1. Consider the ice cream market in Bangkok. In April, the ice cream market demand and supply curves are given by the following equations where Q is the quantity of ice cream units, T is the level of temperature in degree Celsius, and P is the price in dollars per unit of ice cream:

$$\text{Demand: } Q = 10000 + 400 \times (T - 30) - 10P$$
$$\text{Supply: } Q = 2000 + 20P$$

Suppose that T = 40 Celsius, find the equilibrium price and quantity of ice cream in April using the Inverse matrix method.

1. Demand : $Q = 10000 + 400(T-30) - 10p$

$Q = 10000 + 400(40-30) - 10p$

$Q = 10000 + 400(10) - 10p$

$Q = 10000 + 4000 - 10p$

$Q = 14000 - 10p$

$Q + 10p = 14000$ ——— (1)

Supply : $Q = 2000 + 20P$

$Q - 20P = 2000$ ——— (2)

$$\begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix} \begin{bmatrix} Q \\ P \end{bmatrix} = \begin{bmatrix} 14000 \\ 2000 \end{bmatrix}$$

$$A \quad x \quad = \quad d$$

$$A \begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix}$$

$\det(A) = |A| = \begin{vmatrix} 1 & 10 \\ 1 & 20 \end{vmatrix} = (1)(-20) - (1)(10) = -20 - 10 = -30$

$C_{11} = (-1)^{1+1} M_{11} = M_{11} = -20$

$C_{12} = (-1)^{1+2} M_{12} = -M_{12} = -1$

$C_{21} = (-1)^{2+1} M_{21} = -M_{21} = -10$

$C_{22} = (-1)^{2+2} M_{22} = -M_{22} = 1$

$\text{cof}(A) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$\text{cof}(A) = \begin{bmatrix} -20 & -1 \\ -10 & 1 \end{bmatrix}$

$\text{adj}(A) = [\text{cof}(A)]^T$

$\text{adj}(A) = \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj}(A)$

$A^{-1} = \frac{1}{-30} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$

$Ax = d$

$A^{-1}Ax = A^{-1}d$

$x = A^{-1}d$

$$\begin{bmatrix} Q \\ P \end{bmatrix} = \frac{-1}{30} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 14000 \\ 2000 \end{bmatrix}$$

$$\begin{bmatrix} Q \\ P \end{bmatrix} = \frac{-1}{30} \begin{bmatrix} (-20)(14000) + (-10)(2000) \\ (-1)(14000) + (1)(2000) \end{bmatrix}$$

$$\begin{bmatrix} Q \\ P \end{bmatrix} = \frac{-1}{30} \begin{bmatrix} -300000 \\ -12000 \end{bmatrix}$$

$$\begin{bmatrix} Q \\ P \end{bmatrix} = \begin{bmatrix} 10000 \\ 400 \end{bmatrix}$$

$Q^* = 10000$ units → equilibrium quantity

$P^* = \$400$ → equilibrium price

$Y^d = Y - T$

2. Consider a modified version of the IS-LM model where government spending is tied to the level of GDP (Y).

$C = C_0 + C_1 Y_d - C_2 r, \qquad 0 < C_1 < 1$

$I = I_0 + I_1 Y - I_2 r, \qquad 0 < I_1 < 1$

$G = G_0 - G_1 Y, \qquad 0 < G_1 < 1$

$T = T_0$

$M^s = M_0$

$L^d = L_0 + L_1 Y - L_2 r$

where C is the private consumption, I is the private investment, G is the government spending, T is tax, M^s is the level of money supply, L^d is the level of real money demand, and r is the level of nominal interest rate. Suppose that price is fixed equal to 1. All the coefficients are *non-negative*. Additionally, we assume that $I_1 + C_1 - G_1 < 1$.

a. Discuss about the nature of government behavior. Does the assumption over the behavior of government make sense in practice?
b. What does C_2 represent? What does it imply about the behavior of private consumption? Does the assumption make sense?
c. Derive the IS equation. Interpret the meaning of the IS equation. Discuss about the key relation derived from IS equation.
d. Calculate the slope of IS curve. When is the IS curve flat? What does the flat IS curve imply about the sensitivity of real GDP to the interest rate?
e. Use the IS equation and calculate the tax multiplier. How does the tax multiplier depend on slope of IS curve?

a.) $G = G_0 - G_1 Y,$

Government expenditures are often pre-determined as to how much the budget is spent each year. Although there was a change from the set, bet it was at a very small level. The government spending are therefore usually in the form of constants and, in addition, the government expenditures are correlated in the same direction as GDP.

So, this is not make sense.

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b.) $C = C_0 + C_1 Y^d - C_2 r$

C_2 let us know that if the interest rate greater 1 unit, it will affects the lower private consumption to be equal to C_2.

So, this is make sense #

Due to $C_1 > 0$, it means the relationship between GDP (Y) and consumption is getting along with each other, ___ ① and $-C_1 < 0$ so tax and consumption have an opposite relationship. ___ ②

Due to $C_2 > 0$, it means the relationship between interest rate and consumption have an opposite relationship.

c.)

$$C = C_0 + C_1 Y_d - C_2 r,$$

$$I = I_0 + I_1 Y - I_2 r,$$

$$G = G_0 - G_1 Y,$$

$$T = T_0$$

If we plug in G, T into $Y = C + I + G$, it will has only G_0 and T_0 in the Y equation.

$\therefore$ Multiplier of tax means the slope of Y and T_0, and multiplier of government means the slope of Y and G_0.

slope of LS curve $= \dfrac{-(C_2 + I_2)}{1 - (I_1 + C_1 - G_1)}$

because C_2 and I_2 are higher than zero $\therefore C_2 + I_2 > 0$, so $-(C_2 + I_2)$ will less than zero $-(C_2 + I_2) < 0$, and because $I_1 + C_1 - G_1 < 1$, so $1 - (I_1 + C_1 - G_1) > 0$.

$$\frac{\overset{-}{-(C_2 + I_2)}}{\underset{+}{1 - (I_1 + C_1 - G_1)}} < 0$$

- How does this relate to the shape of IS curve:
  - When I_1 is high, the IS curve is flatter!
    - $Y = \frac{1}{1-b(1-t)}[a - bT_0 + I_0 - I_1 r + G_0]$

slope $= \dfrac{-I_1}{1 - b(1-t)}$

IS equation

$Y = C + I + G$

$$Y = [C_0 + C_1 Y_d - C_2 r] + [I_0 + I_1 Y - I_2 r] + [G_0 - G_1 Y]$$

$$Y = C_0 + C_1 [Y - T] - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$$

$$Y = C_0 + C_1 Y - C_1 T_0 - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$$

$$Y - C_1 Y - I_1 Y + G_1 Y = C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0$$

$$(1 - C_1 - I_1 + G_1) Y = C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0$$

$$Y = \frac{C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0}{1 - C_1 - I_1 + G_1}$$

$$Y = \frac{(-C_2 r - I_2 r) + (C_0 - C_1 T_0 + I_0 + G_0)}{1 - C_1 - I_1 + G_1}$$

$$Y = \frac{-(C_2 + I_2) r + (C_0 - C_1 T_0 + I_0 + G_0)}{1 - C_1 - I_1 + G_1}$$

$$Y = \underbrace{\frac{-(C_2 + I_2) r}{1 - C_1 - I_1 + G_1}}_{\text{slope}} + \underbrace{\frac{C_0 - C_1 T_0 + I_0 + G_0}{1 - C_1 - I_1 + G_1}}_{\text{Y- intercept}}$$

considering slope of IS $= \dfrac{-(C_2 + I_2)}{1 - (I_1 + C_1 - G_1}$

due to $C_2, I_2 > 0$, $C_2 + I_2 > 0$, so $-(C_2 + I_2) < 0$

and $I_1 + C_1 - G_1 < 1$ then $1 - (I_1 + C_1 - G_1) > 0$

$\therefore \dfrac{-(C_2 + I_2)}{1 - (I_1 + C_1 - G_1)} < 0$ it's a negative.

Because the slope of IS curve is negative, so the relationship between y and r is opposite. That is when r increases, it will effect the lower in Y.

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d.) From the equation of consumption and investment

$$C = C_0 + C_1 Y_d - C_2 r,$$

$$I = I_0 + I_1 Y - I_2 r,$$

we can see that the IS curve will flat when the sensitivity of C to r (C_2) and I to r (I_2) are high.

e.) Calculate tax multiplier

The multiplier of tax means the slope of Y and T_0.

From IS equation: $Y = \frac{-(C_2 + I_2) r}{(1 - C_1 - I_1 + G_1)} + \frac{C_0 + I_0}{1 - C_1 - I_1 + G_1} - \left(\frac{C_1}{1 - C_1 - I_1 + G_1} \cdot T_0\right) + \left(\frac{1}{1 - C_1 - I_1 + G_1} \cdot G_0\right)$

$$\Delta Y = \frac{-MPC}{1 - MPC - MPI} \cdot \Delta T$$

Tax multiplier shows the lower in y when Tax increases 1 unit.

f. Derive the LM equation. Interpret the meaning of the LM equation. Discuss about the key relation derived from LM equation.

$$\frac{M^s}{P} = L^d$$

$$M_0 = L_0 + L_1 Y - L_2 r$$

$$r = \frac{1}{L_2}\left(L_0 + L_1 Y - \frac{M^s_0}{P}\right)$$

g. Calculate the slope of LM curve. When is the LM curve flat? What does the flat LM curve imply about the sensitivity of interest rate to the real GDP?

slope of LM : $\frac{\Delta r}{\Delta y} = \frac{L_1}{L_2}$

If L_1 is small or L_2 is high, LM curve is flat which mean the change in GDP (y) has small impact on the interest rate.

h. Write both IS and LM equations in terms of the matrix representation.

$$\begin{bmatrix} 1 - C_1 - I_1 + G_1 & C_2 + I_2 \\ L_1 & -L_2 \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} C_0 - C_1 I_0 + I_0 + G_0 \\ M_0 \end{bmatrix}$$

$$A \quad x = d$$

The condition that warrants the uniqueness of the solution is $|A| \neq 0$

A is non-singular matrix

$$\begin{vmatrix} 1 - C_1 - I_1 + G_1 & C_2 + I_2 \\ L_1 & -L_2 \end{vmatrix} \neq 0$$

$$-L_2(1 - C_1 - I_1 + G_1) - L_1(C_2 + I_2) \neq 0$$

$$-[L_2(1 - C_1 - I_1 + G_1) + L_1(C_2 + I_2)] \neq 0$$

i)

$$|A| = -\left[L_2(1-c_1-I_1+G_1) + L_1(c_2+I_2)\right]$$

$$|A_1| = \begin{vmatrix} c_0 - c_1T_0 + I_0 + G_0 & c_2 + I_2 \\ M_0 & -L_2 \end{vmatrix}$$

$$|A_1| = -L_2(c_0 - c_1T_0 + I_0 + G_0) - M_0(c_2+I_2)$$

$$|A_2| = \begin{vmatrix} 1-c_1-I_1+G_1 & c_0 - c_1T_0 + I_0 + G_0 \\ L_1 & M_0 \end{vmatrix}$$

$$|A_2| = M_0(1-c_1-I_1+G_1) - L_1(c_0 - c_1T_0 + I_0 + G_0)$$

$$Y^* = \frac{|A_1|}{|A|} = \frac{-L_2(c_0 - c_1T_0 + I_0 + G_0) - M_0(c_2+I_2)}{-\left[L_2(1-c_1-I_1+G_1) + L_1(c_2+I_2)\right]}$$

$$Y^* = \frac{-\left[-L_2(c_0 - c_1T_0 + I_0 + G_0) + M_0(c_2+I_1)\right]}{-\left[L_2(1-c_1-I_1+G_1) + L_1(c_2+I_2)\right]}$$

$$Y^* = \frac{L_2(c_0 - c_1T_0 + I_0 + G_0) + M_0(c_2+I_1)]}{\left[L_2(1-c_1-I_1+G_1) + L_1(c_2+I_2)\right]}$$ — equilibrium GDP ☺

$$r^* = \frac{|A_2|}{|A|} = \frac{M_0(1-c_1-I_1+G_1) - L_1(c_0 - c_1T_0 + I_0 + G_0)}{-\left[L_2(1-c_1-I_1+G_1) + L_1(c_2+I_2)\right]}$$

$$r^* = \frac{-\left[L_1(c_0 - c_1T_0 + I_0 + G_0) - M_0(1-c_1-I_1+G_1)\right]}{-\left[L_2(1-c_1-I_1+G_1) + L_1(c_2+I_2)\right]}$$

$$r^* = \frac{L_1(c_0 - c_1T_0 + I_0 + G_0) - M_0(1-c_1-I_1+G_1)}{L_2(1-c_1-I_1+G_1) + L_1(c_2+I_2)}$$ — equilibrium interest rate.

J.) The multiplier of G_0 on Y^*

$$A = \frac{\Delta Y^*}{\Delta G_0} = \frac{L_2}{L_2(1-C_1-I_1+G_1)+L_1(C_2+I_2)} \geq 0$$

The multiplier of M_0 on Y^*

$$B = \frac{\Delta Y^*}{\Delta M_0} = \frac{C_2+I_2}{L_2(1-C_1-I_1+G_1)+L_1(C_2+I_2)} \geq 0$$

The multiplier of G_0 on r^*

$$C = \frac{\Delta r^*}{\Delta G_0} = \frac{L_1}{L_2(1-C_1-I_1+G_1)+L_1(C_2+I_2)} \geq 0$$

The multiplier of M_0 on r^*

$$D = \frac{\Delta r^*}{\Delta M_0} = \frac{-(1-C_1-I_1+G_1)}{L_2(1-C_1-I_1+G_1)+L_1(C_2+I_2)} < 0$$

if the government spending is strictly exogenous $(G=G_0)$

$$\frac{\Delta Y^*}{\Delta G_0} = \frac{L_2}{L_2(1-C_1-I_1)+L_1(C_2+I_2)} > A$$

$$\frac{\Delta Y^*}{\Delta M_0} = \frac{C_2+I_2}{L_2(1-C_1-I_1)+L_1(C_2+I_2)} > B$$

$$\frac{\Delta r^*}{\Delta G_0} = \frac{L_1}{L_2(1-C_1-I_1)+L_1(C_2+I_2)} > C$$

$$\frac{\Delta r^*}{\Delta M_0} = \frac{-(1-C_1-I_1)}{L_2(1-C_1-I_1)+L_1(C_2+I_2)} > D$$

∴ The multiplier is smaller than the case that government spending purely exogenous.

The government is seeking for some advices on fiscal and monetary policy implementation. The goal of the government is to (i) *increase the real GDP (Y) by $100*, while (ii) *keeping the current level of interest rate stayed the same*. (That is, the government was thinking that the country is running into an unemployment situation, but the level of interest rate is now optimal.) Following the storyline given here and all your work that you have done before, answer the next two questions.

k. Can the government successfully achieve both goals by simply relying on *a single type of policy implemented*? That is, to achieve the two goals, would it work to either change the *government expenditure* or *money supply*, but not both at the same time? If yes, *under which conditions*?

goals: $\Delta Y^* > 0$ and $\Delta r^* = 0$

fiscal policy ($G\uparrow$) : $\frac{\Delta Y^*}{\Delta G_0} > 0$ if $L_2 > 0$

$\frac{\Delta r^*}{\Delta G_0} = 0$ only if $L_1 = 0$

monetary policy ($M\uparrow$) : $\frac{\Delta Y^*}{\Delta M_0} > 0$ only if $C_2 + I_2 > 0$

$\frac{\Delta r^*}{\Delta M_0} = 0$ not possible as the question assume $1 - C_1 - I_1 + G_1 > 0$

$\frac{\Delta r^*}{\Delta M_0} < 0$, $M_0 \uparrow \rightarrow r^* \downarrow$

the monetary policy cannot keep the current level of interest rate stayed the same.

the government can achieve both goals by using a single type of policy.

that is, to acheive the two goals ($\Delta Y^* > 0$ and $\Delta r^* = 0$), the government can use the expansionary fiscal policy by increasing the government expenditure. This situation is possible only if demand for money is independent of income ($L_1 = 0$) and $L_2 > 0$

l. If the condition that you assumed in (k) does not hold, what would you recommend to the government so that both goals can be simultaneously achieved? (Hint: think about an appropriate mixture of the two policies.)

If $L_1 = 0$ is not true. The government can acheive both goals by using the mixture of the two policies.

$\Delta Y^* =$ multiplier$_G \cdot \Delta G_0$ + multiplier of $M \cdot \Delta M_0$

$\Delta r^* =$ multiplier$_M \cdot \Delta G_0$ + multiplier of $M \cdot \Delta M_0$

$$\Delta Y^* = \frac{\Delta Y^*}{\Delta G} \cdot \Delta G + \frac{\Delta Y^*}{\Delta M_0} \cdot \Delta M_0$$

$$\Delta Y^* = A \cdot \Delta G_0 + B \cdot \Delta M_0$$

$\Delta Y^* = \$100$;

$A \cdot \Delta G_0 + B \cdot \Delta M_0 = 100 \rightarrow 1$

$$\Delta r^* = \frac{\Delta r^*}{\Delta G_0} \cdot \Delta G_0 + \frac{\Delta r^*}{\Delta M_0} \cdot \Delta M_0$$

$$\Delta r^* = C \cdot \Delta G_0 + D \Delta M_0$$

$\Delta r^* = 0$;

$C \cdot \Delta G_0 + D \Delta M_0 = 0 \rightarrow 2$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Delta G_0 \\ \Delta M_0 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$A X = d$$

$$X = A^{-1} d$$

$$\begin{bmatrix} \Delta G_0 \\ \Delta M_0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$