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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

$Y_i - \bar{Y}$	$X_i - \bar{X}$
−0.4125	−14.625
0.1875	−5.625
−0.2125	0.375
0.2875	3.375
0.3875	9.375
−0.2125	−2.625
−0.5125	−2.625
0.4875	12.375

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$\bar{y} = 3.2125$

$\bar{X} = 77.625$

$$\beta_2 = \frac{\Sigma (Y_i - \bar{Y})(X_i - \bar{X})}{\Sigma (X_i - \bar{X})^2}$$

$\beta_1 = \bar{y} - \beta_2 \bar{X}$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

1.1) $\beta_2 = \dfrac{\Sigma(y_i)(x_i)}{\Sigma(x_i)^2}$

$$= \frac{6.0328 - 1.0574 - 0.0797 + 0.9703 + 3.6328 + 0.5578 + 1.3453 + 6.0328}{511.875}$$

$$= \frac{17.4347}{511.875} = 0.0341 \quad \#$$

$\beta_1 = \bar{Y} - \beta_2 \bar{X}$

$= 3.2125 - (0.0341)\, 77.625$

$= 3.2125 - 2.6470$

$= 0.5655 \quad \#$

$Y_i = 0.5655 + 0.0341\, X_i + u_i \quad \#$

1.2) $\hat{Y}_1 = 0.5655 + 0.0341(63)$

$= 2.7138$

$\hat{Y}_2 = 0.5655 + 0.0341(72)$

$= 3.0207$

$\hat{Y}_3 = 3.2253$

$\hat{Y}_4 = 3.3276$

$\hat{Y}_5 = 3.5322$

$\hat{Y}_6 = 3.1230$

$\hat{Y}_7 = 3.1230$

$\hat{Y}_8 = 3.6345$

$\hat{U} = Y_i - \hat{Y}_i$

$\hat{U}_1 = 0.0862$

$\hat{U}_2 = 0.3793$

$\hat{U}_3 = -0.2253$

$\hat{U}_4 = 0.1724$

$\hat{U}_5 = 0.0678$

$\hat{U}_6 = -0.123$

$\hat{U}_7 = -0.423$

$\hat{U}_8 = 0.0655$

$$\sum_{i=1}^{n=8} \hat{u}_i = -0.0001$$

≈ 0 #

1.3) $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \frac{SSR}{n-2} = \frac{\sum_{i=1}^{8} \hat{u}_i^2}{n-2} = \frac{\sum_{i=1}^{8} \hat{u}_i^2}{6}$$

$$= \frac{0.4347}{6} = 0.0725 \quad \#$$

$$var(\hat{\beta}_1) = \frac{\sum (X_i)^2 \, \sigma^2}{n \sum x_i^2} \ ; \ x_i = X_i - \bar{X}$$

$$= \frac{(48717)(0.0725)}{8(511,875)} = 0.8625 \quad \#$$

$$var(\hat{\beta}_2) = \frac{\sigma_u^2}{\sum x_i^2} = \frac{0.0725}{511.875} = 0.0001 \quad \#$$

2. Data is listed in the table

$\bar{X} = 20$

$\bar{Y} = 9.1$

$Y_i - \bar{Y}$	$x_i - \bar{x}$
-9.1	-10
-7.1	-8
-4.1	-6
-3.1	-4
-2.1	-2
0.9	2
0.9	4
5.9	6
6.9	8
10.9	10

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

2.1) $\beta_2 = \dfrac{\sum (x_i)(y_i)}{\sum (x_i)^2}$

$$= \frac{(9.1)(-10) + (-7.1)(-8) + (-4.1)(-6) + (-3.1)(-4) + (-2.1)(-2) + (0.9)(2) + (0.9)(4) + (5.9)(6) + (6.9)(8) + (10.9)(10)}{(-10)^2 + (-8)^2 + (-6)^2 + (-4)^2 + (-2)^2 + (2)^2 + (4)^2 + (6)^2 + (8)^2 + (10)^2}$$

$$= \frac{91 + 56.8 + 24.6 + 12.4 + 4.2 + 1.8 + 3.6 + 35.4 + 55.2 + 109}{440}$$

$$= \frac{394}{440} = 0.8955 \#$$

$$\beta_1 = \bar{y} - \beta_2 \bar{x}$$
$$= 9.1 - 0.8955(20)$$
$$= 9.1 - 17.91$$
$$= -8.81 \ \#$$

$$y_i = -8.81 + 0.8955 X_i + u_i \ \#$$

2.2) $\hat{Y}_1 = -8.81 + 0.8955(10) = 0.145$

$\hat{Y}_2 = -8.81 + 0.8955(12) = 1.936$

$\hat{Y}_3 = -8.81 + 0.8955(14) = 3.727$

$\hat{Y}_4 = -8.81 + 0.8955(16) = 5.518$

$\hat{Y}_5 = -8.81 + 0.8955(18) = 7.309$

$\hat{Y}_6 = -8.81 + 0.8955(22) = 10.891$

$\hat{Y}_7 = -8.81 + 0.8955(24) = 12.682$

$\hat{Y}_8 = -8.81 + 0.8955(26) = 14.473$

$\hat{Y}_9 = -8.81 + 0.8955(28)$
$= 16.264$

$\hat{Y}_{10} = -8.81 + 0.8955(30)$
$= 18.055$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_1 = 0 - 0.145 = -0.145$$

$$\hat{u}_2 = 2 - 1.936 = 0.064$$

$$\hat{u}_3 = 5 - 3.727 = 1.273$$

$$\hat{u}_4 = 6 - 5.518 = 0.482$$

$$\hat{u}_5 = 7 - 7.309 = -0.309$$

$$\hat{u}_6 = 10 - 10.891 = -0.891$$

$$\hat{u}_7 = 10 - 12.682 = -2.682$$

$$\hat{u}_8 = 15 - 14.473 = 0.527$$

$$\hat{u}_9 = 16 - 16.264 = -0.264$$

$$\hat{u}_{10} = 20 - 18.055 = 1.945$$

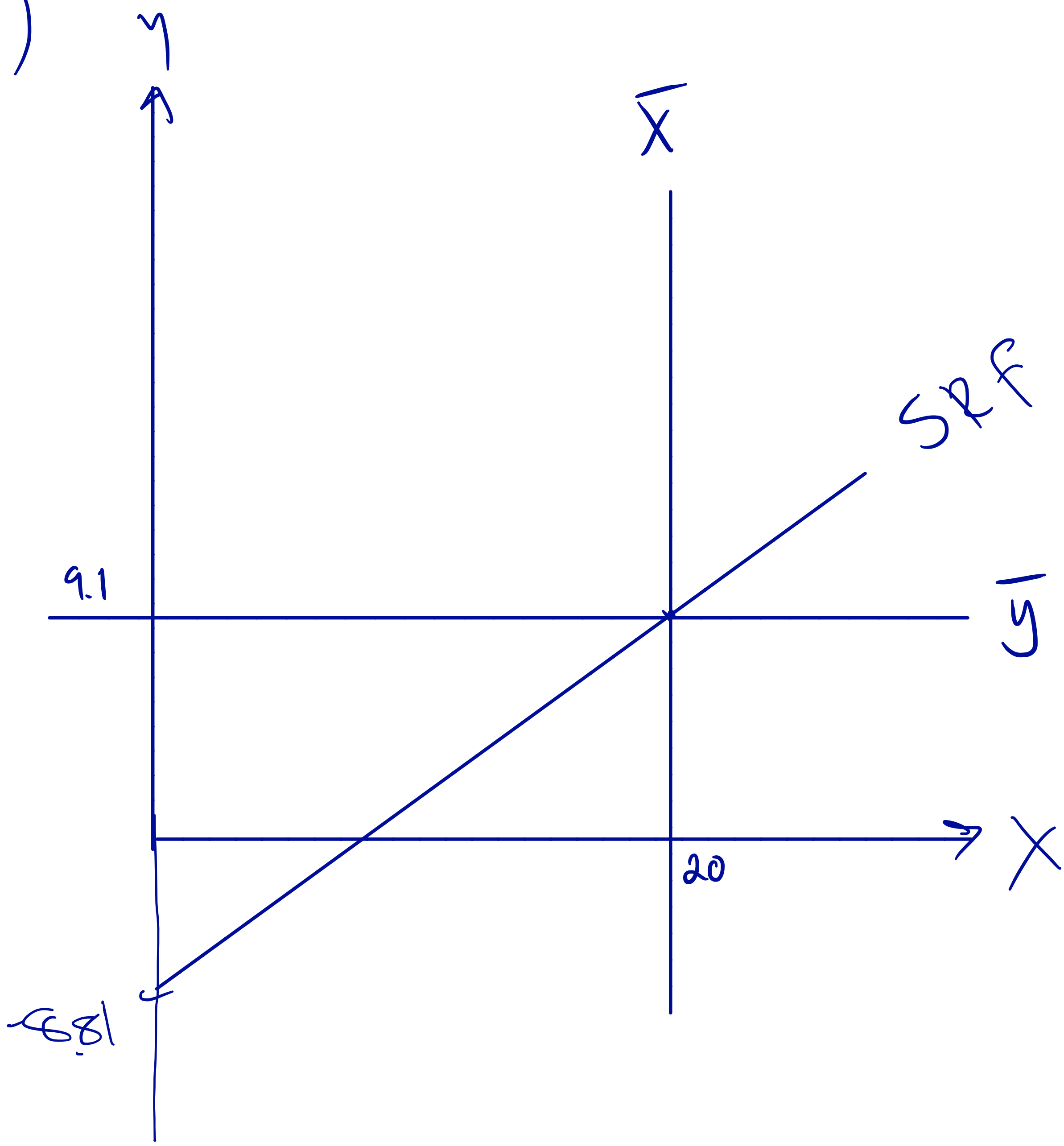
$$\sum u_i = -0.145 + 0.064 + 1.273 + 0.482$$

$$-0.309 - 0.891 - 2.682 + 0.527$$

$$-0.264 + 1.945 = 0$$

#

2.3)



we plug $\bar{x} = 20$ into the function.

$$\bar{y} = -8.81 + 0.8995(20)$$

$$= 9.1$$

#

$\therefore$ this regression function pass through $(\bar{x}, \bar{y})$ #

2.4) $x_i = 18$, $\hat{y}_i = ?$

$$\hat{y}_i = -8.81 + 0.8995(18)$$

$$= 7.309$$

2.5) $var(\hat{u})$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$var(\hat{u}) = \frac{\Sigma u_i^2}{n-2} = \frac{\Sigma (Y_i - \hat{Y})^2}{n-2}$$

$$= \frac{14.0909}{8} = 1.7614$$

$$var(\hat{\beta}_1) = \frac{\Sigma x_i^2}{n \Sigma (x_i - \bar{x})^2} = \frac{4440}{10(440)} = 1.0091$$

$$var(\hat{\beta}_2) = \frac{\delta^2}{\Sigma x_i^2} = \frac{1.7614}{4,440} = 0.004.$$

3.) $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \hat{x}_i + \hat{u}_i$

min $\sum \hat{u}_i^2$ $\qquad \hat{u}_i = \hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 x_i$

$\Downarrow$

$$\sum \hat{u}_i^2 = \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i)^2$$

diff to find min.

$$-2 \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i) = 0$$

$$\sum \hat{y}_i - \sum \hat{\beta}_1 - \beta_2 \sum \hat{x}_i = 0$$

$$\sum \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i$$

$$n \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i \quad \div n$$

$$\beta_1 = \bar{y} - \beta_2 \bar{x} \ \#$$

unbias when $E(\hat{\beta}_1) = \beta_1$

$$\hat{\beta}_1 = \bar{y} - \beta_2 \bar{x}$$

$$\hat{\beta}_1 = \beta_1 + \cancel{\beta_2 \bar{x}} - \cancel{\beta_2 \bar{x}}$$

$$\hat{\beta}_1 = \beta_1$$

$$E(\hat{\beta}_1) = \beta_1$$