EE 422 Mathematical Economics 2 (1/2016)

Assignment 4

The due date for this assignment is (Wednesday) 26th October 2016.

1. (A multiplier-accelerator model of growth)

   Let Y_t denote national income, I_t total investment, and S_t total saving- all in period t. Suppose that savings are proportional to notional income, and that investment is proportional to the change in income from period t to $t+1$. Then, for $t = 0,1,2,...$

   (i) $S_t = \alpha Y_t$

   (ii) $I_{t+1} = \beta(Y_{t+1} - Y_t)$

   (iii) $S_t = I_t$

   The last equation is a familiar equilibrium condition that saving equals investment in each period. Here α and β are positive constants, and we assume that $0 < \alpha < \beta$. Write down a difference equation determining the path of Y_t, given Y_0, and solve it. (20 points)

2. Find the definite solution of the following difference equations. Also draw as well as analyze the time paths, i.e. the time path is oscillatory and convergent: (20 points each)

   a) $y_{t+2} + 3y_{t+1} - \frac{7}{4}y_t = 9 \qquad (y_0 = 6; y_1 = 3)$

   b) $y_{t+2} - 2y_{t+1} + 2y_t = 1 \qquad (y_0 = 3; y_1 = 4)$

   c) $y_{t+2} - y_{t+1} + \frac{1}{4}y_t = 2 \qquad (y_0 = 4; y_1 = 7)$

3. Find the <u>particular solution</u> of the following difference equations: (10 points each)

   a) $y_{t+2} - 2y_{t+1} + 5y_t = t$

   b) $y_{t+2} - 2y_{t+1} + 5y_t = 4 + 2t$

   c) $y_{t+2} + 5y_{t+1} + 2y_t = 18 + 6t + 8t^2$

4. Find the complementary function of the following difference equations: (10 points each)

a) $y_{t+3} - \frac{1}{2} y_{t+2} - y_{t+1} + \frac{1}{2} y_t = 0$

b) $y_{t+3} - 2y_{t+2} + \frac{5}{4} y_{t+1} - \frac{1}{4} y_t = 1$

5. Test the convergence of the solutions of the following difference equations by the Schur theorem: (5 points each)

(a) $y_{t+2} + \frac{1}{2} y_{t+1} - \frac{1}{2} y_t = 3$

(b) $y_{t+2} - \frac{1}{9} y_t = 1$