1 (a)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_2-4R_1 \to R_2 \\ R_3-7R_1 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -11 & -7 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} \frac{R_2}{-3} \to R_2 \\ \frac{R_3}{-6} \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{11}{6} & \frac{7}{6} & 0 & -\frac{1}{6} \end{array}\right]$$

$$R_3-R_2 \to R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \end{array}\right]$$

$$R_3 \times (-6) \to R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$$R_2-2R_3 \to R_2 \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{2}{3} & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$$R_1-2R_2 \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & \frac{7}{3} & -22/3 & 4 \\ 0 & 1 & 0 & -2/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$$R_1-3R_3 \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & -4/3 & 1 \\ 0 & 1 & 0 & -2/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} -2/3 & -4/3 & 1 \\ -2/3 & 11/3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

1 (a)

$$Ax = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$$

$$A^{-1}Ax = A^{-1}\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$$

$$x = \underbrace{\begin{bmatrix} -2/3 & -4/3 & 1 \\ -2/3 & 11/3 & -2 \\ 1 & -2 & 1 \end{bmatrix}}_{3\times 3}\underbrace{\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}}_{3\times 1}$$

3×1

$$= \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

---

1 (b)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 9 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 4R_1 \to R_2$
$R_3 - 7R_1 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -12 & -7 & 0 & 1 \end{array}\right]$$

$\frac{R_2}{-3} \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4/3 & -1/3 & 0 \\ 0 & -6 & -12 & -7 & 0 & 1 \end{array}\right]$$

$R_3 + 6R_2 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right] \Rightarrow \text{No inverse}$$

$\therefore \det \underline{A} = 0$

1 (c)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 6 \end{array}\right]$$

$$\begin{array}{l} R_2-4R_1\to R_2 \\ R_3-7R_1\to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 3 \\ 0 & -6 & -12 & 6 \end{array}\right]$$

$$\begin{array}{l} \frac{R_2}{(-3)}\to R_2 \\ \cancel{R_3+6R_2\to R_3} \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & -6 & -12 & 6 \end{array}\right]$$

$$R_3+6R_2\to R_3 \left[\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 0 \\ 0 & \boxed{1} & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ x_3 is a free variable $= C$

$x_2 + 2x_3 = -1$

$$\underline{x_2 = -1 - 2x_3 = -1-2C} \quad \text{——— (1)}$$

$x_1 + 2x_2 + 3x_3 = 0$

$$\begin{aligned} x_1 &= -2x_2 - 3x_3 \\ &= -2(-1-2C) - 3C \\ &= 2 + 4C - 3C \end{aligned}$$

$$\underline{x_1 = 2 + C} \quad \text{——— (2)}$$

$$\therefore \underline{x} = \begin{bmatrix} 2 + C \\ -1 + (-2)C \\ 0 + C \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + C\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad \text{where } C \in \mathbb{R}$$

1 (d)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & D \end{array}\right] \xRightarrow[R_3-7R_1]{R_2-4R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 3 \\ 0 & -6 & -12 & D \end{array}\right]$$

$$\xrightarrow{\frac{R_2}{(-3)} \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & -6 & -12 & D \end{array}\right] \xRightarrow[\to R_3]{R_3+6R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & D-6 \end{array}\right]$$

The 3^{rd} row is zero row so there will be a solution only if $D-6=0$

consistent when $D=6$

inconsistent when $D \neq 6$

2

x_1 = no of portfolio A

x_2 = " B

x_3 = " C

$$6x_1 + x_2 + 3x_3 = 35$$
$$3x_1 + 2x_2 + 3x_3 = 22$$
$$x_1 + 5x_2 + 3x_3 = 18$$

(a)

$$\left[\begin{array}{ccc|c} 6 & 1 & 3 & 35 \\ 3 & 2 & 3 & 22 \\ 1 & 5 & 3 & 18 \end{array}\right]$$

MA217 Mock Solution

5

(b) $R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 3 & 2 & 3 & 22 \\ 6 & 1 & 3 & 35 \end{array}\right]$$

$R_2 - 3R_1 \to R_2$

$R_3 - 6R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & -13 & -6 & -32 \\ 0 & -29 & -15 & -73 \end{array}\right]$$

$\dfrac{R_2}{-13} \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & 1 & 6/13 & 32/13 \\ 0 & -29 & -15 & -73 \end{array}\right]$$

$R_3 + 29R_2 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & 1 & 6/13 & 32/13 \\ 0 & 0 & -15 + \frac{174}{13} & -73 + 928/13 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & 1 & 6/13 & 32/13 \\ 0 & 0 & \frac{-21}{13} & \frac{-21}{13} \end{array}\right]$$

$$\frac{-21}{13} x_3 = \frac{-21}{13} \Rightarrow x_3 = 1$$

$$x_2 + \frac{6}{13}(x_3) = \frac{32}{13}$$

$$x_2 = \frac{32}{13} - \frac{6(1)}{13} = \frac{26}{13} = 2$$

$$x_1 + 5x_2 + 3x_3 = 18$$

$$x_1 = 18 - 5(2) - 3(1) = 5$$

$\therefore$ $A = 5$

$B = 2$

$C = 1$

(3) $|B| = |B^T|$

$$= \begin{vmatrix} 2a & b & c & 0 \\ c & a+d & 0 & c \\ b & 0 & a+d & b \\ 0 & b & c & 2d \end{vmatrix}$$

$R_1 - R_4 \to R_1 \Rightarrow$

$$\begin{vmatrix} 2a & 0 & 0 & -2d \\ c & a+d & 0 & c \\ b & 0 & a+d & b \\ 0 & b & c & 2d \end{vmatrix}$$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ -2d & c & b & 2d \end{vmatrix}$$

$R_4 + \frac{d}{a} R_1 \to R_4 = \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & \frac{ca+dc}{a} & \frac{ab+db}{a} & 2d \end{vmatrix}$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & \frac{c}{a}(a+d) & \frac{b}{a}(a+d) & 2d \end{vmatrix}$$

3

$R_4 - \frac{c}{a}R_2 \to R_4$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & 0 & \frac{b}{a}(a+d) & \frac{2ad-cb}{a} \end{vmatrix}$$

$R_4 - \frac{b}{a}R_3$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & 0 & 0 & \frac{2ad-2bc}{a} \end{vmatrix}$$

$$|B| = (2a)(a+d)(a+d)(2)\left(\frac{ad-bc}{a}\right)$$

$$= 4(a+d)^2(ad-bc)$$

$|C| =$ $R_3 \leftrightarrow R_4$

$$\begin{vmatrix} c & a+b & 0 & d \\ 0 & b & a & c+d \\ 0 & 0 & a+c & 2a \\ 0 & 0 & 0 & d+b \end{vmatrix}$$

$$= cb(a+c)(d+b).$$

$|D|$ is undefined as det is a property of a square matrix

3(b) $\det B = 0$

$$4(a+d)^2(ad-bc) = 0$$

$\therefore$ either $a+d=0$ or $ad-bc=0$

$a=-d$ or $ad=bc$

(c) $a=1,\ b=2,\ c=1,\ d=0$

$$|B| = 4(1+0)^2(0-(2)(1)) = -8$$

$$\det(2B^2) = 2^4|B||B| = 16(-8)(-8) = 1024$$

$$|C| = (1)(2)(1+1)(0+2) = 2(2)(2) = 8$$

(d) $$|AC^{-1}|^T = |AC^{-1}| = |A||C^{-1}|$$

$$= \frac{|A|}{|C|}$$

$$|C^{\frac{1}{3}}B^{-1}| = \frac{|C|^{1/3}}{|B|} = \frac{2}{(-8)} = \frac{|A|}{8}$$

$$|A| = -2$$

4.

$$\int_{y=2}^{y=3}\int_{x=1}^{x=2} 2xy\,dx\,dy$$

$$= \int_{y=2}^{y=3}\left[2y\frac{x^2}{2}\right]_{x=1}^{x=2} dy$$

$$= \int_{y=2}^{y=3}\left[yx^2\right]_{x=1}^{x=2} dy$$

$$= \int_{y=2}^{y=3} y\overbrace{(4-1)}^{=3y}\,dy$$

$$= \left[\frac{3y^2}{2}\right]_{y=2}^{y=3}$$

$$= 1.5(9-4) = 7.5 \ \text{unit}^3$$

20:02

5.

$$\frac{\iint p\,dx\,dy}{(125-100)(89-70)}$$

$$\int_{100}^{125} p\,dx = \int (x-30)(70+5x-4y) + (y-40)(80-6x+7y)\,dx$$

$$= \int 5x^2 - 80x + 120y - 4xy - 2100 - 3200 + 240x - 200y + 7y^2 - 6xy$$

$$= \int_{100}^{125}\left(5x^2 + 160x - 80y - 10xy - 5300 + 7y^2\right)dx$$

$$= \left[\frac{5x^3}{3} + \frac{160x^2}{2} - 10y\frac{x^2}{2} - 5300x + 7y^2x - 80yx\right]_{100}^{125}$$

5 Average output $= \dfrac{1}{(12-10)(3200-2800)} \iint 50 k^{3/5} L^{2/5} \, dA$

$$= \frac{1}{800} \int_{2800}^{3200} \int_{10}^{12} 50 k^{3/5} L^{2/5} \, dk \, dL$$

$$= \frac{1}{800} \int_{2800}^{3200} 50 L^{2/5} \left[\frac{5}{8} k^{8/5} \right]_{k=10}^{k=12} dL$$

$$= \frac{1}{800} (50)\left(\frac{5}{8}\right) \int_{2800}^{3200} L^{2/5} \left(12^{8/5} - 10^{8/5}\right) dL$$

$$= \frac{1}{800} (50)\left(\frac{5}{8}\right)\left(12^{8/5} - 10^{8/5}\right) \left[\frac{5}{7} L^{7/5} \right]_{K=2800}^{L=3200}$$

$$= \frac{1}{800} (50)\left(\frac{5}{8}\right)\left(\frac{5}{7}\right)\left(12^{8/5} - 10^{8/5}\right)\left[(3200)^{7/5} - (2800)^{7/5} \right]$$

≈ 5181 units