1. Unit root test used to check that the variable are stationary series or not.
The hypotheses are the followings. $H_0: \alpha_i = 0$ and we find out that P-value $= 1$ which is larger than 0.05. Hence, There exist unit root and the series of X is non stationary.
For Y, we test whether $H_0: \gamma = 0$ or not. The result shows P-value $= 0.907$ which is greater than 0.05 thus, We fail to reject null hypothesis. Hence, there exist unit root and the series of Y is non stationary.
To find the order of integration since we know the series are not integrated at order 0. We need to test whether the series are integrate at order 1 or not by performing unit root test of ΔX_t and ΔY_t $H_0: \gamma = 0$ and $H_0: \alpha = 0$ and we got P-value from Mackinnon approximate equals to $0.000 < 0.05$. Thus we reject the null hypothesis so, X and Y series is stationary at I(1)

```
. tsset t
        time variable:  t, 1 to 500
                delta:  1 unit

. dfuller y, trend lag(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             1.000            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 1.0000

------------------------------------------------------------------------------
D.y          |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   .0001178   .0001179     1.00   0.318    -.0001137    .0003494
         LD. |   .6997015   .0248993    28.10   0.000     .6507799    .7486231
      _trend |   2.897751   1.159296     2.50   0.013      .619992    5.175511
       _cons |   1811.233   147.0426    12.32   0.000     1522.327    2100.139

. dfuller d.y, trend lag(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       497

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)           -10.554            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
D2.y         |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         D.y |
         L1. |  -.2787856   .0264156   -10.55   0.000    -.3306866   -.2268845
         LD. |    -.32127   .0373756    -8.60   0.000    -.3947051   -.2478349
      _trend |   3.631984   .3708318     9.79   0.000     2.903379     4.36059
       _cons |   1678.082   154.4788    10.86   0.000     1374.564      1981.6
------------------------------------------------------------------------------


. dfuller x, trend lag(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             0.601            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.9970

------------------------------------------------------------------------------
D.x          |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |
         L1. |   .0001061   .0001764     0.60   0.548    -.0002405    .0004526
         LD. |     .46018   .0349881    13.15   0.000     .3914361    .5289239
      _trend |   4.166909    1.14105     3.65   0.000     1.924999    6.408818
       _cons |   2128.626   140.0551    15.20   0.000     1853.449    2403.803
------------------------------------------------------------------------------
```

```
. dfuller d.x, trend lag(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       497

                              ---------- Interpolated Dickey-Fuller ----------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -10.657            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
D2.x         |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         D.x |
         L1. |  -.4007114   .0376021   -10.66   0.000    -.4745915   -.3268312
         LD. |   -.414172   .0358603   -11.55   0.000    -.4846298   -.3437141
      _trend |   3.508317   .3506567    10.00   0.000     2.819351    4.197283
       _cons |   1596.429   147.0971    10.85   0.000     1307.415    1885.444
------------------------------------------------------------------------------
```

2.

```
. vecrank y x, trend(t) lags(1/1) max
```

→ Linear Trend

```
                       Johansen tests for cointegration
Trend: trend                                        Number of obs =     499
Sample:  2 - 500                                             Lags =       1
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL      eigenvalue  statistic    value
    0       4    -7387.4577        .       790.3357    18.17
    1       7    -6992.3441     0.79477      0.1085*    3.74
    2       8    -6992.2899     0.00022
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL      eigenvalue  statistic    value
    0       4    -7387.4577        .       790.2272    16.87
    1       7    -6992.3441     0.79477      0.1085     3.74
    2       8    -6992.2899     0.00022
-------------------------------------------------------------------------------
```

```
. vecrank y x, trend(rt) lags(1/1) max
```

→ Restricted Trend

```
                       Johansen tests for cointegration
Trend: rtrend                                       Number of obs =     499
Sample:  2 - 500                                             Lags =       1
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL      eigenvalue  statistic    value
    0       2    -8050.4781        .      2116.3764    25.32
    1       6    -7075.2453     0.97993    165.9107    12.25
    2       8    -6992.2899     0.28286
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL      eigenvalue  statistic    value
    0       2    -8050.4781        .      1950.4657    18.96
    1       6    -7075.2453     0.97993    165.9107    12.52
    2       8    -6992.2899     0.28286
-------------------------------------------------------------------------------
```

```
. vecrank y x, trend(c) lags(1/1) max
```

→ Unrestricted Constant

```
                       Johansen tests for cointegration
Trend: constant                                     Number of obs =     499
Sample:  2 - 500                                             Lags =       1
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL      eigenvalue  statistic    value
    0       2    -8050.4781        .      2086.8946    15.41
    1       5    -7083.8611     0.97923    153.6607     3.76
    2       6    -7007.0308     0.26504
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL      eigenvalue  statistic    value
    0       2    -8050.4781        .      1933.2339    14.07
    1       5    -7083.8611     0.97923    153.6607     3.76
    2       6    -7007.0308     0.26504
-------------------------------------------------------------------------------
```

```
. vecrank y x, trend(rc) lags(1/1) max
```

→ Restricted Constant

```
                       Johansen tests for cointegration
Trend: rconstant                                    Number of obs =     499
Sample:  2 - 500                                             Lags =       1
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL      eigenvalue  statistic    value
    0       0    -8811.3759        .      3607.9616    19.96
    1       4    -7091.9244     0.99898    169.0586     9.42
    2       6    -7007.3951     0.28737
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL      eigenvalue  statistic    value
    0       0    -8811.3759        .      3438.9030    15.67
    1       4    -7091.9244     0.99898    169.0586     9.24
    2       6    -7007.3951     0.28737
-------------------------------------------------------------------------------
```

```
. vecrank y x, trend(n) lags(1/1) max
```

→ No trend.

```
                       Johansen tests for cointegration
Trend: none                                            Number of obs =     499
Sample:  2 - 500                                                Lags =       1
-------------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       0    -8811.3759         .       3204.1196   12.53
    1       3    -7211.7591     0.99836        4.8860    3.84
    2       4    -7209.3161     0.00974
-------------------------------------------------------------------------------
                                                        5%
maximum                                        max   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       0    -8811.3759         .       3199.2337   11.44
    1       3    -7211.7591     0.99836        4.8860    3.84
    2       4    -7209.3161     0.00974
-------------------------------------------------------------------------------
```

3. One lag term

```
. vecrank y x, trend(t) lags(1) max

                       Johansen tests for cointegration
Trend: trend                                           Number of obs =     499
Sample:  2 - 500                                                Lags =       1
-------------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       4    -7387.4577         .        790.3357   18.17
    1       7    -6992.3441     0.79477        0.1085*   3.74
    2       8    -6992.2899     0.00022
-------------------------------------------------------------------------------
                                                        5%
maximum                                        max   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       4    -7387.4577         .        790.2272   16.87
    1       7    -6992.3441     0.79477        0.1085    3.74
    2       8    -6992.2899     0.00022
-------------------------------------------------------------------------------
```

Two lag term

```
. vecrank y x, trend(t) lags(2) max

                       Johansen tests for cointegration
Trend: trend                                           Number of obs =     498
Sample:  3 - 500                                                Lags =       2
-------------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       8    -6795.5337         .        187.5771   18.17
    1      11    -6703.3826     0.30932        3.2749*   3.74
    2      12    -6701.7451     0.00655
-------------------------------------------------------------------------------
                                                        5%
maximum                                        max   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       8    -6795.5337         .        184.3022   16.87
    1      11    -6703.3826     0.30932        3.2749    3.74
    2      12    -6701.7451     0.00655
-------------------------------------------------------------------------------
```

Three lag term

```
.
. vecrank y x, trend(t) lags(3) max

                       Johansen tests for cointegration
Trend: trend                                           Number of obs =     497
Sample:  4 - 500                                                Lags =       3
-------------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0      12    -6756.658          .        140.0434   18.17
    1      15    -6688.113      0.24106        2.9534*   3.74
    2      16    -6686.6363     0.00592
-------------------------------------------------------------------------------
                                                        5%
maximum                                        max   critical
  rank    parms       LL       eigenvalue  statistic   value
    0      12    -6756.658          .        137.0900   16.87
    1      15    -6688.113      0.24106        2.9534    3.74
    2      16    -6686.6363     0.00592
-------------------------------------------------------------------------------
```

4. With one lag term, the model with the linear trend is statistically insignificant in 5% critical value $(0.1085 < 3.74)$ $H_0: r=1$ and $H_1: r \geq 1$. Thus we accept null hypothesis. The cointegration is at most 1

With 2 lag term, the model with the linear trend is statistically insignificant in 5% critical value $(3.2749 < 3.74)$ $H_0: r=1$ and $H_1: r \geq 1$. Thus we accept null hypothesis. The cointegration is at most 1

With 8 lag term, the model with the linear trend is statistically insignificant in 5% critical value (2.9584<3.841) Ho: r=1 and H1: r≥1. Thus we accept null hypothesis. The cointegration is at most 1

5.

```
. vec y x

Vector error-correction model

Sample:  3 - 500                                Number of obs     =        498
                                                AIC               =   27.18032
Log likelihood = -6758.899                      HQIC              =   27.21018
Det(Sigma_ml)  =  2.11e+09                      SBIC              =   27.25641

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_y                  4     213.415   0.9995   964110.3   0.0000
D_x                  4     263.876   0.9982   280728.2   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |   -.427199   .0483639    -8.83   0.000    -.5219905   -.3324075
             |
           y |
         LD. |   .4022605   .0224005    17.96   0.000     .3583563    .4461646
             |
           x |
         LD. |   .4814843   .0598769     8.04   0.000     .3641276    .5988409
             |
       _cons |   171.4546   44.03061     3.89   0.000     85.15624    257.7531
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |   .2903827   .0597995     4.86   0.000     .1731779    .4075875
             |
           y |
         LD. |   .5734396   .0276971    20.70   0.000     .5191544    .6277249
             |
           x |
         LD. |   .3676105   .0740347     4.97   0.000     .2225051    .5127159
             |
       _cons |    252.237   54.44157     4.63   0.000     145.5335    358.9405
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                 1   8.09e+08   0.0000
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500176   .0000527 -2.8e+04   0.000     -1.50028   -1.500073
       _cons |  -577.9622          .        .       .            .           .
------------------------------------------------------------------------------
```

```
. vec y x, lag(2)

Vector error-correction model

Sample:  3 - 500                                Number of obs     =        498
                                                AIC               =   27.18032
Log likelihood = -6758.899                      HQIC              =   27.21018
Det(Sigma_ml)  =  2.11e+09                      SBIC              =   27.25641

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_y                  4     213.415   0.9995   964110.3   0.0000
D_x                  4     263.876   0.9982   280728.2   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |   -.427199   .0483639    -8.83   0.000    -.5219905   -.3324075
             |
           y |
         LD. |   .4022605   .0224005    17.96   0.000     .3583563    .4461646
             |
           x |
         LD. |   .4814843   .0598769     8.04   0.000     .3641276    .5988409
             |
       _cons |   171.4546   44.03061     3.89   0.000     85.15624    257.7531
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |   .2903827   .0597995     4.86   0.000     .1731779    .4075875
             |
           y |
         LD. |   .5734396   .0276971    20.70   0.000     .5191544    .6277249
             |
           x |
         LD. |   .3676105   .0740347     4.97   0.000     .2225051    .5127159
             |
       _cons |    252.237   54.44157     4.63   0.000     145.5335    358.9405
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                 1   8.09e+08   0.0000
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500176   .0000527 -2.8e+04   0.000     -1.50028   -1.500073
       _cons |  -577.9622          .        .       .            .           .
------------------------------------------------------------------------------
```

```
. vec y x, lag(3)
```

Vector error-correction model

Sample: 4 - 500		Number of obs	= 497
		AIC	= 27.1245
Log likelihood = -6727.439		HQIC	= 27.16771
Det(Sigma_ml) = 1.96e+09		SBIC	= 27.23459

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	6	211.673	0.9995	980003.4	0.0000
D_x	6	253.098	0.9984	305162.2	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D_y						
_ce1						
L1.	-.3193909	.058155	-5.49	0.000	-.4333726	-.2054092
y						
LD.	.3553209	.0559202	6.35	0.000	.2457194	.4649225
L2D.	.0174858	.0312998	0.56	0.576	-.0438607	.0788323
x						
LD.	.6042668	.0763396	7.92	0.000	.4546439	.7538896
L2D.	.0521968	.0634683	0.82	0.411	-.0721988	.1765925
_cons	181.5838	45.38519	4.00	0.000	92.63049	270.5372
D_x						
_ce1						
L1.	.4434064	.0695359	6.38	0.000	.3071185	.5796943
y						
LD.	.4713427	.0668638	7.05	0.000	.3402922	.6023932
L2D.	.012763	.0374252	0.34	0.733	-.060589	.0861151
x						
LD.	.5220109	.0912792	5.72	0.000	.3431069	.700915
L2D.	.0911602	.0758891	1.20	0.230	-.0575797	.2399001
_cons	130.797	54.26707	2.41	0.016	24.43548	237.1585

Cointegrating equations

Equation	Parms	chi2	P>chi2
_ce1	1	6.64e+08	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

beta	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_ce1						
y	1	.	.	.	.	.
x	-1.500049	.0000582	-2.6e+04	0.000	-1.500163	-1.499935
_cons	-92.13439	.	.	.	.	.

1. In all the model, the overall test for cointegrated equations are statistically significant P > Chi2 is 0.00. The cointegration relation exist.

2. The optimal lags is lag 3 since it has lowest of SBIC of 27.23459

3. The long-term relationship or cointegrating equation is $Y_t = \beta_0 + \beta_1 x_t$. Then, the CE from lag-3 model is

$$y = 92.13439 + 1.500049 x_t.$$

4. The speed of adjustment of 3-lag model is -0.3193909 for y and 0.4434064 for x.

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