

Quiz 4

1. Find the solution set for

$$|x + |x + 2|| < 3.$$

Answer: From $|x + 2| = \begin{cases} -(x + 2), & x + 2 < 0 \Leftrightarrow x < -2 \\ x + 2, & x + 2 \geq 0 \Leftrightarrow x \geq -2 \end{cases}$, we will consider 2 cases.

Case I For $x \in (-\infty, -2)$, $|x + 2| = -(x + 2)$ and $|x + |x + 2|| < 3$ becomes

$$|x - x - 2| < 3 \quad \text{or} \quad |-2| < 3 \quad \text{or} \quad 2 < 3,$$

which is always true for all $x \in \mathbb{R}$. Hence, the solution set is $(-\infty, -2) \cap \mathbb{R} = (-\infty, -2)$.

Case II For $x \in [-2, \infty)$, $|x + 2| = (x + 2)$ and $|x + |x + 2|| < 3$ becomes

$$|x + x + 2| < 3 \quad \text{or} \quad |2x + 2| < 3,$$

which implies

$$\begin{aligned} -3 &< 2x + 2 < 3 \\ \frac{-3 - 2}{2} &< x < \frac{3 - 2}{2} \\ \frac{-5}{2} &< x < \frac{1}{2}. \end{aligned}$$

That is, $x \in (-\frac{5}{2}, \frac{1}{2})$, and the solution set for this case is $[-2, \infty) \cap (-\frac{5}{2}, \frac{1}{2}) = [-2, \frac{1}{2})$.

From cases I and II, the solution set is $(-\infty, -2) \cup [-2, \frac{1}{2}) = (-\infty, \frac{1}{2})$. ■