Solution: Quiz 1

1. Define the truth table for a connective operator $\oplus$ as follow.

p	q	$p \oplus q$
T	T	T
T	F	T
F	T	F
F	F	F

Determine whether or not $p \oplus (q \oplus \sim p) \equiv (\sim p \oplus q) \oplus p$.

Solution:

$p \oplus (q \oplus \sim p)$ and $(\sim p \oplus q) \oplus p$ are not equivalent. . From the truth table,

p	q	$\sim p$	$q \oplus \sim p$	$\sim p \ \oplus q$	$p \oplus (q \oplus \sim p)$	$(\sim p \ \oplus q) \oplus p$
T	T	F	T	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	F	T	F	T	F	T

the truth values of $p \oplus (q \oplus \sim p)$ and $(\sim p \oplus q) \oplus p$ in the last two columns are different. Therefore, they are not equivalent.

2. Determine whether the following argument form is valid or not by using the truth table.

I eat vegetable or I am not healthy. If I eat vegetable, then I am healthy.
Therefore, I am healthy if and only if I eat vegetable.

Answer:
Let p be *"I eat vegetable"* and q be *"I am healthy."*
The above argument can be written in the following form.

$$p \vee \sim q$$
$$p \rightarrow q$$
$$\therefore\ q \leftrightarrow p$$

Truth table:

p	q	$\sim q$	$p \vee \sim q$	$p \rightarrow q$	$q \leftrightarrow p$	
T	T	F	[T]	[T]	T	←-- critical row
T	F	T	T	F		
F	T	F	F	T		
F	F	T	[T]	[T]	T	←-- critical row:

The critical rows are rows 1 and 4. Since all rows with true premises (critical rows) have true conclusion, then this argument is **valid**.