
$$= (20q - 2q^2 - q^2) \times 2$$
$$= 40q - 6q^2$$

FOC: $\frac{\partial \Pi}{\partial q} = 0 = 40 - 12q \Rightarrow q = \frac{40}{12} = \frac{10}{3}$

$$P = 20 - 2\left(\frac{10}{3}\right) = \frac{40}{3}$$

$$\Pi^{industry} = 40\left(\frac{10}{3}\right) - 6\left(\frac{10}{3}\right)^2$$
$$= \frac{400 - 200}{3} = \frac{200}{3}$$

$$\Pi^{firm} = \frac{200}{3} \div 2 = \frac{100}{3}$$

c) Because firms are facing diseconomy of scale, e.g. avg. cost increases the more firm produce. So, it is cheaper (more cost-saving) for firms to split production between two plants.

d). No ... try calculating a profit-maximization problem of one firm given the other producing at $\frac{10}{3}$ units.