Macroeconomics

Lecture 6

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Asset Prices and Consumption

- To apply recursive structure for the construction of consumption and asset price theories.

Asset Prices and Consumption

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \qquad 0 < \beta < 1, \qquad (1)$$

$$s.t. \quad A_{t+1} = R_t (A_t + y_t - c_t), \qquad A_0 \ \ given \qquad (2)$$

$\{R_t\}$ *is a random process,*

R_t *becomes known to the agent only at the beginning of period* $(t+1)$.

$\{y_t\}$ *is also a random process.*

The Euler equation is

$$U'(c_t) = E_t \beta R_t U'(c_{t+1}), \qquad t = 0, 1, \ldots. \qquad (3)$$

Asset Prices and Consumption

- Special cases of the Euler equation (3) provide the basis for several theories:
- 1) Hall's Random Walk Theory of Consumption;
- 2) The Random Walk Theory of Stock Prices;
- 3) Lucas's Model of Asset Prices.

Hall's Random Walk Theory of Consumption

Suppose that $R_t = R > 1$, *then* (3) *implies that*

$$E_t U'\left(c_{t+1}\right) = \left(\beta R\right)^{-1} U'\left(c_t\right), \qquad (4)$$

The marginal utility of consumption follows a univariate first-order Markov process and that no other variables in the information set help to predit $U'\left(c_{t+1}\right)$, *once lagged* $U'\left(c_t\right)$ *has been included.*

For example, in the case of constant relative risk aversion utility function $U\left(c_t\right) = c_t^{\gamma} / \gamma$, *equation* (4) *becomes*

$$E_t c_{t+1}^{\gamma-1} = \left(\beta R\right)^{-1} c_t^{\gamma-1}$$

The Random Walk Theory of Stock Prices

- To interpret the asset as a share of an enterprise that sells for price p_t, measured in consumption goods at period t per share during period t.
- This asset pays d_t ,which is a nonnegative random dividends, to the owner of the share at the beginning of t.

The Random Walk Theory of Stock Prices

- Assume that d_t is governed by a Markov process, with time invariant transition density $f(d',d)$ where $prob(d_{t+1} \leq d' \mid d_t = d) = \int_0^{d'} f(s,d)ds$.
- s_t denote the number of share owned by the consumer at the beginning of t and let
- $(p_t + d_t)s_t = A_{t,}$ then from eq.(2)

$$(p_{t+1}+d_{t+1})s_{t+1}=\frac{p_{t+1}+d_{t+1}}{p_t}\left[(p_t+d_t)s_t+y_t-c_t\right] \qquad (5)$$

where $\dfrac{p_{t+1}+d_{t+1}}{p_t}=R_t$

Then Euler equation (3) *can be rewritten as*

$$1=\beta E_t\left(\frac{p_{t+1}+d_{t+1}}{p_t}\right)\frac{U'(c_{t+1})}{U'(c_t)} \qquad (6)$$

$\because E_t(xy)=E_t x E_t y+\mathrm{cov}_t(x,y), \quad \mathrm{cov}_t \equiv E_t(x-E_t x)(y-E_t y),$

$\therefore$ *Eq.* (6) *becomes*

$$1=\beta E_t\left(\frac{p_{t+1}+d_{t+1}}{p_t}\right)E_t\frac{U'(c_{t+1})}{U'(c_t)}+\beta\,\mathrm{cov}_t\left[\frac{U'(c_{t+1})}{U'(c_t)},\frac{p_{t+1}+d_{t+1}}{p_t}\right] \qquad (7)$$

To obtain the random walk theory of stock prices, it is necessary to assume, first, that $E_t\dfrac{U'(c_{t+1})}{U'(c_t)}$ *is a constant and, second that*

$$\mathrm{cov}_t\left[\frac{U'(c_{t+1})}{U'(c_t)},\frac{p_{t+1}+d_{t+1}}{p_t}\right]=0.$$

For convenience, suppose that $E_t\dfrac{U'(c_{t+1})}{U'(c_t)}=1$

and

$$\operatorname{cov}_t\left[\frac{U'(c_{t+1})}{U'(c_t)},\frac{p_{t+1}+d_{t+1}}{p_t}\right]=0.$$

If $U(c_t)$ *is linear in* c_t, *so that* $U'(c_t)$ *is independent of* c_t.

Eq (7) *implies that*

$$E_t\left(p_{t+1}+d_{t+1}\right)=\beta^{-1}p_t \qquad (8)$$

- **Eq (8) states that, when the share price is adjusted for dividends and discounting, it follows a first-order univariate Markov process.**

The Random Walk Theory of Stock Prices

- The stochastic difference equation (8) has the following solution

$$p_t = \sum_{j=1}^{\infty} \beta^j E_t d_{t+j} + \gamma_t \left(\frac{1}{\beta}\right)^t, \qquad (9)$$

$where \quad \{\gamma_t\} \; is \; any \; random \; process \; that \; obeys$

$E_t \gamma_{t+1} = \gamma_t.$

$If \quad \gamma_t = 0, \; eq.(9) \; becomes \; a \; formula \; linking$

$share \; prices \; with \; \exp ected \; future \; dividends.$

$$From \quad eq.(8),$$

$$p_t = \beta E_t\left[\left\{\beta E_{t+1}\left(p_{t+2}+d_{t+2}\right)\right\}+d_{t+1}\right]$$

$$= \beta^2 E_t.E_{t+1}\left(p_{t+2}\right)+\beta^2 E_t d_{t+2}+\beta E_t d_{t+1}$$

$$= \beta^2 E_t.E_{t+1}\left(p_{t+2}\right)+\sum_{j=1}^{2}\beta^j E_t d_{t+j}$$

$$= \beta^3 E_t.E_{t+1}E_{t+2}\left(p_{t+3}\right)+\sum_{j=1}^{3}\beta^j E_t d_{t+j}$$

$$\bullet$$

$$\bullet$$

$$= \lim_{j\to\infty}\left\{E_t.E_{t+1}\ldots.E_{t+j-1}\left(p_{t+j}.\beta^j\right)\right\}+\sum_{j=1}^{\infty}\beta^j E_t d_{t+j}$$

$$= \left[\left(\frac{1}{\beta}\right)^t \times \lim_{j\to\infty}\left\{E_t.E_{t+1}\ldots.E_{t+j-1}\left(p_{t+j}.\beta^{t+j}\right)\right\}\right]+\sum_{j=1}^{\infty}\beta^j E_t d_{t+j} \quad \ldots\ldots\ldots(9)$$