

Instructions

- (1) Please read the instruction carefully.
- (2) Please read each question carefully and answer the questions straightforwardly. Always provide economic reasons at least a paragraph for your analysis, or a graph when necessary, even when the question does not indicate so.
- (3) Handing and submitting assignments are only available via BE Moodle.

Answering the questions and preparing answer sheets

- (1) Answers are to be handwritten, in either digital or analog form, in a blank canvas or any clean paper. Make sure that your handwriting is clearly visible and readable.
- (2) There is no need to rewrite the question. Just indicate the question number clearly for each of the answer, such as 1.a).
- (3) Default decimal point is 4.
- (4) Choose precise wordings, especially when you want to interpret the meaning of a test, confidence interval, or coefficients.
- (5) When done, for the digital case, collage all the pages into a single PDF file. For those who write on sheets of paper, take photo of all pages then convert all of them into a single PDF file as well.
- (6) Name your PDF file as StudentID_YourNickname, such as 640123456_Bo.

Submitting your answers

- (1) Make sure your file does not exceed 10MB. This is the maximum file size for BE Moodle upload.
- (2) Login to BE Moodle, head into the course, then the assignment topic.
- (3) Choose your file to submit. Done. There will be timestamp for your upload date and time, so please make sure to not submit later than that.

Question 1. (12 points) Economic model of Crime.

1.a) Based on the regression results provided, write out the estimated coefficients in the form of regression equation (1.1). Interpret the estimated coefficients associated with *avgsen*. Based on Model (1.1), test whether the average sentence served from prior convictions has an impact on the number of arrests in the current year (1986). Show your work. (Use $\alpha = 0.05$)

1.b) What is the overall significance of the regression from Model (1.1) and Model (1.2)? What test do you use? (Use $\alpha = 0.01$)

1.c) If we are interested in testing whether “ethnic background and legal income” has an impact on the number of arrests in the current year (1986), what kind of null/alternative hypothesis would we be testing? Perform the test and discuss your finding. (Use $\alpha = 0.05$)

Estimate the model (1.1) reports in the Table 1.1

$$narr86_i = \beta_1 + \beta_2 pcnv_i + \beta_3 avgsen_i + \beta_4 tottime_i + \beta_5 ptime86_i + \beta_6 qemp86_i + u_i \quad (1.1)$$

Table 1.1

Source	SS	df	MS	Number of obs	=	2,725
Model	85.9532425	5	17.1906485	F(5, 2719)	=	24.29
Residual	1924.39391	2,719	.707757967	Prob > F	=	0.0000
Total	2010.34716	2,724	.738012906	R-squared	=	0.0428
				Adj R-squared	=	0.0410
				Root MSE	=	.84128

1719

narr86	Coefficient	Std. err.	t	P> t	[95% conf. interval]
pcnv	-.1512246	.040855			Omitted for the purpose of this exam
avgsen	-.0070487	.0124122			
tottime	.0120953	.0095768			
ptime86	-.0392585	.0089166			
qemp86	-.1030909	.0103972			
_cons	.7060607	.0331524			

$narr86 = 0.7061 - 1.5122 pcnv - 0.0070 avgsen + 0.121 tottime - 0.3926 ptime86 - 1.0309 qemp86$

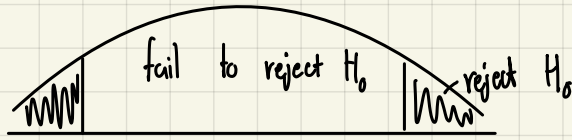
1.a) Based on the regression results provided, write out the estimated coefficients in the form of regression equation (1.1). Interpret the estimated coefficients associated with *avgsen*. Based on Model (1.1), test whether the average sentence served from prior convictions has an impact on the number of arrests in the current year (1986). Show your work. (Use $\alpha = 0.05$)

$$narr86 = 0.7061 - 1.5122 pcnv - 0.0070 avgsen + 0.121 tottime - 0.3926 ptime86 - 1.0509 qemp86$$

$\beta_3 = -0.0070$ If the average sentence served from prior conviction increase for 1 month, The number of arrests in the current year will decrease by 0.007 times (in 1986).

$$H_0: \beta_3 = 0 \quad \alpha = 0.05 \quad t_{0.025, 2719} = 1.96$$

$$H_1: \beta_3 \neq 0 \quad \frac{\alpha}{2} = 0.025 \quad t_{cal} = \frac{-0.007 - 0}{0.0124} = -0.5645$$



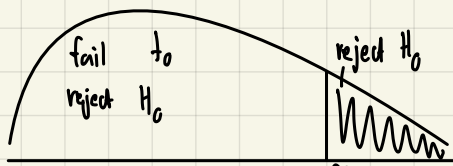
t_{cal} fall into fail to reject H_0 region, so β_3 is not significant with 95% confident level.

\therefore Average sentence served from prior convictions do not significant impact to the number of arrests in the current year (1986).

1.b) What is the overall significance of the regression from Model (1.1) and Model (1.2)? What test do you use? (Use $\alpha = 0.01$)

$$\text{Model 1 } H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$

H_1 : Otherwise (At least one of β_i not equal to 0)



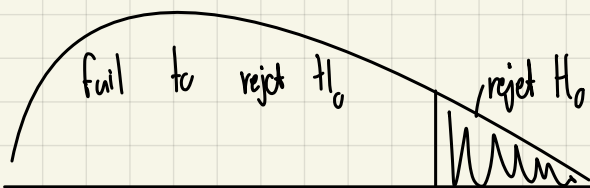
$$F_{cal} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.0428/5}{(1-0.0428)/2719} = 29.3153$$

F_{cal} fall into reject H_0 region. So at least one of β_i will not equal to 0 with 99% confident level.

$$\text{Model 2 } narr86 = -0.1332 pcnv - 0.1132 avgsen + 0.0120 tottime - 0.0408 ptime86 - 0.0505 qemp86 - 0.0015 inc86 + 0.3265 black + 0.1939 hispan.$$

$$H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = 0$$

H_1 : Otherwise (At least one of β_i not equal to 0)



$$F_{cal} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.0723/8}{(1-0.0723)/2716}$$

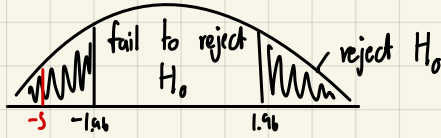
$$= 26.4588$$

F_{cal} fall into reject H_0 region. So at least one of β_i will not equal to 0 with 99% confident.

1.c) If we are interested in testing whether "ethnic background and legal income" has an impact on the number of arrests in the current year (1986), what kind of null/alternative hypothesis would we be testing? Perform the test and discuss your finding. (Use $\alpha = 0.05$)

$$H_0: \beta_7 = 0 \quad \alpha = 0.05 \quad t_{0.025, 2716} = 1.96$$

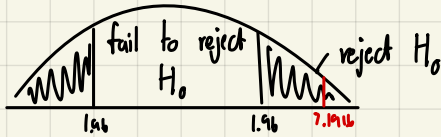
$$H_1: \beta_7 \neq 0 \quad \alpha = 0.025 \quad t_{cal} = \frac{-0.0015 - 0}{0.0003} = -5$$



t_{cal} fall into reject H_0 region, So β_7 is significant with 95% confident level.
 \therefore If legal income (1986) increase by 1 hundred dollar, Number of arrests will decrease by 0.0015.

$$H_0: \beta_8 = 0 \quad \alpha = 0.05 \quad t_{0.025, 2716} = 1.96$$

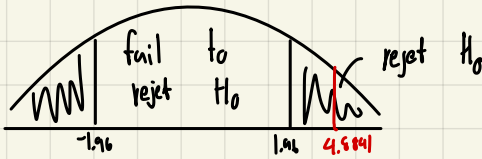
$$H_1: \beta_8 \neq 0 \quad \alpha = 0.025 \quad t_{cal} = \frac{0.3265 - 0}{0.0459} = 7.1916$$



t_{cal} fall into reject H_0 region, So β_8 is significant with 95% confident level.
 \therefore If black ethnic background is significant impact on number of arrests in 1986.

$$H_0: \beta_9 = 0 \quad \alpha = 0.05 \quad t_{0.025, 2716} = 1.96$$

$$H_1: \beta_9 \neq 0 \quad \alpha = 0.025 \quad t_{cal} = \frac{0.1999 - 0}{0.0397} = 4.9541$$



t_{cal} fall into reject H_0 region, So β_9 is significant with 95% confident level.
 \therefore If

Estimate the model (1.2) reports in the Table 1.2

$$narr86_i = \beta_1 + \beta_2pcnv_i + \beta_3avgsen_i + \beta_4tottime_i + \beta_5ptime86_i + \beta_6qemp86_i + \beta_4inc86_i + \beta_5black_i + \beta_6hispan_i + u_i \tag{1.2}$$

where

- $narr86_i$ = the number of arrests in the current year (1986)
- $pcnv_i$ = the proportion of prior arrests that led to a conviction
- $avgsen_i$ = the average sentence served from prior convictions (in months)
- $tottime_i$ = months spent in prison since age 18 prior to 1986
- $ptime86_i$ = months spent in prison in 1986
- $qemp86_i$ = the number of quarters that the man was legally employed in 1986
- $inc86_i$ = legal income, 1986, (hundred dollars)
- $black_i$ = 1 if black ethnic background
- $hispan_i$ = 1 if Hispanic ethnic background

Table 1.2

Source	SS	df	MS	Number of obs	=	2,725
Model	145.390104	8	18.173763	F(8, 2716)	=	26.47
Residual	1864.95705	2,716	.686655763	Prob > F	=	0.0000
				R-squared	=	0.0723
				Adj R-squared	=	0.0696
Total	2010.34716	2,724	.738012906	Root MSE	=	.82865

narr86	Coefficient	Std. err.	t	P> t	[95% conf. interval]
pcnv	-.1332344	.0403502			Omitted for the purpose of this exam
avgsen	-.0113177	.0122401			
tottime	.0120224	.0094352			
ptime86	-.0408417	.008812			
qemp86	-.0505398	.0144397			
inc86	-.0014887	.0003406			
black	.3265035	.0454156			
hispan	.1939144	.0397113			
_cons	.5686855	.0360461			

$narr86 = -0.1332 pcnv - 0.1132 avgsen + 0.0120 tottime - 0.0408 ptime86 - 0.0505 qemp86 - 0.0015 inc86 + 0.3265 black + 0.1939 hispan.$

Question 2. (12 points) Dummy variables and interaction terms.

Using the Thailand labor force survey (LFS) in quarter 2 of 2019 and 2020, employees log of wage is modeled as follows. (Number of observations is 97,878 in total)

$$\ln wage_i = \beta_1 + \beta_2 civil_i + \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i$$

where

$\ln wage_i$	= natural logarithmic scale of monthly wage
$civil_i$	= 1; civil servant and state employee = 0; otherwise
$year_i$	= 1; year 2020 = 0; otherwise (2019)

This model is also known as Difference-in-Differences (DiD) and its intention is to capture the effect of COVID-19 since March of 2020 on different types of employment. During the pandemic, we assume that civil servant and state employee's wage is not reduced (control group) while others', namely employees in private firms or freelance, etc., is suspected to be reduced (treatment group). The estimation result is shown below with standard errors in parentheses. Answer the following questions.

$$\ln \widehat{wage}_i = 9.1748 + 0.587 civil_i - 0.0336 year_i + 0.0444 civil_i \cdot year_i + u_i$$

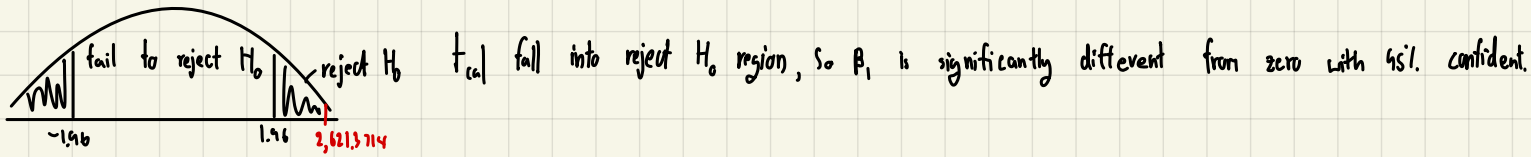
(0.0035)	(0.0072)	(0.005)	(0.0102)
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- 2.a)** Test all the parameters individually if each of them is significantly different from zero or not.
- 2.b)** How much on average does a civil servant and state employee earn more or less than the others disregarding the year?
- 2.c)** How much on average does the pandemic affect wage overall?
- 2.d)** Are the control group and the treatment group better-off or worse-off during the pandemic. Discuss each group separately, show your work and explain with economic reasons according to the intention of this model.

2.a) Test all the parameters individually if each of them is significantly different from zero or not.

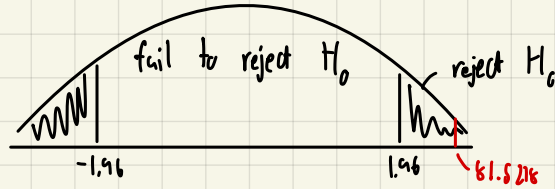
$$H_0: \beta_1 = 0 \quad \alpha = 0.05 \quad t_{0.025, 97874} = 1.96 \quad |t_{cal}| = \frac{9.1798 - 0}{0.0035} = 2,621.3714$$

$$H_1: \beta_1 \neq 0 \quad \alpha_2 = 0.025 \quad n = 97878 \quad K = 4$$



$$H_0: \beta_2 = 0 \quad \alpha = 0.05 \quad t_{0.025, 97874} = 1.96$$

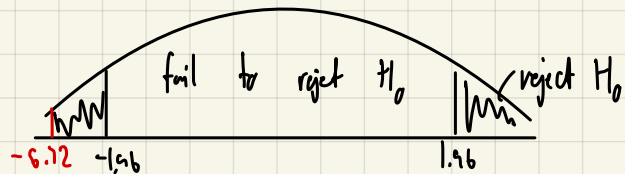
$$H_1: \beta_2 \neq 0 \quad \alpha_2 = 0.025 \quad |t_{cal}| = \frac{0.587 - 0}{0.0072} = 81.5278$$



t_{cal} fall into reject H_0 region, so β_2 is significantly different from zero with 95% confident level.

$$H_0: \beta_3 = 0 \quad \alpha = 0.05 \quad t_{0.025, 97874} = 1.96$$

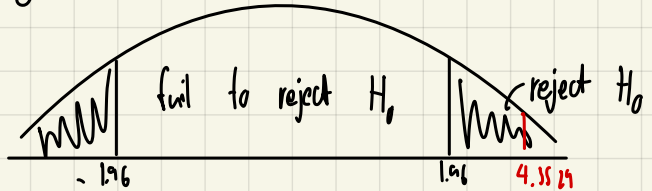
$$H_1: \beta_3 \neq 0 \quad \alpha_2 = 0.025 \quad |t_{cal}| = \frac{-0.0336 - 0}{0.005} = -6.72$$



t_{cal} fall into reject H_0 region, so β_3 is significantly different from zero with 95% confident level.

$$H_0: \beta_4 = 0 \quad \alpha = 0.05 \quad t_{0.025, 97874} = 1.96$$

$$H_1: \beta_4 \neq 0 \quad \alpha_2 = 0.025 \quad |t_{cal}| = \frac{0.0444 - 0}{0.0102} = 4.3529$$



t_{cal} fall into reject H_0 region, so β_4 is significantly different from zero with 95% confident level.

\therefore All parameter is different from zero with 95% confident level.

2.b) How much on average does a civil servant and state employee earns more or less than the others disregarding the year?

$$\ln \widehat{wage}_i = 9.1748 + 0.587 \text{ civil}_i - 0.0336 \text{ year}_i + 0.0444 \text{ civil}_i \cdot \text{year}_i + u_i$$

(0.0035) (0.0072) (0.005) (0.0102)

$$\text{Civil} = 1 \quad \text{year} = 1 \text{ (2020)}$$

$$\quad \quad \quad = 0 \text{ (2019)}$$

2020

$$\ln \widehat{wage} = 9.1748 + 0.587 - 0.0336 + 0.0444 = 9.7726$$

$$\text{wage} = 17546.3284 \text{ Baht}$$

2019

$$\ln \widehat{wage} = 9.1748 + 0.587 - 0 + 0 = 9.7618$$

$$\text{wage} = 17357.8471 \text{ Baht}$$

\therefore Mean of wage in 2020 more than 2019 = 188.4807 Baht

2.c) How much on average does the pandemic affect wage overall?

not civil servant & state employee (2020)

$$\text{civil} = 0$$

$$\text{year} = 1$$

$$\ln \widehat{wage} = 9.1748 - 0.0336 = 9.1412$$

$$\text{wage} = 9331.9568 \text{ Baht}$$

not civil servant & state employee (2019)

$$\text{civil} = 0$$

$$\text{year} = 0$$

$$\ln \widehat{wage} = 9.1748$$

$$\text{wage} = 9650.8377 \text{ Baht}$$

From the pandemic effect to average in 2020 between not civil servant and civil servant by 8214.716 Baht. And in 2019 civil servant has more wage than not civil servant by 7707.01 Baht.

2.d) Are the control group and the treatment group better-off or worse-off during the pandemic. Discuss each group separately, show your work and explain with economic reasons according to the intention of this model.

From 2.c), the pandemic effect normal citizen who are not civil servant and state employee from the average wage in 2020 is lower than 2019 by 318.8009 Bakt. Because the pandemic cause to close in Business, it make people will not receive any welfare from firm and fire. In another case, the average wage of civil servant & state employee in 2020 increase from 2019. As this group may receive any welfare from government.

Question 3. (8 points) Multicollinearity.

As cheese ages, several chemical processes take place that determine the taste of the final product. The data given pertain to concentrations of various chemicals in a sample of 30 mature cheddar cheeses and subjective measure of taste for each sample.

Estimate the model (3.1) reports in the Table 3.1

$$Taste = \beta_0 + \beta_1 acetic + \beta_2 h2s + \beta_3 lactic + u \tag{3.1}$$

Where $Taste$ = Measures of taste for each sample
 $acetic$ = The natural logarithm of concentration of acetic
 $h2s$ = The natural logarithm of concentration of hydrogen sulfide
 $lactic$ = Lactic

Table 3.1 $t_{est} = -34.13491 + 1.5386Acetic + 3.915 H_2S + 18.8024 Lactic$

Source	SS	df	MS	Number of obs	=	30
Model	5020.64468	3	1673.54823	F(3, 26)	=	16.47
Residual	2642.24237	26	101.624706	Prob > F	=	0.0000
				R-squared	=	0.6552
				Adj R-squared	=	0.6154
Total	7662.88705	29	264.237485	Root MSE	=	10.081

taste	Coefficient	Std. err.	t	P> t	[95% conf. interval]
acetic	1.538645	3.000501			Omitted for the purpose of this exam
h2s	3.915242	1.153106			
lactic	18.80235	8.342614			
_cons	-34.13491	15.67628			

	acetic	h2s	lactic	Variable	VIF	1/VIF
acetic	1.0000			lactic	1.83	0.546648
h2s	r_{12} 0.2700	1.0000		h2s	1.72	0.582609
lactic	r_{13} 0.3607	r_{23} 0.6448	1.0000	acetic	1.15	0.867477
				Mean VIF	1.57	

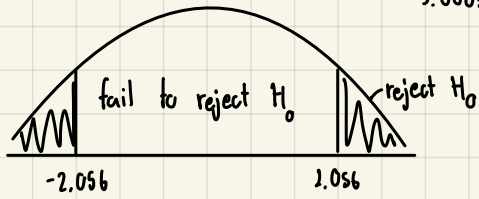
from table r_{12} and r_{23} low and r_{13} moderate and mean VIF is low.

3.a

$$t_{\text{test}} = -34.13491 + 1.5386 \text{ Acetic} + 3.9152 \text{ H}_2\text{S} + 18.8024 \text{ Lactic}$$

$$H_0: \beta_2 = 0 \quad t_{0.025, 26} = 2.056$$

$$H_1: \beta_2 \neq 0 \quad t_{\text{cal}} = \frac{1.5386 - 0}{3.0005} = 0.5132 \quad \text{not sig}$$



$$H_0: \beta_3 = 0 \quad t_{0.025, 26} = 2.056$$

$$H_1: \beta_3 \neq 0 \quad t_{\text{cal}} = \frac{3.9152 - 0}{1.1531} = 3.3954 \quad \text{sig}$$

$$H_0: \beta_4 = 0 \quad t_{0.025, 26} = 2.056$$

$$H_1: \beta_4 \neq 0 \quad t_{\text{cal}} = \frac{18.8024 - 0}{8.3426} = 2.2538 \quad \text{sig}$$

Only Acetic is not significant it maybe low multicollinearity

from table r_{13} and r_{24} low and r_{34} moderate and mean VIF is low

$$\text{TOL} = \frac{1}{\text{VIF}} = 0.6369 \quad (\text{high TOL})$$

3.b) The OLS estimators are unbiased. But unbiasedness is a multisample or repeated sampling property. Colinearity does not destroy the property of minimum variance. All linear unbiased estimators, the OLS estimators have minimum variance - they are efficient. But this does not mean the variance of OLS estimator will necessarily be small. As a result, it is still BLUE.

3.a) Is there evidence of multicollinearity in the data? How do you know? Explain your answers in detail and state the critical value for hypothesis testing to receive full points.

3.b) What is the property of BLUE? If there is the multicollinearity problem, is the OLS estimators still retain the property of BLUE? If not, which properties are violated?

Question 4. (8 points) Heteroscedasticity.

The data on U.S. inflation rates (%) and unemployment rates (%), 1948-2006

Estimate the model (4.1) reports in the Table 4.1

$$Inf_t = \beta_1 + \beta_2 unem_t + u_t \tag{4.1}$$

where Inf_t = inflation rates (%)

$unem_t$ = unemployment rates (%)

Table 4.1

Source	SS	df	MS	Number of obs	=	59
Model	32.3284496	1	32.3284496	F(1, 57)	=	3.85
Residual	478.096987	57	8.38766644	Prob > F	=	0.0545
Total	510.425437	58	8.80043856	R-squared	=	0.0633
				Adj R-squared	=	0.0469
				Root MSE	=	2.8961

inf	Coefficient	Std. err.	t	P> t	[95% conf. interval]
unem	.5054734	.2574699			
_cons	1.010847	1.491583			

White's general test statistic: 1.0266 Chi-sq (2)

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity

chi2(1) = 1.12

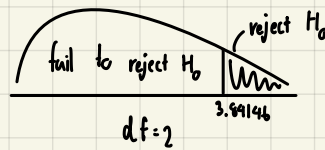
$$\ln f_t = 1.010847 + 0.5054734 unem_t$$

$$4.a \quad \ln f_t = 1.010847 + 0.5054734 \text{ unem}_t$$

β_1 : When the unemployment equal to 0%, the inflation will equal to 1.010847%

β_2 : If the unemployment rate increase by 1%, the inflation will increase by 0.5054734%

4.b. H_0 : Homo white test $\hat{u} = \beta_1 + \beta_2 \text{ unem} + \beta_3 \text{ unem}^2$
 H_1 : Hetero



$n \cdot R_K^2 < 3.99146 \therefore$ fall into fail to Reject H_0 region, so it is homo at $\alpha = 0.05$.

4.c Since we cannot reject the null hypothesis in 4.b.) therefore, we can make sure that BLUE property is not violated.

Answer the following questions.

4.a) Interpret the intercept and slope coefficients.

4.b) According to the test statistics given after Table 4.1 below, is there any sufficient evidence to conclude that there is heteroscedasticity problem? Show your work on the hypothesis testing. (Use $\alpha = 0.05$)

4.c) Given your test results in a), do the OLS estimators still retain the property of BLUE? If not, which properties are violated?
