(1)

HW2

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1.1) $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$\hat{\beta}_2 = \dfrac{\sum x_i y_i}{\sum x_i^2}$

$\bar{Y} = \dfrac{2.8 + 3.4 + 3 + 3.5 + 3.6 + 3 + 2.7 + 3.7}{8} = 3.2125$

$\bar{X} = \dfrac{63+72+78+91+97+75+75+90}{8} = 77.625$

$x_i = X_i - \bar{X}$, $y_i = Y_i - \bar{Y}$

$x_1 = -14.625$ $y_1 = -0.4125$ $x_1y_1 = 6.0328$

$x_2 = -5.625$ $y_2 = 0.1875$ $x_2y_2 = -1.0547$

$x_3 = 0.375$ $y_3 = -0.2125$ $x_3y_3 = -0.0797$

$x_4 = 3.375$ $y_4 = 0.2875$ $x_4y_4 = 0.9703$

$x_5 = 9.375$ $y_5 = 0.3875$ $x_5y_5 = 3.6327$

$x_6 = -2.625$ $y_6 = -0.2125$ $x_6y_6 = 0.5578$

$x_7 = -2.625$ $y_7 = -0.5125$ $x_7y_7 = 1.3453$

$x_8 = 12.375$ $y_8 = 0.4875$ $x_8y_8 = 6.0328$

$x_1^2 = 213.8906$

$x_2^2 = 31.6406$

$x_3^2 = 0.1406$

$x_4^2 = 11.3906$

$x_5^2 = 87.8906$

$x_6^2 = 6.8906$

$x_7^2 = 6.8906$

$x_8^2 = 153.1406$

$\sum x_i^2 = 511.8748$ $\sum x_i y_i = 17.4374$

$\hat{\beta}_2 = \dfrac{17.4374}{511.8748} = 0.0341$ *

$\hat{\beta}_1 = 3.2125 - (0.0341)(77.625) = 0.5655$ *

1.2 $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_i = 0.5655 + 0.0341 X_i$

$\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{Y}_1 = 0.5655 + 0.0341(63) = 2.7138$ $\hat{u}_1 = 2.8 - 2.7138 = 0.0862$ $\hat{u}_1^2 = 0.0074$

$\hat{Y}_2 = 0.5655 + 0.0341(72) = 3.0207$ $\hat{u}_2 = 3.4 - 3.0207 = 0.3793$ $\hat{u}_2^2 = 0.1439$

$\hat{Y}_3 = 0.5655 + 0.0341(78) = 3.2253$ $\hat{u}_3 = 3 - 3.2253 = -0.2253$ $\hat{u}_3^2 = 0.0508$

$\hat{Y}_4 = 0.5655 + 0.0341(91) = 3.3276$ $\hat{u}_4 = 3.5 - 3.3276 = 0.1724$ $\hat{u}_4^2 = 0.0297$

$\hat{Y}_5 = 0.5655 + 0.0341(97) = 3.5322$ $\hat{u}_5 = 3.6 - 3.5322 = 0.0678$ $\hat{u}_5^2 = 0.0046$

$\hat{Y}_6 = 0.5655 + 0.0341(75) = 3.123$ $\hat{u}_6 = 3 - 3.123 = -0.1230$ $\hat{u}_6^2 = 0.0151$

$\hat{Y}_7 = 0.5655 + 0.0341(75) = 3.123$ $\hat{u}_7 = 2.7 - 3.123 = -0.4230$ $\hat{u}_7^2 = 0.1789$

$\hat{Y}_8 = 0.5655 + 0.0341(90) = 3.6345$ $\hat{u}_8 = 3.7 - 3.6345 = 0.0655$ $\hat{u}_8^2 = 0.0043$

$\sum \hat{u}_i = 0.00001 \approx 0$ $\sum \hat{u}_i^2 = 0.4347$ *

1.3 $var(\hat{\beta}_1) = \dfrac{\sum X_i^2}{n \sum x_i^2} \sigma_u^2 = \dfrac{[(63)^2+(72)^2+(78)^2+(91)^2+(97)^2+(75)^2+(75)^2+(90)^2](0.0725)}{(8)(511.8748)} = 0.8625$ *

$var(\hat{\beta}_2) = \dfrac{\sigma_u^2}{\sum x_i^2} = \dfrac{0.0725}{511.8748} = 1.416 \times 10^{-4}$ *

σ_u^2 estimate by $\hat{\sigma}_u^2 = \dfrac{\sum \hat{u}_i^2}{n-2} = \dfrac{0.4347}{8-2} = 0.0725$ *

(2) $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

2.1) $\hat{\beta}_2 = \dfrac{\sum x_i y_i}{\sum x_i^2}$

$\bar{Y} = \dfrac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$

$\bar{X} = \dfrac{10+12+14+16+18+22+24+26+28+30}{10} = 20$

$x_i = X_i - \bar{X}$, $y_i = Y_i - \bar{Y}$

$x_1 = -10$ $x_1^2 = 100$

$x_2 = -8$ $x_2^2 = 64$

$x_3 = -6$ $x_3^2 = 36$

$x_4 = -4$ $x_4^2 = 16$

$x_5 = -2$ $x_5^2 = 4$

$x_6 = 2$ $x_6^2 = 4$

$x_7 = 4$ $x_7^2 = 16$

$x_8 = 6$ $x_8^2 = 36$

$x_9 = 8$ $x_9^2 = 64$

$x_{10} = 10$ $x_{10}^2 = 100$

$\sum x_i^2 = 440$

$y_1 = -9.1$ $x_1y_1 = 91$

$y_2 = -7.1$ $x_2y_2 = 56.8$

$y_3 = -4.1$ $x_3y_3 = 24.6$

$y_4 = -3.1$ $x_4y_4 = 12.4$

$y_5 = -2.1$ $x_5y_5 = 4.2$

$y_6 = 0.9$ $x_6y_6 = 1.8$

$y_7 = 0.9$ $x_7y_7 = 3.6$

$y_8 = 5.9$ $x_8y_8 = 35.4$

$y_9 = 6.9$ $x_9y_9 = 55.2$

$y_{10} = 10.9$ $x_{10}y_{10} = 109$

$\sum x_i y_i = 394$

$\hat{\beta}_2 = \dfrac{394}{440} = 0.8954$ *

$\hat{\beta}_1 = 9.1 - 0.8954(20) = -8.808$ *

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_i = -8.808 + 0.8954 X_i$

$\hat{u}_i = Y_i - \hat{Y}_i$

2.2)

$\hat{Y}_1 = -8.808 + 0.8954\,(10) = 0.146$ $\quad \hat{u}_1 = 0 - 0.146 = -0.146$ $\quad \hat{u}_1^2 = 0.0213$

$\hat{Y}_2 = -8.808 + 0.8954\,(12) = 1.9368$ $\quad \hat{u}_2 = 2 - 1.9368 = 0.0632$ $\quad \hat{u}_2^2 = 0.0039$

$\hat{Y}_3 = -8.808 + 0.8954\,(14) = 3.7276$ $\quad \hat{u}_3 = 5 - 3.7276 = 1.2724$ $\quad \hat{u}_3^2 = 1.6190$

$\hat{Y}_4 = -8.808 + 0.8954\,(16) = 5.5184$ $\quad \hat{u}_4 = 6 - 5.5184 = 0.4816$ $\quad \hat{u}_4^2 = 0.2319$

$\hat{Y}_5 = -8.808 + 0.8954\,(18) = 7.3092$ $\quad \hat{u}_5 = 7 - 7.3092 = -0.3092$ $\quad \hat{u}_5^2 = 0.0956$

$\hat{Y}_6 = -8.808 + 0.8954\,(22) = 10.8908$ $\quad \hat{u}_6 = 10 - 10.8908 = -0.8908$ $\quad \hat{u}_6^2 = 0.7935$

$\hat{Y}_7 = -8.808 + 0.8954\,(24) = 12.6816$ $\quad \hat{u}_7 = 10 - 12.6816 = -2.6816$ $\quad \hat{u}_7^2 = 7.1909$

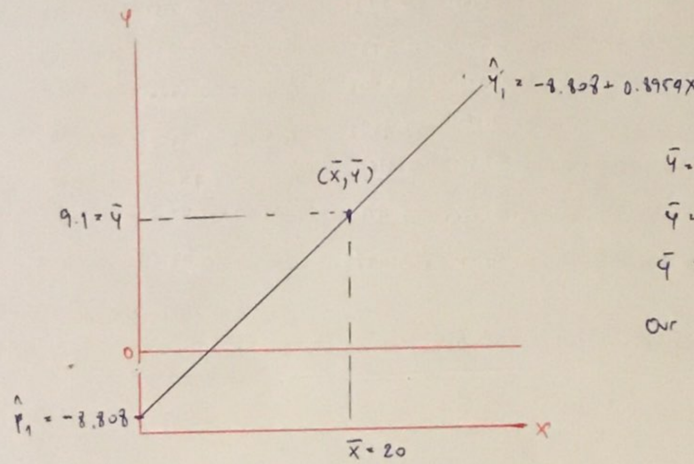
$\hat{Y}_8 = -8.808 + 0.8954\,(26) = 14.4724$ $\quad \hat{u}_8 = 15 - 14.4724 = 0.5276$ $\quad \hat{u}_8^2 = 0.2784$

$\hat{Y}_9 = -8.808 + 0.8954\,(28) = 16.2632$ $\quad \hat{u}_9 = 16 - 16.2632 = -0.2632$ $\quad \hat{u}_9^2 = 0.0693$

$\hat{Y}_{10} = -8.808 + 0.8954\,(30) = 18.054$ $\quad \hat{u}_{10} = 20 - 18.054 = 1.946$ $\quad \hat{u}_{10}^2 = 3.7869$

$\sum \hat{u}_i = 0$ ※ $\qquad \sum \hat{u}_i^2 = 14.0907$

2.3)



$\bar{Y} = -8.808 + 0.8954\,(\bar{X})$

$\bar{Y} = -8.808 + 0.8954\,(20)$

$\bar{Y} = 9.1$

Our SRF will pass through the $(\bar{X}, \bar{Y})$ ※

2.4) If $X_i = 18$ the predicted of Y is $-8.808 + 0.8954\,(18) = 7.3092$ ※

2.5) $var(\hat{u}_1)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2}\sigma^2_u = \frac{(10^2 + 12^2 + 14^2 + 16^2 + 18^2 + 22^2 + 24^2 + 26^2 + 28^2 + 30^2)(1.7613)}{(10)(440)} = 1.7773 \text{ ※}$$

$$var(\hat{\beta}_2) = \frac{\sigma^2_u}{\sum x_i^2} = \frac{1.7613}{440} = 4.0029 \times 10^{-3} \text{ ※}$$

$\sigma^2 u$ estimated by $\hat{\sigma}_u^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0907}{8} = 1.7613$ ※

3 As $\hat{\beta}_1$ is an estimator of β_1, $E(\hat{\beta}_1)$ should be equal to β_1

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$, $\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$ let $k = \frac{x_i}{\sum x_i^2}$

$\hat{\beta}_1 = \bar{Y} - \sum k Y_i \bar{X}$ and $Y_i = \beta_1 + \beta_2 X_i + u_i$

$$\hat{\beta}_1 = \frac{\sum Y_i}{n} - \sum k Y_i \bar{X} = \sum\left(\frac{1}{n} - k\bar{X}\right)Y_i$$

$$= \sum\left(\frac{1}{n} - k\bar{X}\right)(\beta_1 + \beta_2 X_i + u_i)$$

$$= \sum\left(\frac{\beta_1}{n} + \frac{\beta_2 X_i}{n} + \frac{u_i}{n} - k\bar{X}\beta_1 - k\bar{X}\beta_2 X_i - k\bar{X}u_i\right)$$

$$\hat{\beta}_1 = \beta_1 + \beta_2\bar{X} + \cancel{\frac{\sum u_i}{n}} - \beta_2\bar{X} - \bar{X}\sum k_i u_i$$

as k is $f(x)$, k would be constant $\sum k_i$ = constant

$E(\hat{\beta}_1) = E(\beta_1) + \bar{X}\sum k_i \cancel{E(u_i)}^{0}$

$E(\hat{\beta}_1) = \beta_1$

$\hat{\beta}_1$ is unbiased estimated of β_1 ※

take $E() \rightarrow E(\hat{\beta}_1) = E(\beta_1) + \bar{X}\,E(\sum k_i u_i) \rightarrow$ assumption 3: zero mean value of disturbance treat x as given $E(u_i|X_i)$