EE325 Introductory Econometrics: Section 2
Semester 1, Academic year 2020
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Additional practice ☺

1. What is the meaning of Time series data, Cross sectional data, and Panel data?

2. What are the meanings of Independent Events and Mutually Exclusive Events?

3. One bag contains 3 kinds of fruits; 3 oranges, 2 mango and 3 mangosteen. If 4 fruits are randomly selected at the same time, X represents the number of oranges picked and Y represents the number of mangos picked. Answer the following questions:

3.1. Find the probability of getting one mango, given that all the remaining fruit is mangosteen.

3.2. Find $P(X+Y>2), P(XY \leq 3), P\left(\frac{X}{Y}>1\right)$

3.3. Find $E(X)$ and $E(Y)$

3.4. Find $Var(X)$ and $Var(Y)$

3.5. Find $\text{cov}(X,Y)$

3.6. Find $E(3X+4Y+5)$ and $Var(3X+4Y+5)$

3.7. Find the correlation coefficient between X and Y.

3.8. Find $E(Y|X=2)$ and $Var(Y|X=2)$

3.9. Are random variables X and Y independent of each other? Why?

4. Let X and Y to be discrete random variables with joint probability distribution function according to the following table. Answer the following questions:

		X		
		1	2	3
Y	2	0.1	0.2	0.1
	4	0.3	0.2	a

4.1. Find the value of a in the table.

4.2. Find $E(X)$ and $E(Y)$

4.3. Find $Var(X)$ and $Var(Y)$

4.4. Find the value $E(Y|X=1)$ and $Var(Y|X=2)$

4.5. Find $Var(X-Y)$

4.6. Find $E(E(Y|X))$ and show that $E(E(Y|X))=E(Y)$. (law of iterated expectations)

5. The density function of X is given by

$$f(x)=\begin{cases} a+bx^2 & ;0\leq x<1 \\ 0 & ;elsewhere \end{cases}$$

Suppose that $E(X)=\frac{3}{5}$, Find a and b .

6. Let X and Y be continuous random variables which have a function of joint density:

$$f(x,y)=\begin{cases} x^2+\frac{xy}{k} & ;0<x<1,0<y<2 \\ 0 & ;elsewhere \end{cases}$$

where k is a constant. Answer the following questions

6.1. Find the value of k .

6.2. Find $P\left(\frac{1}{4}<X<\frac{1}{2},\frac{1}{2}<Y<1\right),P\left(X>\frac{1}{2}\right)$

6.3. Find $P\left(X>\frac{1}{2}\middle|Y=1\right),P\left(\frac{1}{4}<X<\frac{1}{2}\middle|\frac{1}{2}<Y<1\right)$

6.4. Find the correlation coefficient between X and Y

6.5. Let $Z=2X-3Y+5$. Find the $E(Z)$ and $Var(Z)$ values.

6.6. Find $E\left(Y\middle|X=\frac{1}{2}\right)$ and $Var\left(Y\middle|X=\frac{1}{2}\right)$

6.7. Find $Var\left(Y^2\right)$

6.8. Are random variables X and Y statistical independence? Why?

6.9. Find $E\left(E\left(Y|X\right)\right)$ and show that $E\left(E\left(Y|X\right)\right)=E\left(Y\right)$. (law of iterated expectations)

6.10. Show that $Var(Y)=E\left[Var\left(Y|X\right)\right]+Var\left[E\left(Y|X\right)\right]$.

7. Let X and Y to be continuous random variables representing the amount of raw materials of type X (kg) and type Y (kg) for producing a product, respectively. According to the researchers' studies, X and Y have a joint probability density function:

$$f(x,y)=\begin{cases} Ax^{\alpha}y^{1-\alpha} & ;0\le x\le 1,0\le y\le 1 \\ 0 & ;elsewhere \end{cases}$$

where A is a constant and $0\le\alpha\le 1$

7.1. Find the range of A .

7.2. The probability of using X and Y raw materials for producing that product are statistical independence or nor?

8. Let X_1,X_2,X_3,X_4,X_5 be an independent random sample derived from a population with a standard normal distribution. If the random variable $T=\sum_{i=1}^{5}X_i^2$, then what does the random variable T mean?, What is the variance? and what form of distribution will it be?

9. According to the age data of the elderly in nursing homes which have the normal distribution. In 2011, found that 5% of the total elderly were under 60.5 years, 25% of the total elderly were over 72 years and there was no change in the number of elderly from 2011 to 2013. If one elderly person was randomly assigned in a nursing home in 2013, answer the following questions:

9.1. Find the probability of getting older people between 70 and 72 years old.
9.2. Find the probability that the elderly will be older than the average age of not more than 3 years in percentage.
9.3. Find the probability that the elderly will age different from the average age of less than 3 years in percentage.

10. Let $X_1, X_2, ..., X_n$ be an independent random variable that has the mean that is equal to μ and the variance that is equal to σ^2.
10.1. Let $W_a = a_1X_1 + a_2X_2 + ... + a_nX_n$ where a_i is a constant, find the condition of a_i that makes W_a unbiased estimator of μ.
10.2. Show that $Var(W_a) \geq Var(\bar{Y})$ (Hint: use the inequality, $\left(\sum_{i=1}^{n} a_i\right)^2 \Big/ n \leq \sum_{i=1}^{n} a_i^2$)

11. Let X and Y be positive random variables, where $E(Y|X) = \theta X$. And θ is unknown parameters.
11.1. Set the random variable Z, $Z = Y/X$. Show that $E(Z) = \theta$.
11.2. Let W be the estimator of θ, calculated from $W = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i}{X_i}\right)$. Show that W is an unbiased estimator of θ.