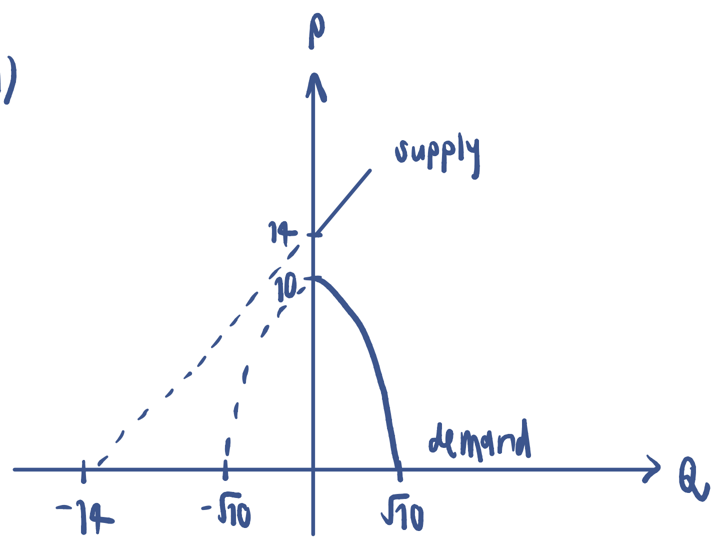
<u>EE320 Placement test</u>

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1)



1.2) $P = 10 - Q^2$ —— 1

$Q = -14 + P$

$P = Q + 14$ —— 2

1 = 2 ; $10 - Q^2 = Q + 14$

$0 = Q^2 + Q + 4$

It cannot be solved

$\because b^2 - 4ac$ in $y = ax^2 + bx + c < 0$

$1 - 4(1)(4) = -15$

1.3) The supply curve will shift to the right but the equilibrium still cannot be solved.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right)$$

$$\left(\frac{Q}{Q+1}\right)' = \frac{(Q+1)(1)-(Q)(1)}{(Q+1)^2} = \frac{1}{(Q+1)^2}$$

$$R'(Q) = \frac{1}{Q^2+1}(2Q) + \frac{3}{(Q+1)^2} \quad \#$$

<u>note</u> $Q \geq 0$

$Q^2+1 \geq 1 > 0$

$(Q+1)^2 > 0$

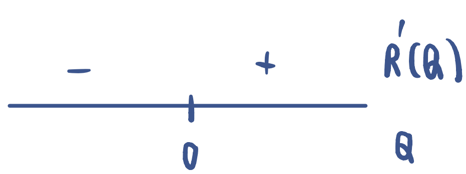
$\therefore\ R'(Q) = \text{DNE} \quad \varnothing$

set $R'(Q) = 0$

$R'(Q) = 0$

$$\frac{1}{Q^2+1}(2Q) + \frac{3}{(Q+1)^2} = 0$$

$Q = 0$



when $Q \geq 0$ $\quad R'(Q) > 0$

so, $R(Q)$ is an increasing function #

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$\pi'(Q) = -Q^2 - 2Q + 8$

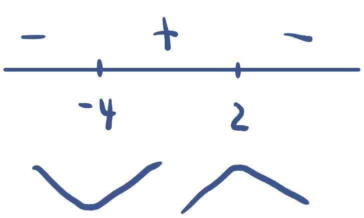
find critical point ; $\pi'(Q) = 0$

$-Q^2 - 2Q + 8 = 0$

$Q^2 + 2Q - 8 = 0$

$(Q+4)(Q-2) = 0$

$Q = -4, 2$



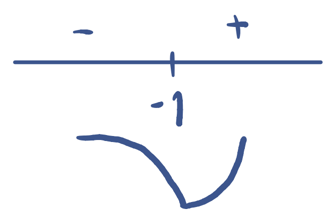
profit maximizing when $Q = 2$

$\pi(2) = \frac{25}{3}$ #

$\pi''(Q) = -2Q - 2$

Concave up $\pi''(Q) > 0$ −∪+

concave down $\pi''(Q) < 0$ +∩−



point of inflection

$(-1, \pi(-1)) = (-1, -\frac{29}{3})$

<u>2^{nd} derivative</u>

find $\pi'(Q) = 0 \Rightarrow Q = -4, 2$

$\pi'(Q) = DNE \Rightarrow \emptyset$

$\pi''(-4) = 6 > 0 \Rightarrow$ Rel. min

$\pi''(2) = -6 < 0 \Rightarrow$ Rel. max

4. Suppose that $\underset{2\times2}{A} = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $\underset{2\times3}{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $\underset{3\times3}{C} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B   size are not equal

4.2 A*B $\begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$ #

4.3 det(A) = $|A| = \begin{vmatrix} 8 & 9 \\ 10 & 11 \end{vmatrix} = (8\times11)-(10\times9) = -2$ #

4.4 det(B)   B is not a square matrix

4.5 det(C)

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \underset{R_2-R_1}{\overset{R_3-R_2}{=}} \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix} \underset{R_2-R_1}{\overset{R_3-R_1}{=}} \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$

cofactor = $\begin{bmatrix} -3 & 6 & -3 \\ 6 & -12 & 6 \\ -3 & 6 & -3 \end{bmatrix}$

$|C| = 0$ #

5. Suppose that $U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial U}{\partial x} = a x^{a-1}(y^b) + \frac{x+y}{x}\left[\frac{(x+y)(1)-(x)(1)}{(x+y)^2}\right]$$

$$= a x^{a-1} \cdot y^b + \frac{(x+y)y}{x(x+y)^2} \quad \#$$

$$\frac{\partial U}{\partial y} = x^a \cdot b y^{b-1} + \frac{x+y}{y}\left[\frac{(x+y)(0)-(x)(1)}{(x+y)^2}\right]$$

$$= b x^a y^{b-1} - \frac{(x+y)(x)}{y(x+y)^2} \quad \#$$