Exercise 2 (Part 1)

1. Determine the truth value of each of these statements. Explain your answer.

   (a) $\forall n \in \mathbb{Z},\ \ n-1<n$ (b) $\forall n \in \mathbb{Z},\ \ n \leq 10n$

   (c) $\exists n \in \mathbb{Z},\ \ 2n=-n$ (d) $\exists n \in \mathbb{Z}^{-},\ \ n=\frac{1}{n}$

2. Let $\mathbb{R}$ be the domain of x. Determine the **truth set** for each of these statements.

   (a) $P(x)$: "$x+1<2x$" (b) $P(x)$: "$x^2<4$ and $x \leq 0$"

3. Let $Q(x,y)$ be the statement "$x+y=x-y$." If the domain for both variables consists of all integers, determine the truth values of the following statements. Explain you answer.

   (a) $Q(1,1)$ (b) $Q(2,0)$ (c) $\forall y, Q(1,y)$

   (d) $\exists x, Q(x,2)$ (e) $\forall x \exists y, Q(x,y)$ (f) $\forall y \exists x, Q(x,y)$

4. Let $Q(x,y,z)$ be the statement "$x+y=z$." Let the domain of all variables be the set of all real numbers. Determine the truth value of the statement $\exists z \forall x \forall y,\ Q(x,y,z)$. Explain your answer.

5. Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is colorful," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ and determine whether this argument is valid or invalid by applying the equivalences of statement forms and the inference rule(s).

   "All hummingbirds are colorful."
   "No large birds live on honey."
   "Birds that do not live on honey are not colorful."
   $\therefore$ "Hummingbirds are not large birds."

6. Let $\mathbb{Z}^{+}$ be the domain of x. Determine whether the following statements are true or false. Give a counterexample for each false statement.
   (a) $\sqrt{x}>1 \Rightarrow x^2>23$, (b) $\sqrt{x}>2 \Leftrightarrow x^2>23$.

7. Write a negation for each statement without using *the negation symbol "*$\sim$*."*

   (a) $\exists x \in \mathbb{R}$, $(x-2)(x+1)>0$ if and only if $x>2$ or $x<-1$.
   (b) $\forall \varepsilon \exists \delta \forall x\,(|x-a|<\delta \rightarrow |f(x)-L|<\varepsilon)$

8. Show that each of the following arguments is valid by **universal modus ponens** , **universal modus tollens** or **universal transitivity**, or show that it is invalid from the **converse error** or the **inverse error**. In addition, use also the **diagram** to confirm that each argument is valid or invalid.

   (a) All rabbits like vegetable. My pet is not a rabbit. Therefore, my pet does not like vegetable.
   (b) Everyone who eats fruit every day is healthy. Linda is not healthy. Therefore, Linda does not eat fruit every day.