Student Name Pavita kriathkungwalkai ID 6104640674

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i)$ $= (a+bx_1)+(a+bx_2)+(a+bx_3)+(a+bx_4)+(a+bx_5)$

   $= 5a+bx_1+bx_2+bx_3+bx_4+bx_5$

   b. $\sum_{y=0}^{5} f(x+y)$ $= f(x)+f(x+1)+f(x+2)+f(x+3)+f(x+4)+f(x+5)$

   c. $\sum_{i=1}^{10} i^2$ $= 1+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2$

   $= 1+4+9+16+25+36+49+64+81+100$

   $= 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$ $= (2(1)+2)+(2(2)+(3))$

   $= 4+7$

   $= 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b

   $0.5b+b+2.25b+2b+1.5b+0.5b+0.25b = 1$

   $8b = 1$

   $b = \frac{1}{8}$

   b. Find the answer for $P(X \le 2)$

   $= 1-P(X=3)-P(X=4)$

   $= 1-0.0625-0.03125$

   $= 0.90625$

   c. Find the answer for $P(-2 \le X \le 3)$

   $= 1-P(X=4)$

   $= 1-0.3125$

   $= 0.96875$

   d. Find the answer for $P(X \ge 1)$

   $= P(X=1)+P(X=2)+P(X=3)+P(X=4)$

   $= 0.25+0.1875+0.0625+0.03125$

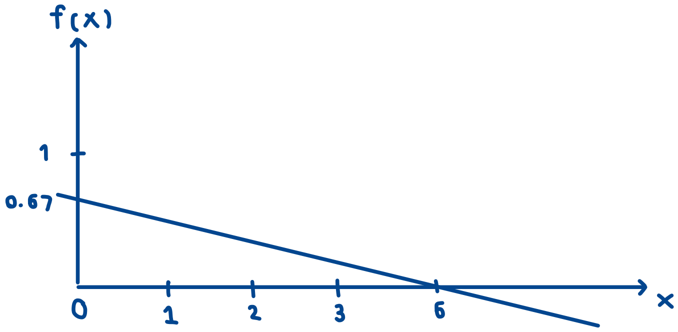
   $= 0.53125$

Student Name Pavita kriathkungwalkai ID 6104640674

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$P(1 \leq x \leq 3) = \int_1^3 f(x)dx$

$= \int_1^3 -\frac{1}{9}x + \frac{6}{9} dx$

$= -\frac{x^2}{18} + \frac{6}{9}x \Big|_1^3$

$= \left[-\frac{9}{18} + \frac{18}{9}\right] - \left[\frac{-1}{18} + \frac{6}{9}\right]$

$= -\frac{9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9}$

$= -\frac{8}{18} + \frac{12}{9}$

$= \frac{16}{18}$

c. Find the answer for $P(X \ge 2)$

$P(X \geq 2) = \int_2^3 f(x)dx$

$= \int_2^3 -\frac{1}{9}x + \frac{6}{9} dx$

$= -\frac{x^2}{18} + \frac{6}{9}x \Big|_2^3$

$= \left[\frac{-9}{18} + \frac{18}{9}\right] - \left[\frac{-4}{18} + \frac{12}{9}\right]$

$= -\frac{5}{18} + \frac{6}{9}$

$= \frac{7}{18}$

d. Find the expected value of X

$E(x) = \int_0^3 x f(x)dx$

$= \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx$

$= \int_0^3 -\frac{1x^2}{9} + \frac{6x}{9} dx$

$= -\frac{x^3}{27} + \frac{6x^2}{18} \Big|_0^3$

$= -1 + \frac{54}{18} = \frac{36}{18} = 2$

Student Name Pavita Kriathkungwalkai ID 6104640674

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

Y/X	1	2	3	4	5	6
0	1/12	1/12	1/12	1/12	1/12	1/12
1	1/12	1/12	1/12	1/12	1/12	1/12

   b. Find the marginal probability distribution function (PDF) of X

X	1	2	3	4	5	6	total
P_i	2/12	2/12	2/12	2/12	2/12	2/12	1

marginal probability of X = 1

   c. Find the marginal probability distribution function (PDF) of Y

Y	0	1	total
P_i	6/12	6/12	1

marginal probability of Y = 1

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$P(X|Y=1)$

X	1	2	3	4	5	6
$P(X=x|Y=1)$	1/6	1/6	1/6	1/6	1/6	1/6

   e. Find the expected value of X given Y is equal to 1

$$P(X|Y=1)\ \Sigma x_i P(X=x_i|Y=1) = \frac{\Sigma x_i P(X=x_i, Y=1)}{P(Y=1)} = \frac{1}{P(Y=1)} \Sigma x_i P(X=x_i, Y=1)$$

$$= \frac{1}{0.5}\left[(1\cdot 1/12) + (2\cdot 1/12) + (3\cdot 1/2) + (4\cdot 1/2) + (5\cdot 1/2) + (6\cdot 1/2)\right] = 7/2$$

   f. Find the variance of X given Y is equal to 1

$$\mathrm{var}(X|Y=1) = \Sigma (X - E(X|Y=1^2) \,.\, P(X|Y=1)$$

$$= \left[((1-7/2)^2\cdot 1/6) + ((2-7/2)^2\cdot 1/6) + ((3-7/2)^2\cdot 1/6) + ((4-7/2)^2\cdot 1/6)\right.$$

$$\left. + ((5-7/2)^2\cdot 1/6) + ((6-7/2)^2\cdot 1/6)\right)$$

$$= 10/3$$

Student Name Pavita kriathkungwalkai ID 6104640674

5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$

$= \frac{1}{N} E(x_1 + x_2 + x_3)$

$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3)\right]$

$= \frac{1}{3}\left[\mu x + \mu x + \mu x\right]$

$= \frac{1}{3} \cdot 3\mu x$

$= \mu x$

$\text{var}(\bar{x}) = \text{var}\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$

$= \frac{1}{N}\text{var}(x_1 + x_2 + x_3)$

$= \frac{1}{N^2}\left[\text{var}(x_1) + \text{var}(x_2) + \text{var}(x_3) + 2\text{cov}(x_1, x_2) + 2\text{cov}(x_1, x_3) + 2\text{cov}(x_2, x_3)\right]$

$= \frac{1}{3^2}\left[\frac{1}{4}6_x^2 + \frac{1}{4}6_x^2 + \frac{1}{4}6_x^2 + \frac{1}{2}6_x^2 + \frac{1}{2}6_x^2 + \frac{1}{2}6_x^2\right]$

$= \frac{1}{3^2} \cdot \frac{9}{4}6_x^2 = \frac{6_x^2}{4}$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$

$= \frac{1}{N} E(x_1 + x_2 + x_3 + x_4)$

$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3) + E(x_4)\right]$

$= \frac{1}{4}\left[\mu x + \mu x + \mu x + \mu x\right]$

$= \frac{1}{4} \cdot 4\mu x$

$= \mu x$

$\text{var}(\bar{x}) = \text{var}\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$

$= \frac{1}{N^2}\text{var}(x_1 + x_2 + x_3 + x_4)$

$= \frac{1}{N^2}\left[\text{var}(x_1) + \text{var}(x_2) + \text{var}(x_3) + \text{var}(x_4)\right]$

$= \frac{1}{4^2}\left[6_x^2 + 6_x^2 + 6_x^2 + 6_x^2\right]$

$= \frac{1}{16} \cdot 46_x^2 = \frac{6_x^2}{4} = 0.256_x^2$

Student Name Pavita kriathkungwalkai ID 6104640674

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$\tilde{x} = \frac{1}{4}(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$E(\tilde{x}) = E\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$

$= \frac{1}{4}E(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$= \frac{1}{4}\left[E(0.5x_1) + E(x_2) + E(0.5x_3) + E(2x_4)\right]$

$= \frac{1}{4}(0.5\mu x + \mu x + 0.5\mu x + 2\mu x)$

$= \frac{1}{4} \cdot 4\mu x = \mu x$

$var(\tilde{x}) = var\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$

$= \frac{1}{4^2} var(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$= \frac{1}{4^2}\left[var(0.5x_1) + var(x_2) + var(0.5x_3) + var(2x_4)\right]$

$= \frac{1}{4^2}\left[0.25\, var(x_1) + var(x_2) + 0.25\, var(x_3) + 4\, var(x_4)\right]$

$= \frac{1}{4^2}\left[0.25\,6^2_x + 6^2_x + 0.25\,6^2_x + 4\,6^2_x\right]$

$= \frac{5.5\,6^2_x}{16} = 0.34\,6^2_x$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$\bar{x}$ is a more efficient estimator of μx than $\tilde{x}$ because it has smaller variance.