Chapter 11 Consumption: Utility, Indifference Curve and Budget Line

Economics
Doing your best under some limitation

Consumer's Problem The consumer wants to maximize satisfaction (utility) by deciding what and how much to consume under a limitation of income

2 Types of Utility

old idea.

1. Cardinal Utility—the consumer assigns a numerical value to denote his level of satisfaction. For example, a noodle for lunch gives the consumer a utility of 3 utils (units of utility).

This numerical value of 3 is entirely up to the consumer. He can assign value of 30, 300, or 17. However, if a pizza (=6) gives him twice the satisfaction he gets from noodle, then he will assign a number double that given to the noodle.

A total utility of consuming an increasing quantity (= x) of a product ~~always~~ may increases at the beginning. Beyond a quantity, the total utility will decrease.

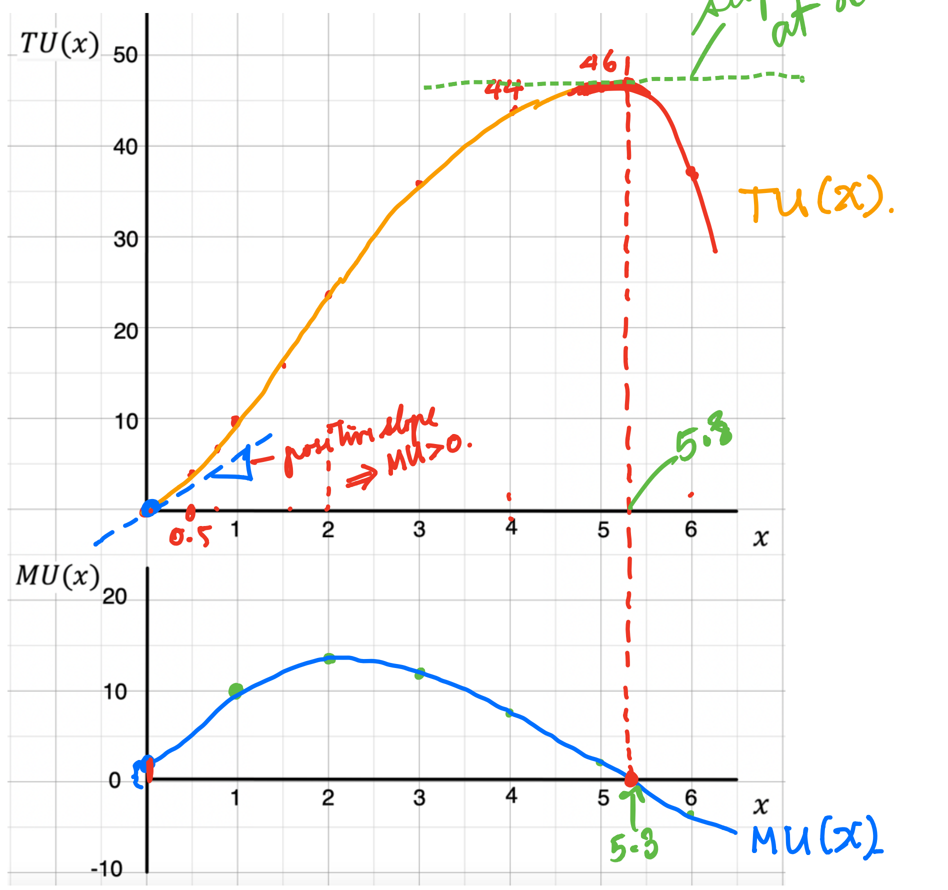
$$TU(x) = Total\ Utilty\ from\ consuming\ x\ units$$
$$MU(x) = Marginal\ Utilty\ from\ consuming\ x\ units$$
$$= the\ change\ of\ TU\ per\ 1\ unit\ change\ of\ x$$
= rate of change of $TU(x)$.

If the consumer consumes 1, 2, 3, … units of x, we may have an example of $TU(x)$ and $MU(x)$ as given in the following table

Boat Noodle.

x	$TU(x)$	$MU(x)$
0	0	
1	10	10
2	24	14
3	36	12
4	44	8
5	46	2
6	42	-4

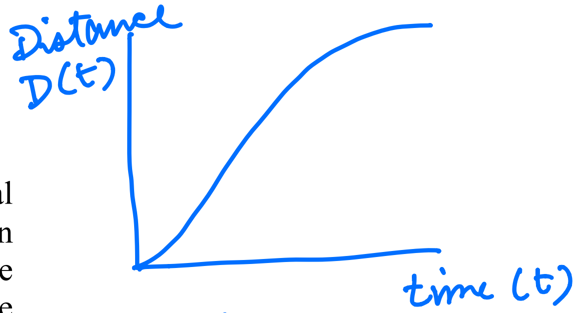
$$MU(x) = \frac{d\,TU(x)}{dx}.$$



If the consumer is allowed to consume any decimal of x,

0, 1, 2, ... bowls of noodles.
0.173 bowls of noodles.

$$\boxed{MU(x)} = marginal\ utilty\ from\ consuming\ x\ units$$
$$= rate\ of\ change\ of\ TU(x)\ per\ unit\ of\ x$$
$$= \frac{d}{dx}TU(x)$$



After certain amount of consumption x, the Marginal Utility of x always is decreasing. This is one explanation why the Demand curve has a negative slope. To induce the consumer to buy the another unit, the price has to be lowered.

$\frac{dD(t)}{dt}$ = speed.
= rate of change of distance per unit change of time.
60 km/h.

equality.

Given the value of $MU(x)$ is known, if we allow a change in $x = \Delta x$, then we can approximate the change in $TU(x)$ as follows:

If $\Delta x \to 0$ dx. $dTU(x) = MU(x).dx$

$$\boxed{\Delta TU(x) \approx MU(x)\Delta x}$$

To be used later.

For example, when $x_0 = 10$, we have $MU(x) = 12$. If we increase x by $\Delta x = 0.1$, then TU will change approximately by

$MU(x) = \frac{d\,TU(x)}{dx}$.

$$\Delta TU(x) \approx MU(x)\Delta x$$
$$= 12(0.1) = 1.2.$$

If we know speed we can estimate ΔD change in distance when we allow small change in time. Δt.

$\Delta D \approx$ speed. Δt.

0.6 km $\approx$ 60 × 0.01 hr.

Approximately because the speed might not be constant.

- The eventual decline of Marginal Utility can explains the Law of Demand.

2. Ordinal Utility Assume the consumer has 2 products X and Y to consume. A ***bundle*** is a quantity of X and a quantity of Y to be consumed by a consumer. A bundle (x_0, y_0) is represented by a point in the XY plane.

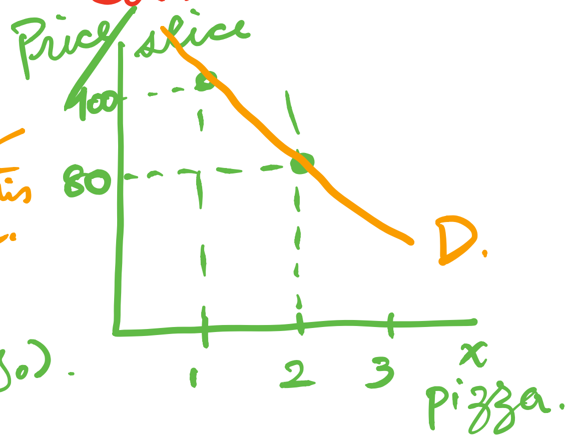
Ordinal utility approach assumes that the consumer is 'rational' according to the following assumptions:

Assumptions:

1) Given two bundles $A = (x_0, y_0)$ and $B = (x_1, y_1)$, the consumer can say one of these:

$A \succ B$ $A \prec B$

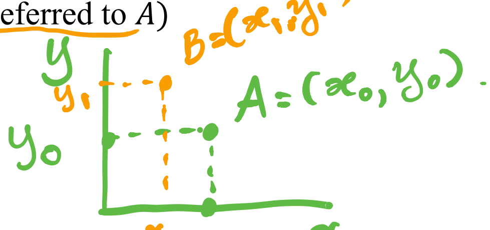
1) $A \succ B$ (A is preferred to B)
2) $B \succ A$ (B is preferred to A)
3) $A \sim B$ (A and B are indifferent) → A & B give the same level of satisfaction to this consumer.



Equivalently, we can say:

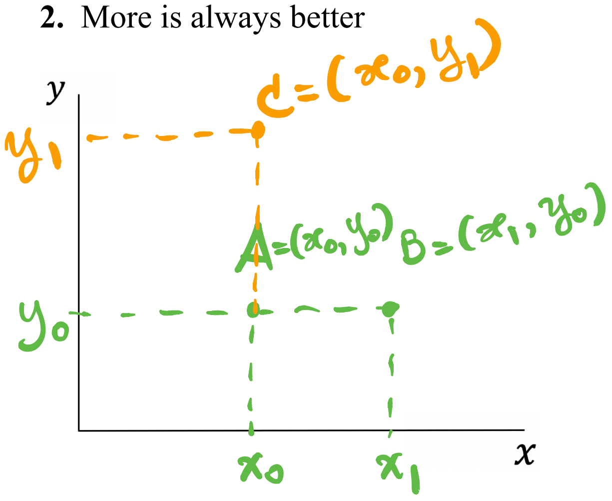
$\succsim$

1) $A \succcurlyeq B$ (A is not less preferred to B)
2) $B \succcurlyeq A$ (B is not less preferred to A)

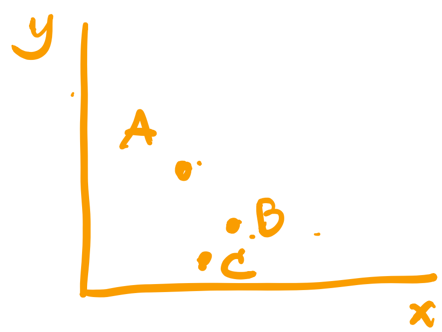


3) $A \sim B$ (A and B are indifferent)

2. More is always better

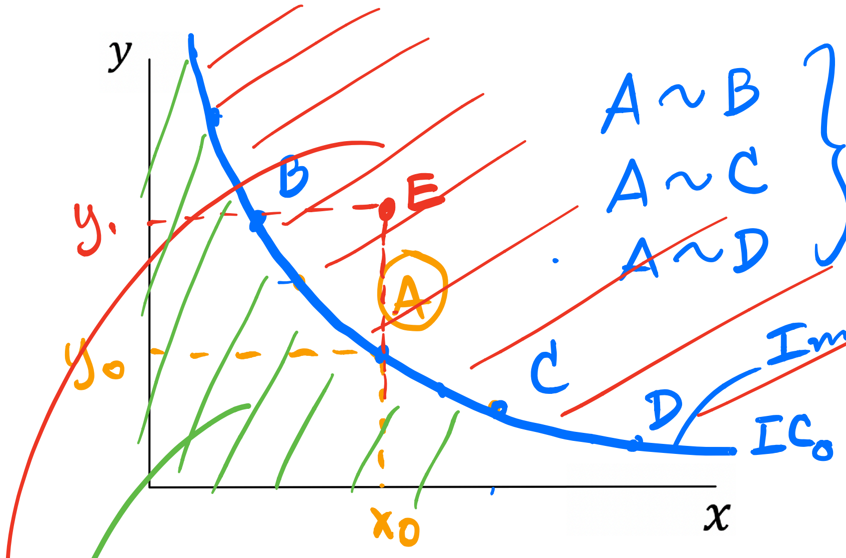


$B \succ A.$
$C \succ A.$



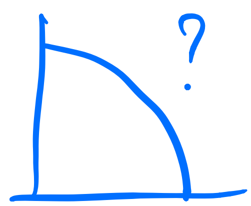
3. Transitivity: If $A \succ B$, and $B \succ C$, then $A \succ C$.

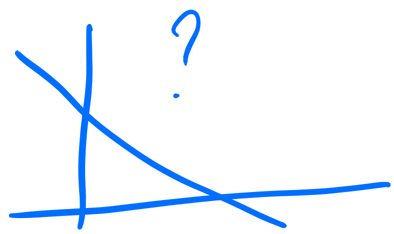
4. Given a bundle A, the consumer can tell all other bundles that are equally preferred. This creates an Indifference Curve (IC) passing through point A.



$A \sim B$
$A \sim C$
$A \sim D$
$\rightarrow B \sim C.$

Indifference Curve.



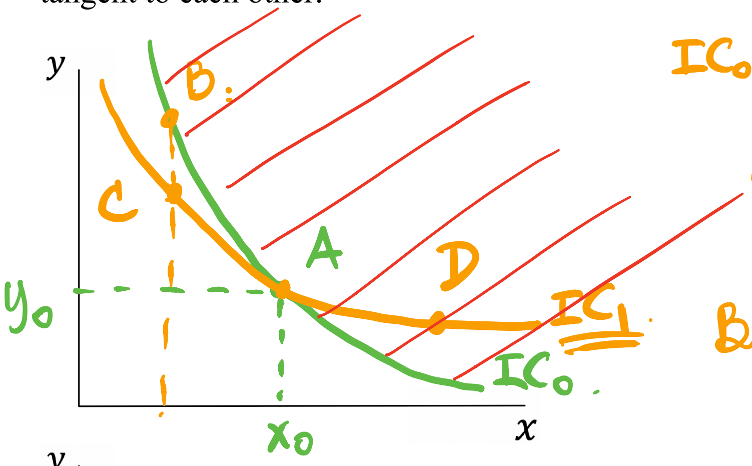


Indifference Curve (IC) is a curve whose every point gives the same satisfaction (utility) to the consumer.

- An IC separates the graph into 3 parts
  a) All the bundles ***on*** the IC
  b) All the bundles ***above*** the IC – each point gives higher satisfaction than a point on IC_0
  c) All the bundles ***below*** the IC – lower

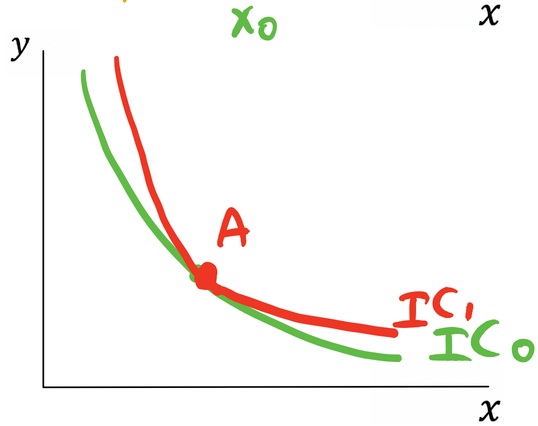
Properties of Indifference Curves (IC).

1. For any bundle $A = (x_0, y_0)$, there is exactly one IC passing through it. That is, no two IC's can intersect nor be tangent to each other.



$IC_0 + IC_1$ passing through A.

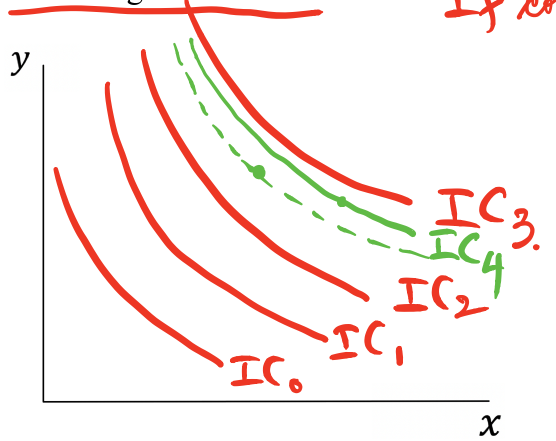
$B \succ C$, $C \sim D$. $\Big\}$ $B \succ D$ $\Big\}$ inconsistent
But. $D \succ B$.



$IC_0 + IC_1$ are tangent at A.
why? – We will also have inconsistency.

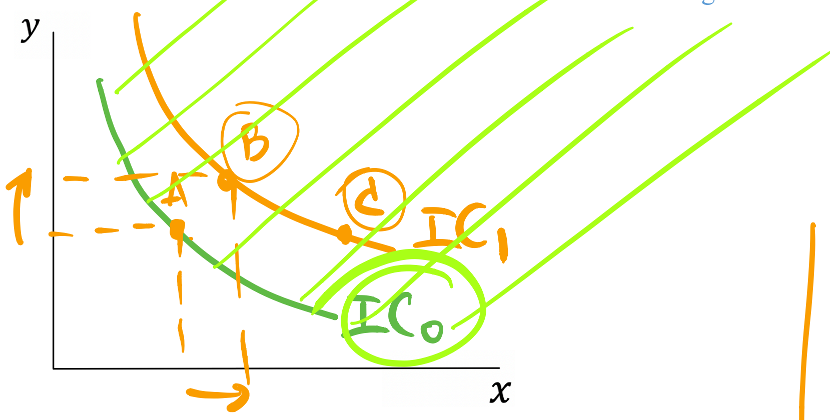
2. There are infinite number of IC's, each never intersect nor is tangent to another

If consumer can consume any fraction of a unit of x + y.



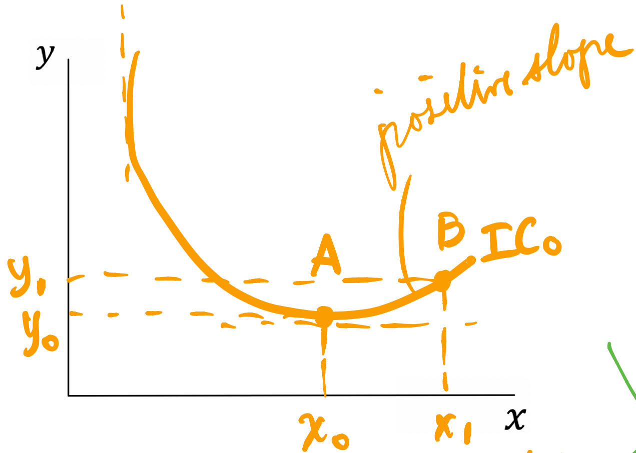
Between any 2 IC's, there is always another IC.

3. Higher IC means higher satisfaction.



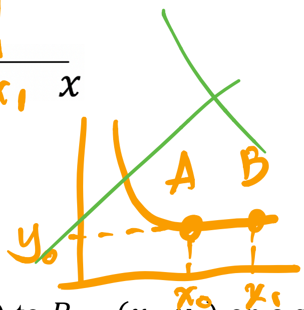
4. Each IC always has negative slope

• What if there is a part of an IC with positive slope?



A + B are on same IC_0
so $A \sim B$.
but we have both more of x & y at B
so according to our assumption
of consumer being rational
$B \succ A$.
$\therefore$ Not consistent!

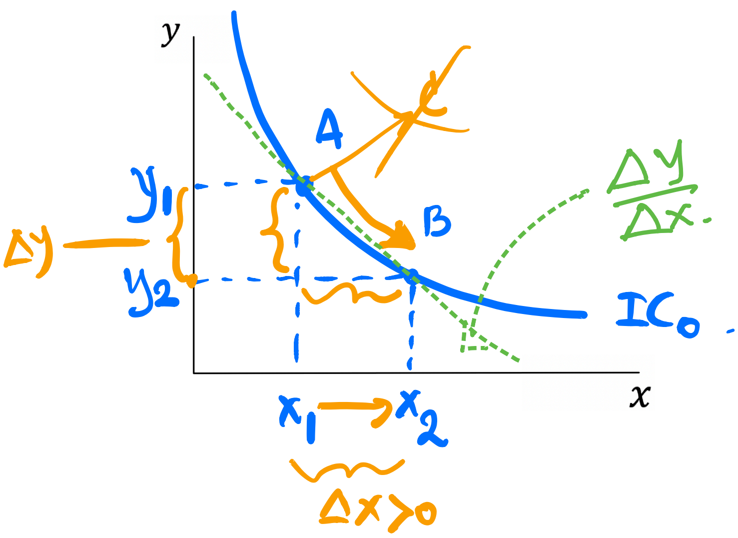
• Can an IC have zero slope?



• If we move from $A = (x_1, y_1)$ to $B = (x_2, y_2)$ on a same IC, there is

new old

• an increase in $x = x_2 - x_1 = \Delta x > 0$
• a decrease in $y = y_2 - y_1 = \Delta y < 0$



The ratio $\boxed{\frac{\Delta y}{\Delta x}}$ is the exchange rate the between x and y with no change in the satisfaction level.

$\frac{\Delta y}{\Delta x}$

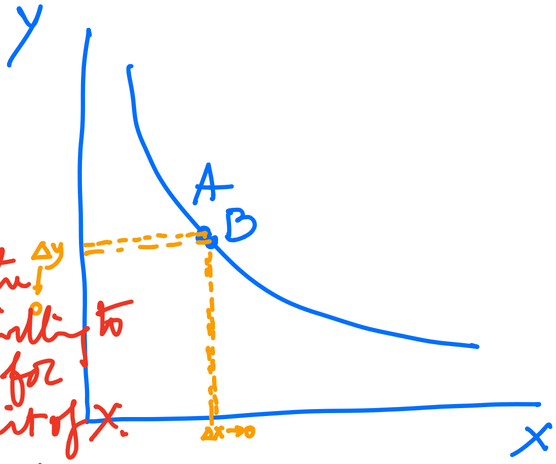
Δx causes an increase in $TU \approx MU_x(x_1, y_1)\Delta x$ — more TU

Δy causes a decrease in $TU \approx MU_y(x_1, y_1)\Delta y$ — less TU

With total change being zero, we have

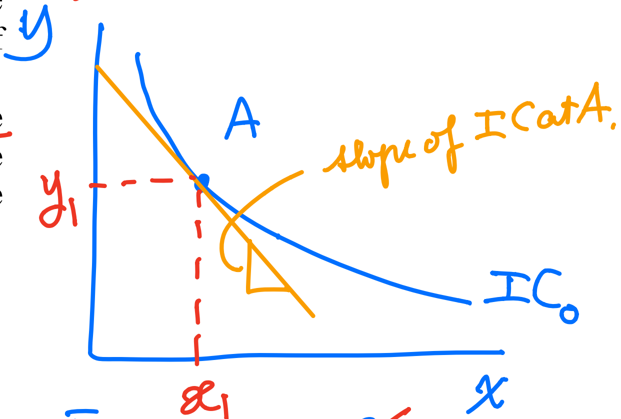
$$MU_x(x_1, y_1)\Delta x + MU_y(x_1, y_1)\Delta y \approx 0$$
$$MU_y(x_1, y_1)\Delta y \approx -MU_x(x_1, y_1)\Delta x$$
$$\frac{\Delta y}{\Delta x} \approx -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}$$

When we move from A to B that is very close together. So close that $\Delta x \to 0$, and we have the slope of IC at A as



slope of IC at A. — $$\boxed{\frac{dy}{dx}} = -\boxed{\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}} =$$ how much the consumer is willing to sacrifice y for 1 more unit of x.

- At point A the slope can be found by drawing a line tangent to A. The slope of IC at A is equal to the slope of the tangent line.
- Slope at point A = (x_1, y_1) is called the **Marginal Rate of Substitution** (MRS). It is the **instantaneous exchange rate** between x and y such that **in the mind of the consumer** his satisfaction (utility) is unchanged.

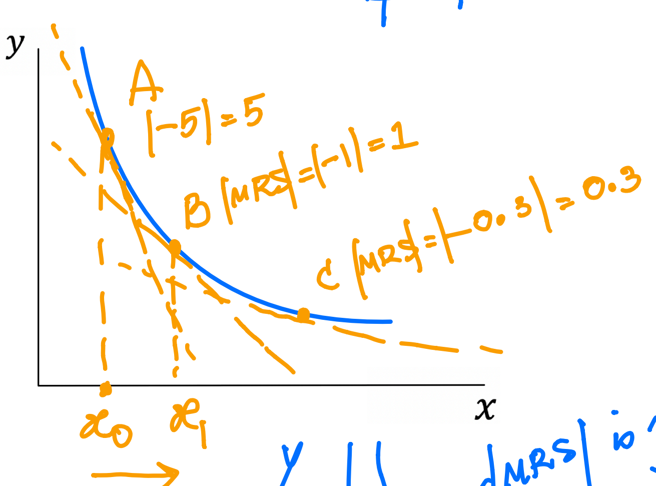


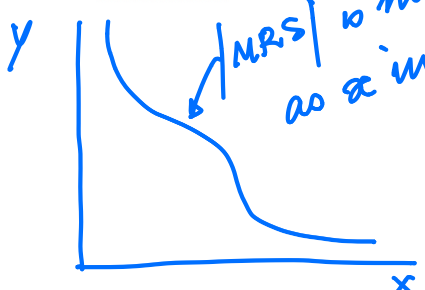
- $MRS = -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}$

- This leads to the 5th property of IC

MRS is negative.

- **5. Diminishing** MRS On a given IC, as the consumption of x increases, the value of |MRS| decrease.





|MRS| is not diminishing as x increases

Ex $MU_x = \cancel{2}\ 4$

$MU_y = 2$ ✓

1 more of x + ~~2~~ 4 util.

2 ~~1~~ less of y − ~~2~~ 4 util.

exchange rate between x & y is 1 to 1.

$$\frac{dy}{dx} = -\frac{MU_x}{MU_y} = -\frac{2}{2} = -1$$

1 x for 2 of y

$$\frac{dy}{dx} = -\frac{MU_x}{MU_y} = -\frac{4}{2} = -2$$

Consumption Problem: maximizing satisfaction under a budget constraint

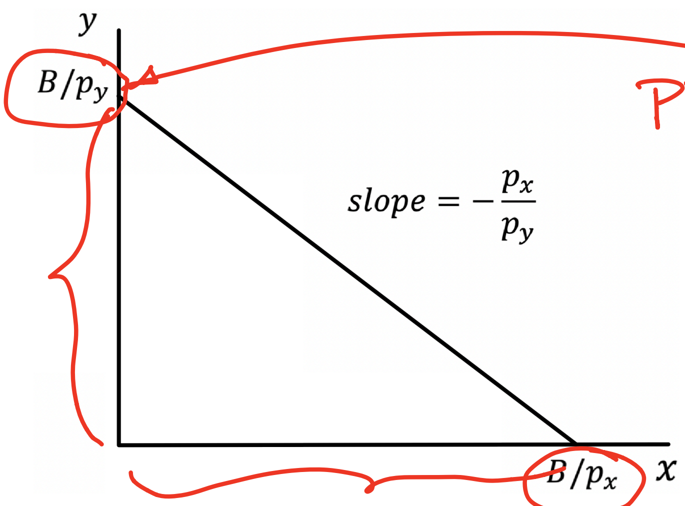
IC's.

Budget Line A consumer is assumed to have a fixed income or budget B that he can buy x and y at fixed prices p_x and p_y, respectively. The equation for budget line is given by,

spent on x   spent on y.

$$p_x x + p_y y = B$$

— consumer must spend exactly = B.
— consumer does not save any money.



PPC. — constant cost.

$P_x x + P_y y = B.$

y-intercept

when $x=0$, $y = \frac{B}{P_y}$ — $(0, B/P_y)$

$y=0$, $x = \frac{B}{P_x}$ — $(\frac{B}{P_x}, 0)$

x-intercept

$$slope = -\frac{B/P_y}{B/P_x} = -\frac{P_x}{P_y}.$$

$$slope = -\frac{p_x}{p_y} = relative\ price\ x\ interms\ of\ y$$
$$= exchange\ rate\ between\ x\ and\ y\ in\ the\ market$$

$P_x = \cancel{10}\ 20\ B$

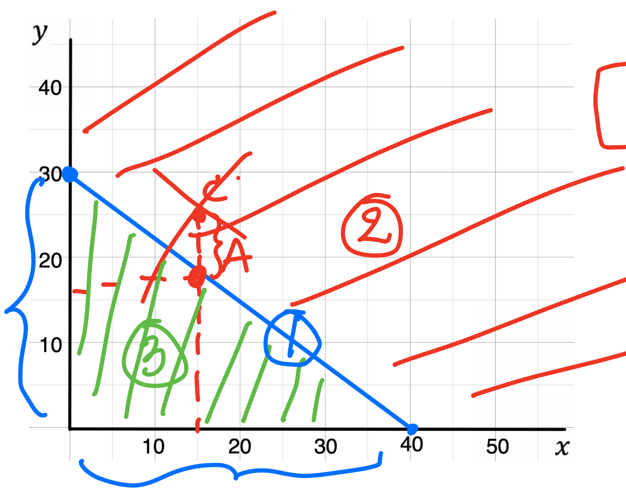
$P_y = 10\ B$

1 more of x $\Rightarrow$ ~~1~~ 2 less of y.

$-\frac{P_x}{P_y} = \frac{-20}{10} = -2.$

- Every point on the budget line is a bundle the consumer can afford.

Example: $p_x = 3, p_y = 4, B = 120$



$\boxed{3x+4y=120.}$

$x=0$, $y = \frac{120}{4} = 30$.

$y=0$, $x = \frac{120}{3} = 40.$

$slope = -\frac{30}{40} = -\frac{3}{4}$

$$slope = -\frac{p_x}{p_y} = -\frac{3}{4}$$

Note. Slope of Budget line does not depend on B.

$P_x = 300$
$P_y = 400$ } slope of Budget line $= -\frac{P_x}{P_y} = -\frac{300}{400} = -\frac{3}{4}$

A given budget line divides the x-y plane into 3 parts.

1) The Budget line — every point on a given budget line is a bundle that the consumer can buy at price P_x & P_y with the given Budget B. $P_x X + P_y Y = B$

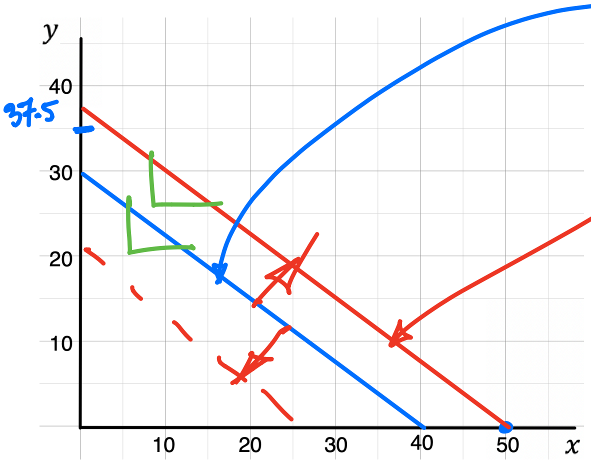
- Budget line gives us all the bundles that the consumer can afford and spend all income B.

2). Area above the budget line are all the bundles the consumers cannot afford at budget B and prices P_x & P_y.

3) Area under the budget line, are bundles the consumer has enough income to buy at P_x & P_y prices but with some money left over.

Changes of Budget Line

1. Income increases: Income increases from, $B = 120$ to $B' = 150$.



$3x+4y=120.$
$3x+4y=150.$

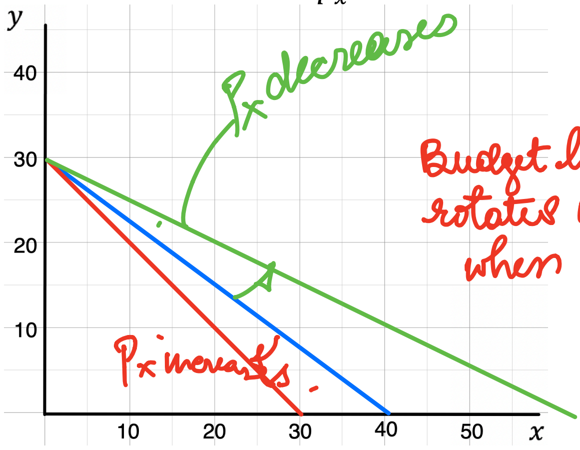
$\frac{B'}{P_y} = \frac{150}{4} = 37.5.$

$\frac{B'}{P_x} = \frac{150}{3} = 50$

The old + new Budget lines are parallel → same slope.

Higher income ⇒ higher Budget (for same P_x + P_y)

2. Price of *x* increases $p_x = 3$ increases to $p_x = 4$



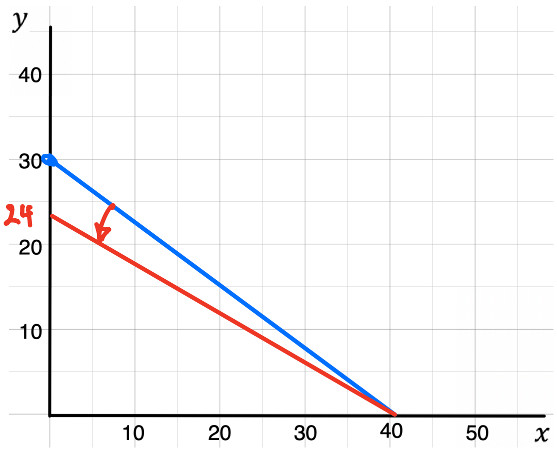
Budget line rotates inward when P_x increases.

$3x+4y=120.$
$4x+4y=120.$

$\frac{B}{P_y}$ - same.

$\frac{B}{P_x'} = \frac{120}{4} = 30.$

Price of *y* decreases $p_y = 4$ increases to ~~$p_x = 3$~~ $P_y = 5$



$3x + 4y = 120$

$3x + 5y = 120$.

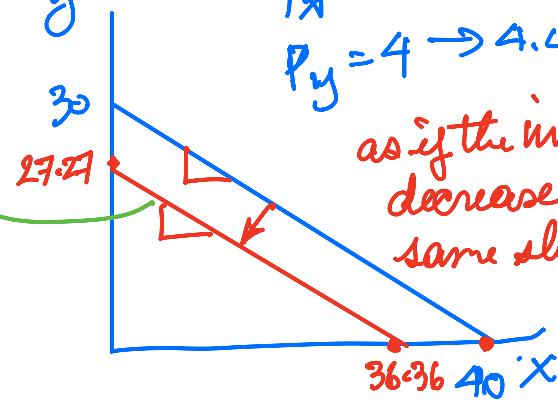
$\frac{120}{5} = 24$

$B \to \frac{B}{1.1}$ and same P_x, P_y.

$P_x = 3 \to 3.3$

$P_y = 4 \to 4.4$

as if the income decreases! same slope.



- In Economics, only one thing is allowed to change at a time. That is, either income or price of a product changes.
- If we have inflation so that the prices p_x and p_y increases at the same percentage of 10%, what is the effect on the budget line?
- If we have 10% inflation and at the same time income also increase by 10%, how will the budget line change?

$$\left.\begin{array}{l} P_x \to P_x(1.1) \\ P_y \to P_y(1.1) \\ B \to B(1.1) \end{array}\right\} \text{no change in Budget line.}$$

$$\left.\begin{array}{l} \frac{B(1.1)}{P_x(1.1)} = \frac{B}{P_x} \\ \frac{B(1.1)}{P_y(1.1)} = \frac{B}{P_y} \end{array}\right\} \text{no change}$$

$$\frac{B}{P_x(1.1)} = \frac{120}{3.3} = 36.36.$$

$$\frac{B}{P_y(1.1)} = \frac{120}{4.4} = 27.27$$

$$\text{slope} = -\frac{\cancel{B}/P_y\cancel{(1.1)}}{\cancel{B}/P_x\cancel{(1.1)}} = -\frac{P_x}{P_y}.$$

When we have inflation it is the same as if we have lower income — i.e. real income decreases

measure the amount of goods we can buy.