Formula Sheet

Properties of Absolute-Value Inequities

Let x and y be real numbers.

(1) If $x \neq 0$, then $|x| > 0$.

(2) $|x| = 0$ if and only $x = 0$

(3) $|-x| = |x|, \quad |x| \geq |x|, \quad |x| \leq |x|$

(4) $-|x| \leq x \leq |x|$

(5) $|x + y| \leq |x| + |y|$

(6) $|x - y| \leq |x| + |y|$

(7) $||x| - |y|| \leq |x - y|$

(8) $|xy| = |x||y|$

(9) $\left|\frac{x}{y}\right| \leq \frac{|x|}{|y|}$

(10) $|x|^2 = x^2$

(11) Let a be a positive constant.
$|x| < a$ if and only if $-a < x < a$.
$|x| \leq a$ if and only if $-a \leq x \leq a$.

$|x| > a$ if and only if $x < -a$ or $x > a$.
$|x| \geq a$ if and only if $x \leq -a$ or $x \geq a$.

Definition: Let $F : X \to Y$ be a function from a set X to a set Y.

- F is **one-to-one**: $\forall x_1, x_2 \in X$, if $F(x_1) = F(x_2)$, then $x_1 = x_2$.
- F is **onto**: $\forall y \in Y, \exists x \in X$ such that $F(x) = y$.