Part 3 (25 points): Problem Set

Suppose market demand for gasoline is given by P = 1000 – Q. There are 2 firms competing in this market. Both firms have an identical total cost function of TC = Q^2. Firms are **Cournot competitors** (meaning that they choose quantity as the strategic variable) and play a supergame. The collusive agreement being considered is for the two firms to produce 1/2 of the industry output. To sustain the collusion, firms adopt the grim punishment strategy of stop colluding forever.

a) (5 points) If collusion is successful, what should be the market price and quantity?

b) (10 points) What should be the value of δ to sustain collusion if detection of deviation requires 2 periods? Show the derivation of δ to receive full credits.

c) (5 points) Suppose firm 1 discovers a more cost-efficient way to produce. The total cost function of firm 1 now becomes $Q^2/2$. (Firm 2 still have the same total cost function, which is $TC = Q^2$). If detection of deviation requires 2 periods, which firm is more likely to cheat? Explain in words, no calculation needed.

d) (5 points) Suppose firm 2 announces that it would adopt the Bertrand (rather than Cournot) as the grim punishment strategy. Do you think this can help enforce the collusion? Explain in words, no calculation needed.

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a) $\pi_{industry} = \pi_{firm\,1} + \pi_{firm\,2}$

$= (TR_1 - TC_1) + (TR_2 - TC_2)$

$= 2(TR - TC)$ << as firm are identical

$= 2(P \cdot q - q^2)$

$= 2((1000 - Q) \cdot q - q^2)$

$= 2((1000 - 2q)q - q^2)$

$= 2000q - 4q^2 - 2q^2$

$= 2000q - 6q^2$

$FOC = \frac{\partial \pi}{\partial q} = 0 = 2000 - 12q$

$q = \frac{2000}{12} = \frac{500}{3}$

$Q = \frac{2000}{6}$

$P = \frac{2000}{3}$

$\pi_{industry} = 2000(q) - 6q^2$

$= 2000\left(\frac{2000}{12}\right) - 6\left(\frac{2000}{12}\right)^2$

$= \frac{500{,}000}{3}$

$\therefore$ market price $= \frac{2000}{3}$

market quantity $= \frac{2000}{6}$

$\pi_i^C = \frac{250{,}000}{3} \approx 83333.33$

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b) The collusion will successful when

$V_i^* > V_i^r$

$V_i^* = \pi_i^* + \delta\pi_i^* + \delta^2\pi_i^* + \ldots + \delta^\infty \pi^*$

$= \frac{\pi_i^*}{1-\delta}$

$V_i^r = \pi_i^r + \delta\pi_i^r + \delta^2\pi_i^C + \delta^3\pi_i^C + \ldots + \delta^\infty \pi_i^C$

$= \pi_i^r + \delta\pi_i^r + \frac{\delta^2\pi_i^C}{1-\delta}$

$V_i^* \geq V_i^r$

$\frac{\pi_i^*}{1-\delta} \geq \pi_i^r + \delta\pi_i^r + \frac{\delta^2\pi_i^C}{1-\delta}$

$\pi_i^* \geq \pi_i^r(1-\delta) + \delta(1-\delta)\pi_i^r + \delta^2\pi_i^C$

$\pi_i^* \geq \pi_i^r - \delta\pi_i^r + \delta\pi_i^r - \delta^2\pi_i^r + \delta^2\pi_i^C$

(cancel out)

$\delta^2(\pi_i^r - \pi_i^C) \geq \pi_i^r - \pi_i^*$

$\delta^2 \geq \frac{\pi_i^r - \pi_i^*}{\pi_i^r - \pi_i^C}$

plug in $\pi_i^* = 500{,}000/3$, $\pi_i^C = 80{,}000$, $\pi_i^r = 781$

$\delta^2 \geq \frac{781250/9 - 250000/3}{781250/9 - 80000}$

$\delta^2 \geq \frac{25}{49}$

$\delta \geq \frac{5}{7} = 0.714$

$\pi_i^* = \frac{500{,}000}{3}$

$\pi_i^r = TR - TC$

$= P(q_i^r) - q_i^{r2}$

$= (1000 - Q)(q_i^r) - q_i^{r2}$

$= (1000 - q_i^r - \frac{500}{3}q_i - q_i^{r2}$

$= \frac{2500}{3}q_i^r - q_i^{r2} - q_i^{r2}$

$= \frac{2500}{3}q_i^r - 2q_i^{r2}$

$FOC = \frac{\partial \pi_i^r}{\partial q_i^r} = \frac{2500}{3} - 4q_i^r = 0$

$q_i^r = \frac{625}{3}$, $Q_i^r = \frac{625}{3} + \frac{500}{3} = \frac{1125}{3}$

π_i^C:

Firm 1 $\pi = TR - TC$

$= P \cdot q_1 - q_1^2$

$= (1000 - q_1 - q_2)q_1 - q_1^2$

$= 1000q_1 - q_1^2 - q_1q_2 - q_1^2$

$FOC = \frac{\partial \pi_1^C}{\partial q_1} = 1000 - q_2 - 4q_1 = 0$ — ①

Firm 2 $\pi = P \cdot q_2 - q_2^2$

$= 1000q_2 - q_1q_2 - 2q_2^2$

$FOC = \frac{\partial \pi^C}{\partial q_1} = 1000 - q_1 - 4q_2 = 0$ — ②

$q_1 = 1000 - 4q_2$

solve for q_2 cournot

put in $1000 - 2q_2 = q_1$ in ①

$1000 - q_2 - 4(1000 - 4q_2) = 0$

$1000 - q_2 - 4000 + 16q_2 = 0$

$15q_2 = 3000$

$q_2^C = 200 = q_1^C$

$Q^C = 400$

$P^C = 1000 - 400 = 600$

$\pi^C = P \cdot q_1 - q_1^2$

$= 600 \cdot 200 - 200^2$

$= 80000$

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c) The cost effective firm may have more incentive to cheat since he/she face to lower cost. If firm with lower cost cheat they can produce a lot and gain large profit.

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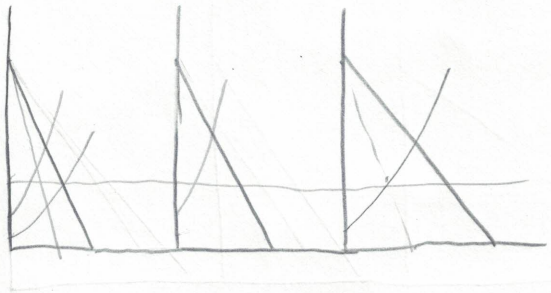
Moreover, even after another firm detect the cheating, firm with lower cost still benefit from lower cost structure to become dominant firm instead.



d) To use bertrand instead of cournot may not be an effective way as bertrand strategy, firm will continue cut the price to fight back. When firm 1 cut price, he/she will gain all the customer. In the next period another firm will fight back and make the price lower that price firm 1 set. In the end, price cut will continue until $P = MC$ where there is no positive profit.

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Both firm know that given that zero profit another firm won't retaliate using bertrand strategy. Then, it won't effective to do.