

Long-Run Cost & Production

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(1) Meaning of Long-Run Production

(2) Isoquant

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1. Meaning of Long-Run Production

1.1 Definitions and assumptions

5.1

Definition – Long-Run Production

is a production when all factors can be variable.

5.2

Definition – Long-Run Production Function

$$Q = f(\text{capital, labor}) = f(K, L)$$

Assumptions

- (1) Factors and output can be infinitely divisible. (Continuous property)
- (2) Combinations of factors are transitive.

2. Isoquant

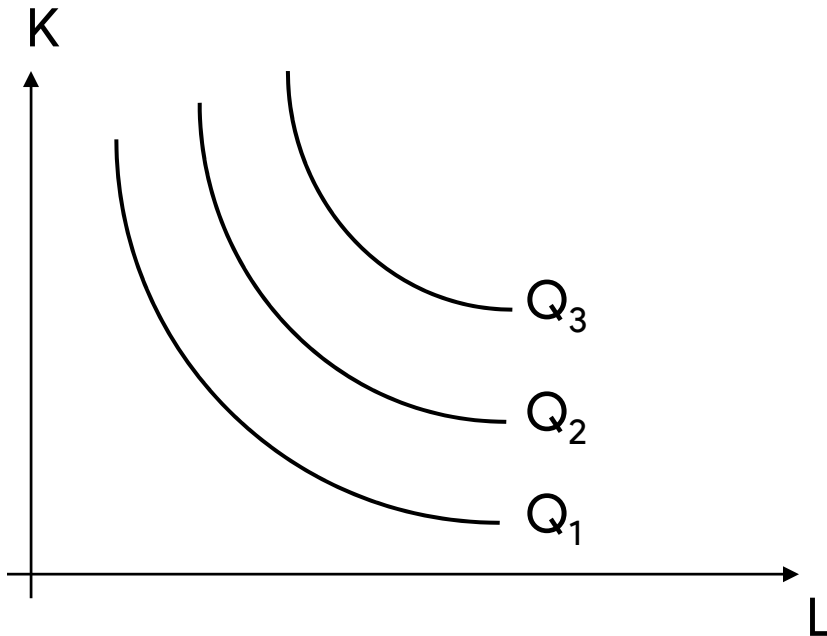
2.1 Isoquant properties

5.3

Definition – Isoquant

Isoquant is a curve showing different combinations of factors that yield the same amount of output.

Isoquant



Notes

- (1) Level of output
- (2) Isoquants do not intersect
- (3) Convex to the origin

2. Isoquant

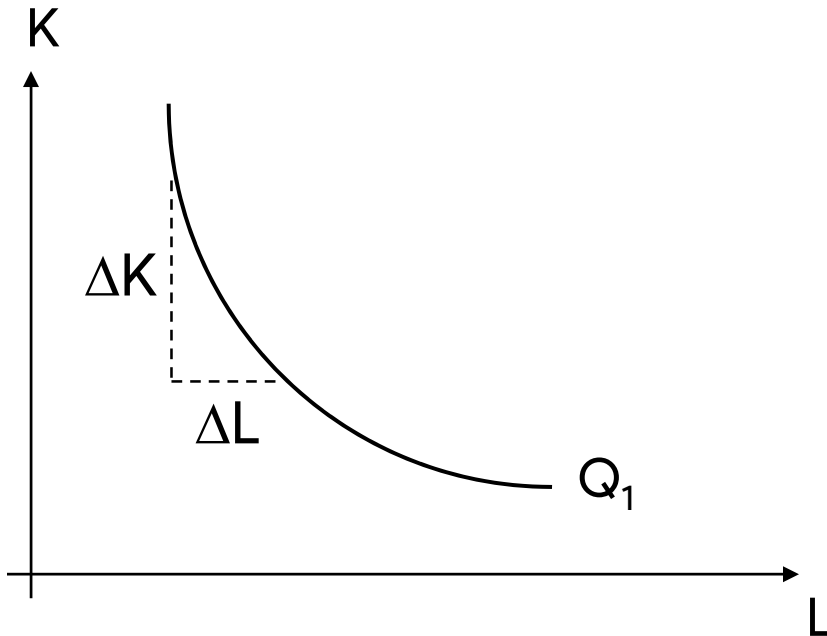
2.2 Marginal Rate of Technical Substitution

5.4

Definition – Marginal Rate of Technical Substitution

is a rate of two substituting factors that yields the same amount of output.

Isoquant



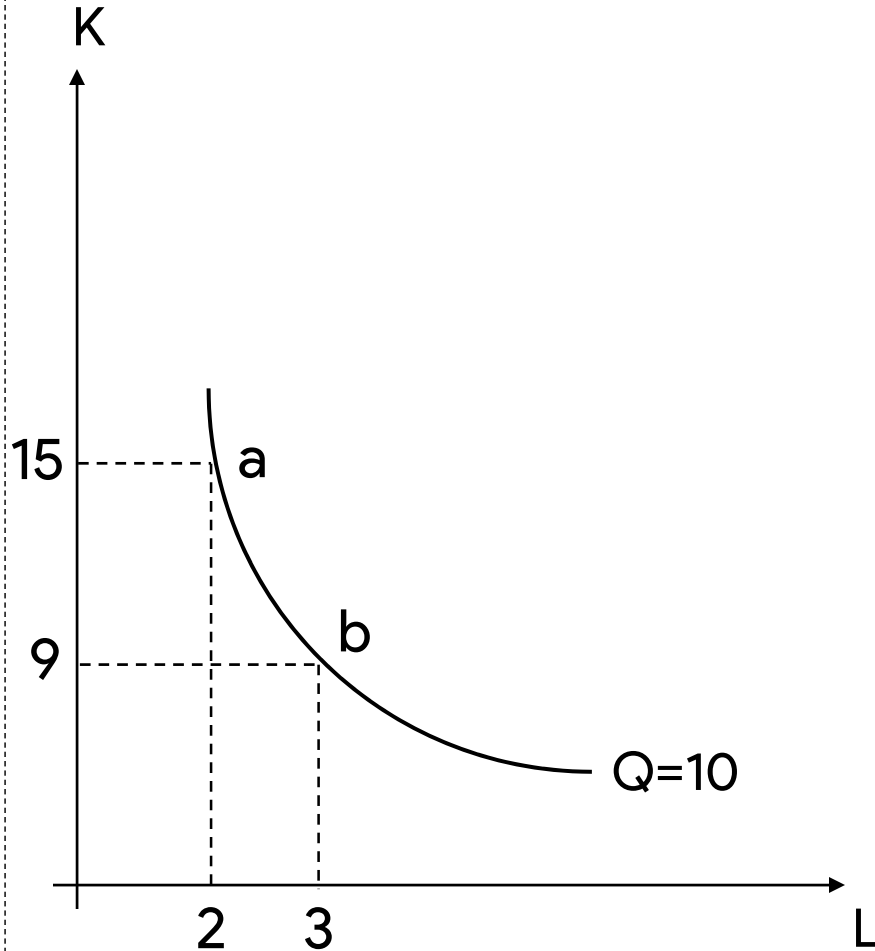
Notes

$$MRTS_{LK} = \frac{\Delta K}{\Delta L}$$

2. Isoquant

2.2 Marginal Rate of Technical Substitution

Isoquant



Example

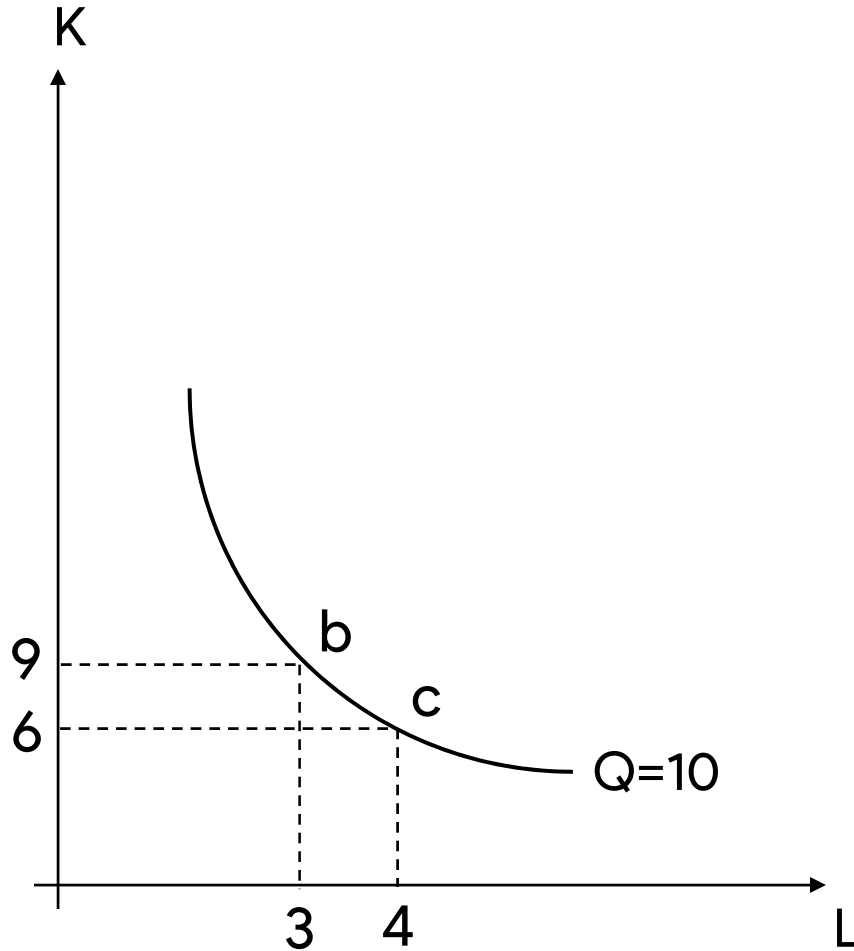
Combination a and b yield 10 units of output.

$$MRTS_{LK(b)} =$$

2. Isoquant

2.2 Marginal Rate of Technical Substitution

Isoquant



Example

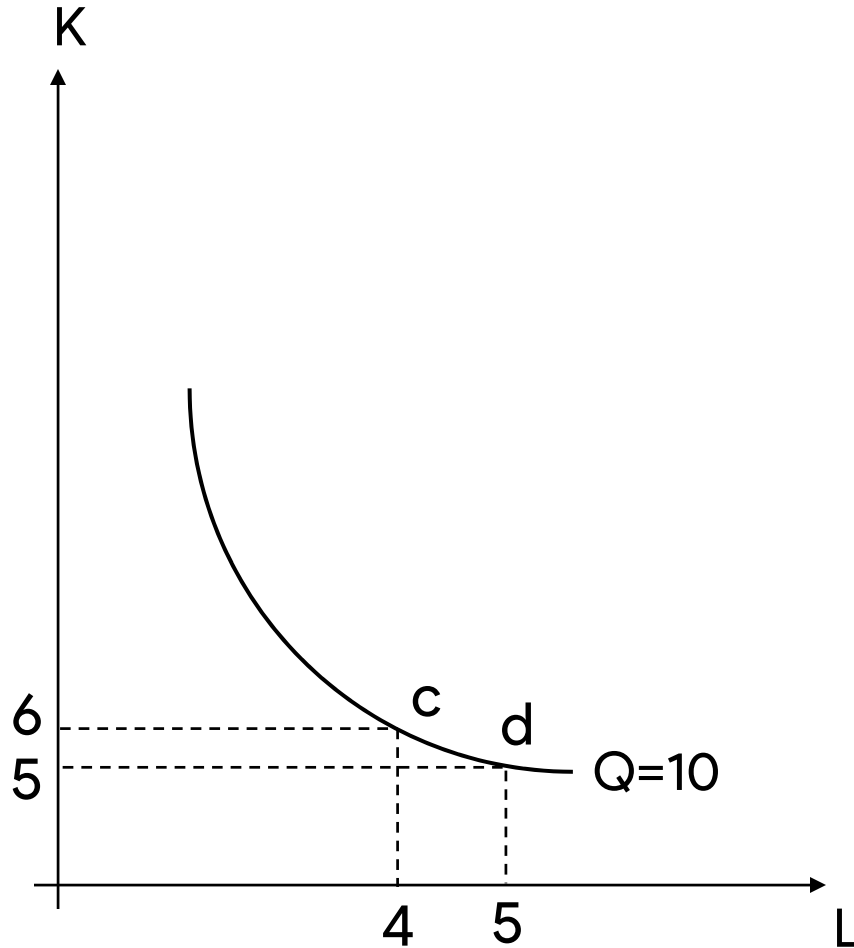
Combination b and c yield 10 units of output.

$$MRTS_{LK(c)} =$$

2. Isoquant

2.2 Marginal Rate of Technical Substitution

Isoquant



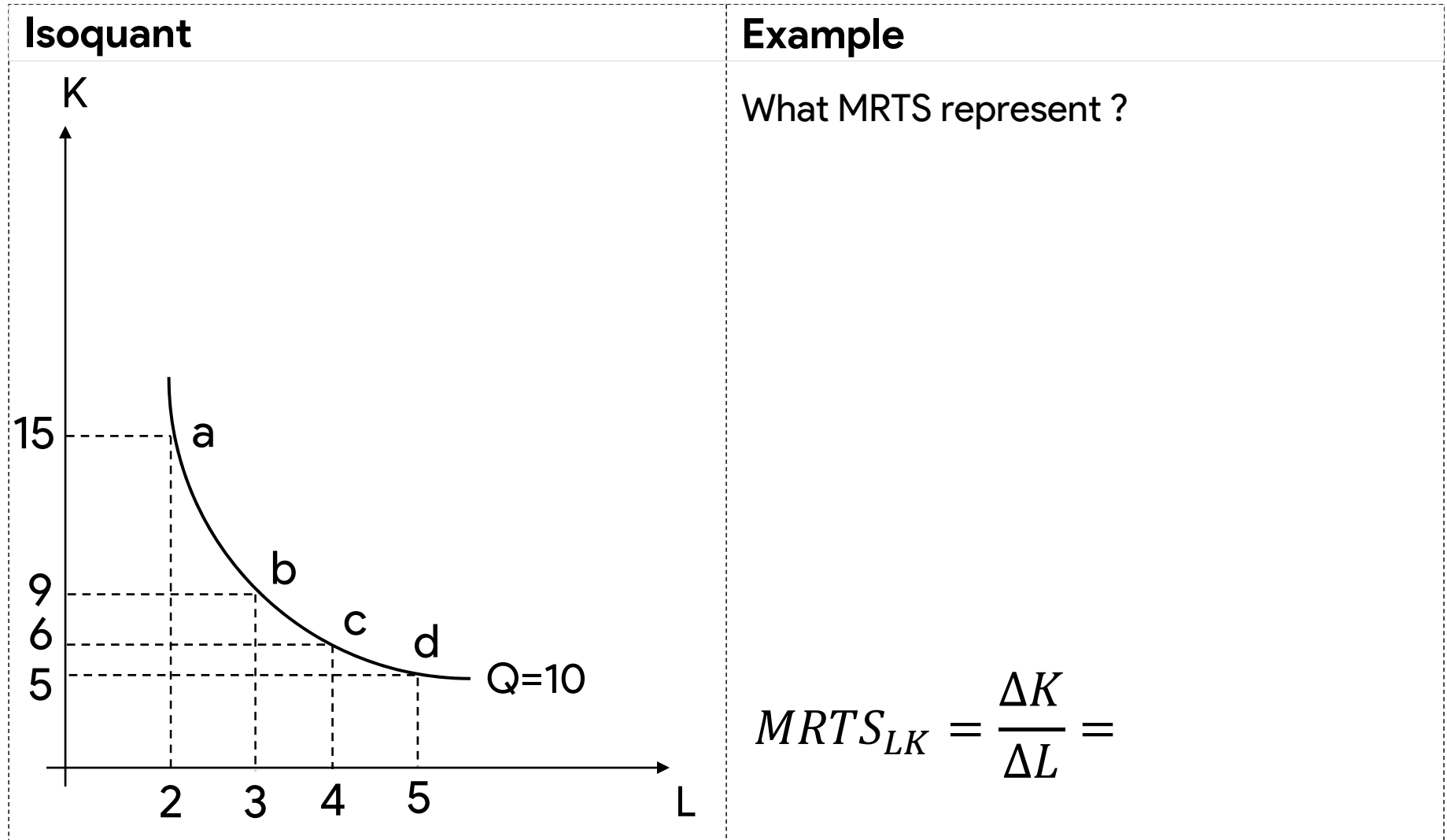
Example

Combination c and d yield 10 units of output.

$$MRTS_{LK(d)} =$$

2. Isoquant

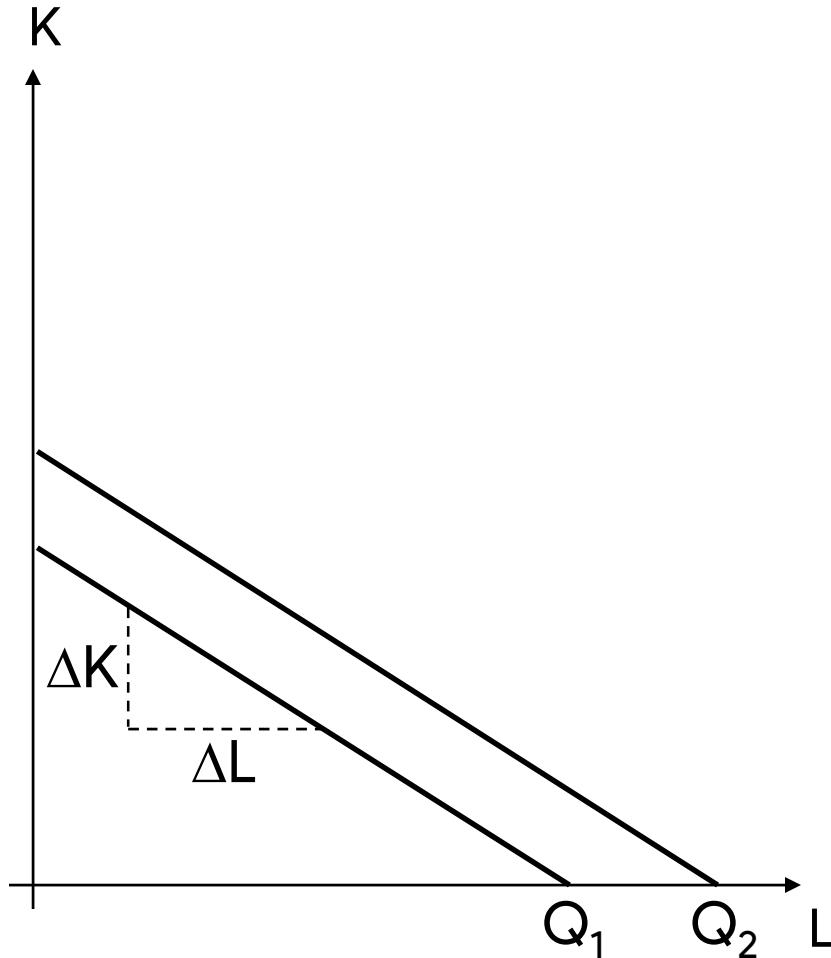
2.2 Marginal Rate of Technical Substitution



2. Isoquant

2.3 Other types of isoquant

Perfectly substitutable



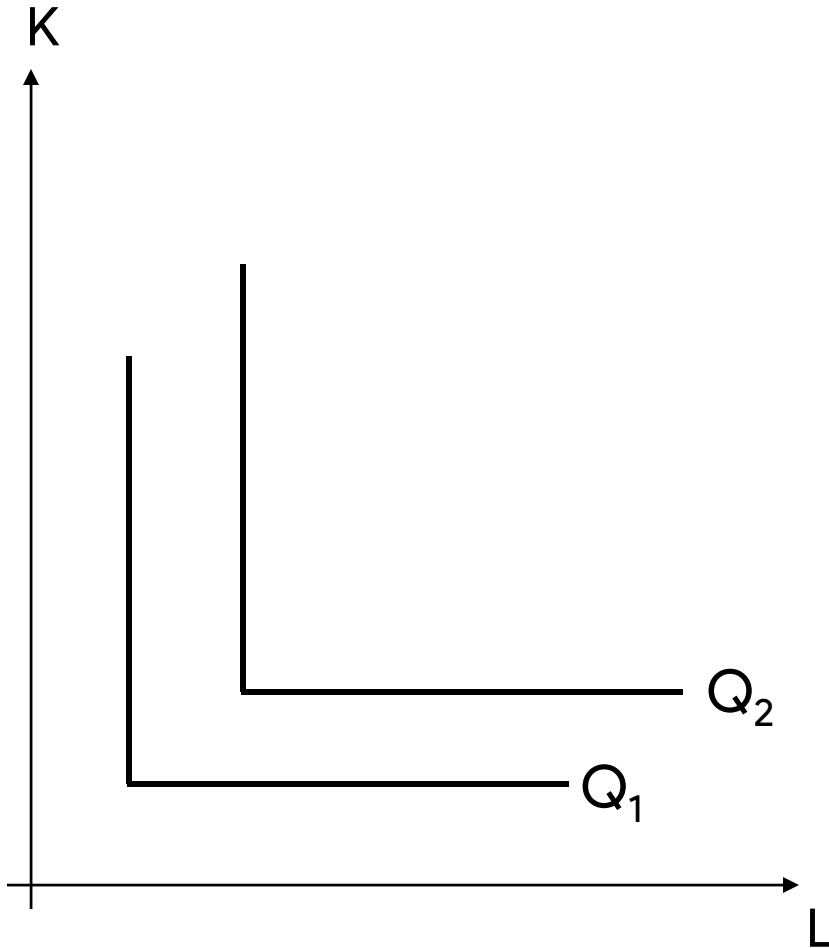
Note

MRTS is constant for every point.

2. Isoquant

2.3 Other types of isoquant

Perfectly complementary



Note

MRTS is either 0 or undefined.

3. Isocost

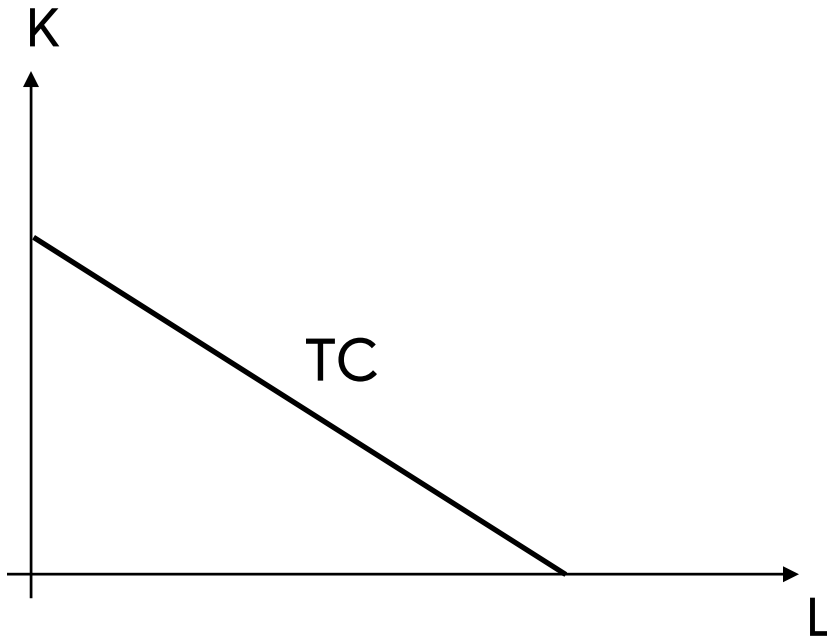
3.1 Definition

5.4

Definition – Isocost

Isocost is a line representing combinations of utilizing two factors of production and results in incurred cost.

Isocost



Note

$$TC = wL + rK$$

3. Isocost

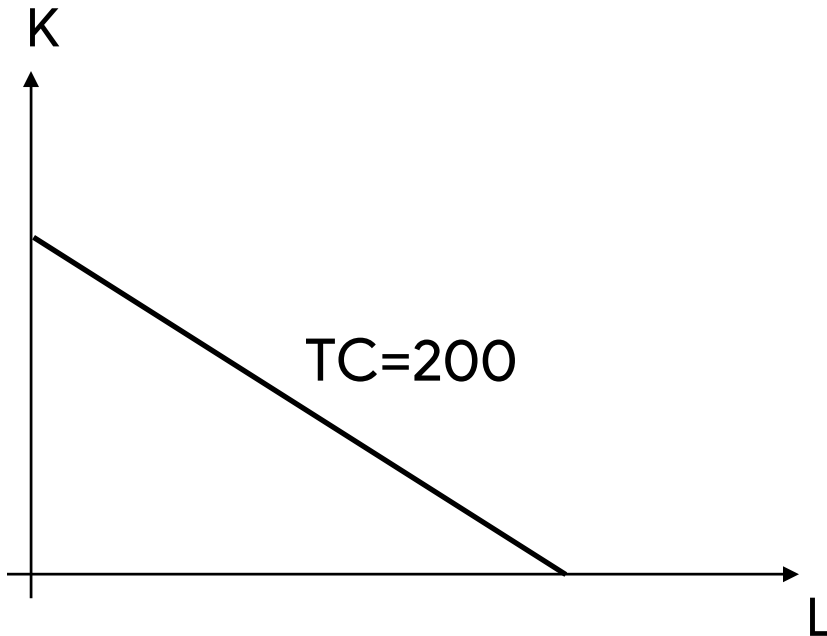
3.2 Marginal Rate of Market Substitution

5.5

Definition – Marginal Rate of Market Substitution

is a rate of substituting one and another factor of production that yield the same amount of cost.

Isocost



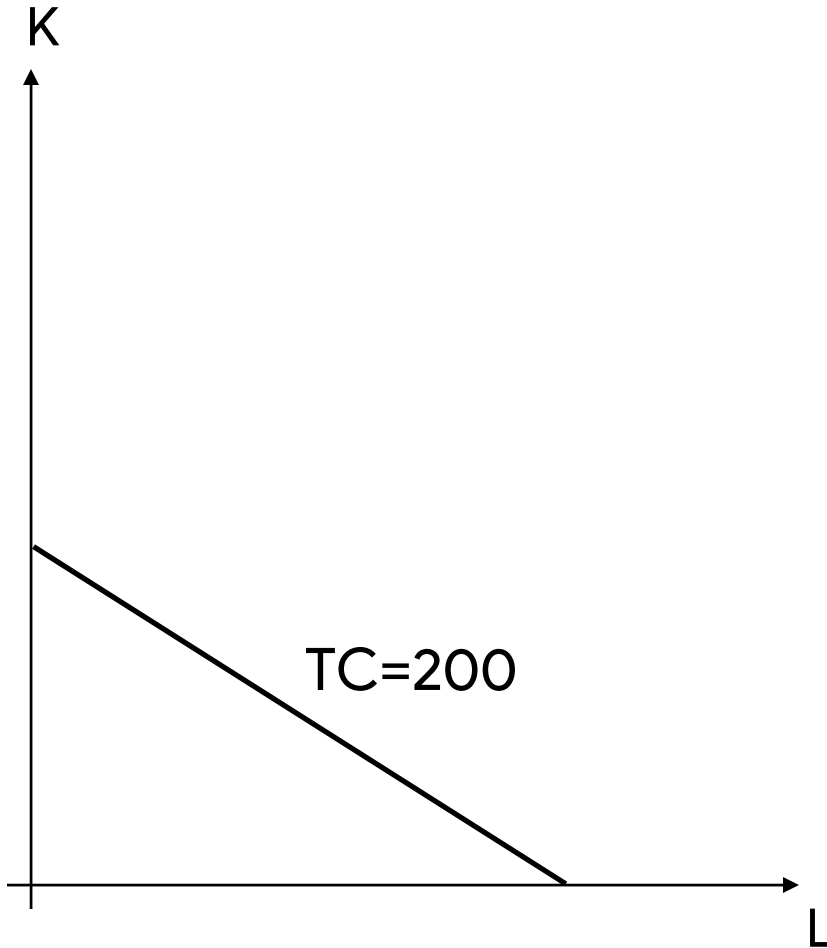
Example

Supposed that a producer can bear 200 baht cost daily when daily wage is 25 baht and daily capital interest is 50 baht.

3. Isocost

3.2 Marginal Rate of Market Substitution

Change in cost



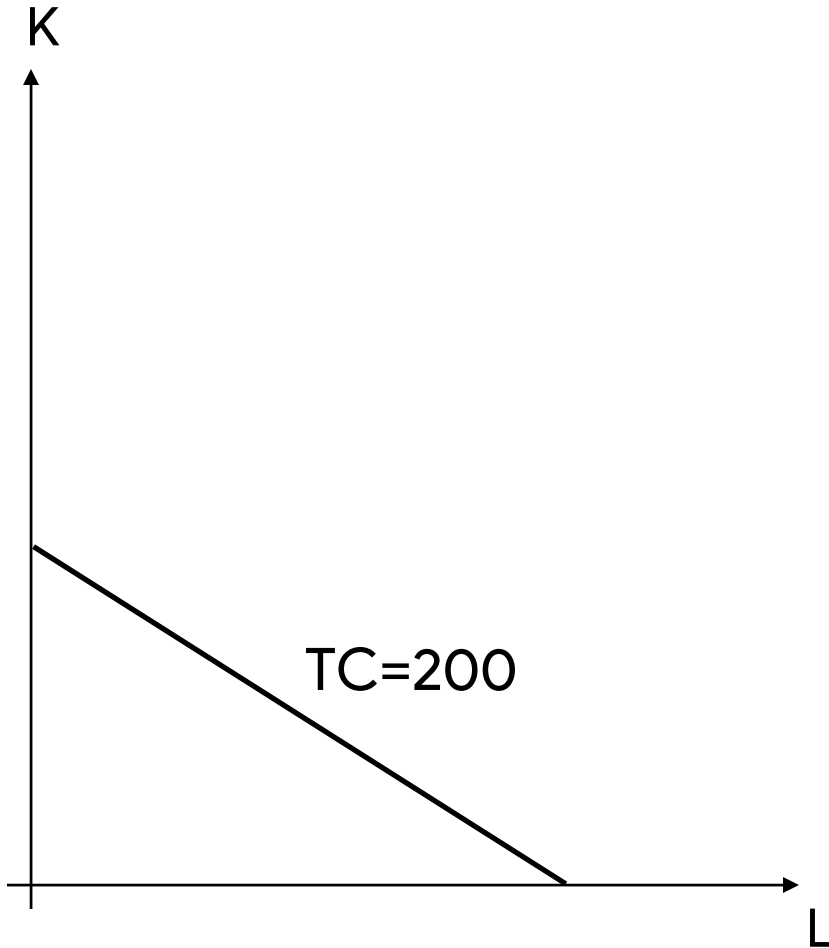
Example

Given that all factors cost remains unchanged, if this producer can bear 300 baht and 100 baht of daily cost.

3. Isocost

3.2 Marginal Rate of Market Substitution

Change in cost



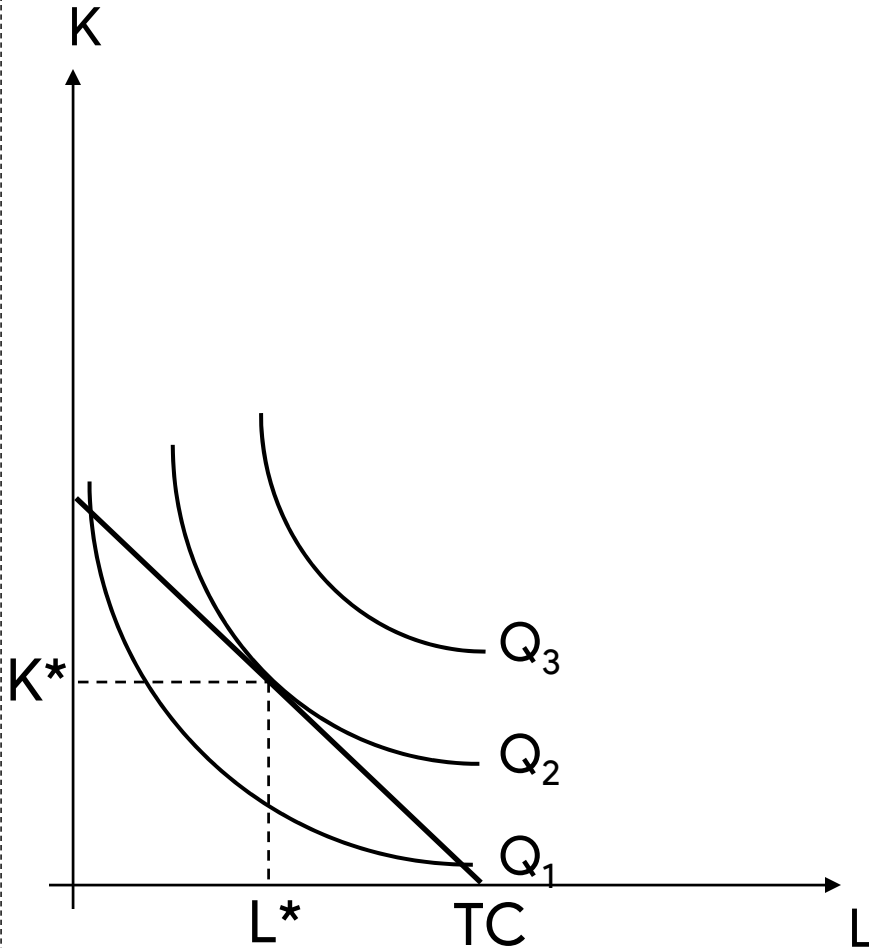
Example

Supposed that wage is raised from 25 to 50 baht daily and capital interest drops from 50 to 20 baht daily.

4. Cost Minimization

4.1 Definition

Cost minimization



Note

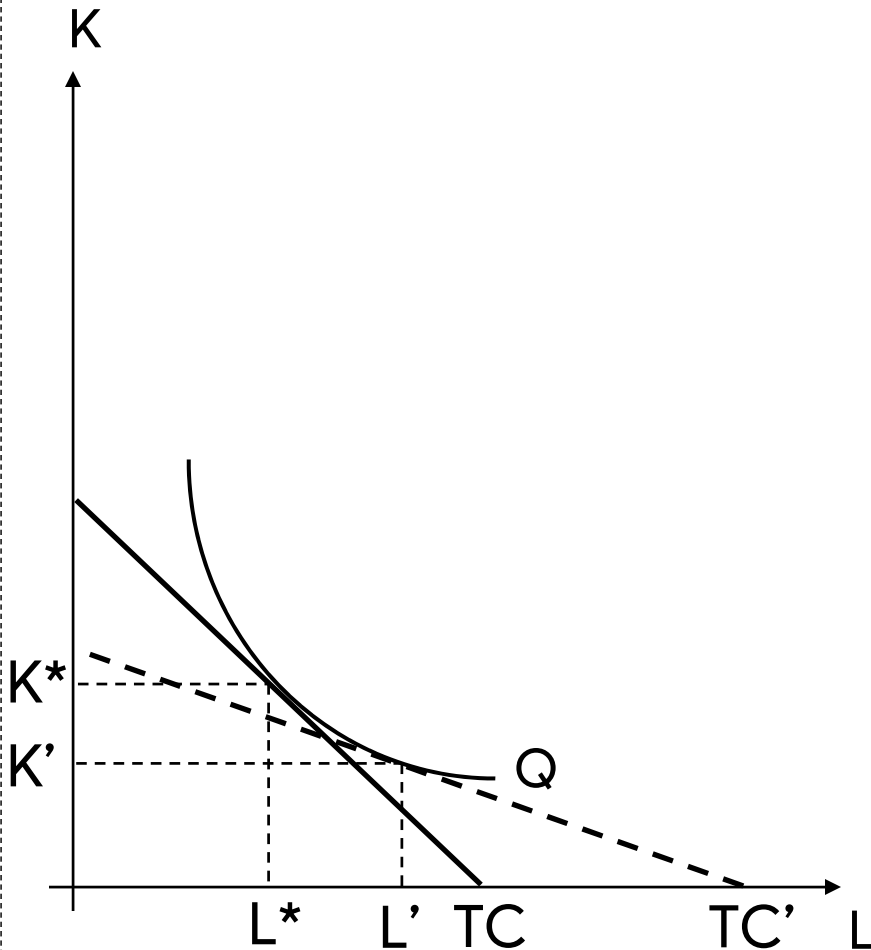
A producer has the least combination where an isoquant touches with an isocost.

$$MRTS_{LK} =$$

4. Cost Minimization

4.2 Changes of least cost combination

Cost minimization



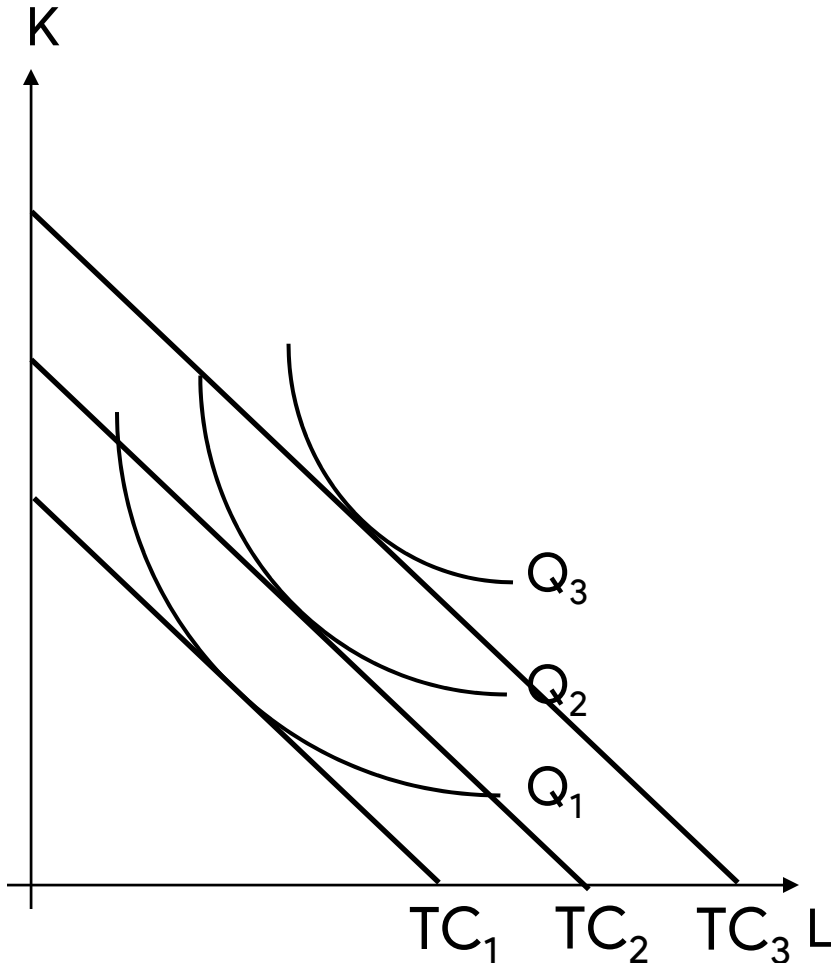
Note

Supposed that wage decreases while interest rate rises.

4. Cost Minimization

4.3 Expansion path

Cost minimization



Note

Expansion path is a curve connecting least cost combinations altogether.

4. Cost Minimization

4.4 Returns to scale

5.6

Definition – Law of Returns to Scale

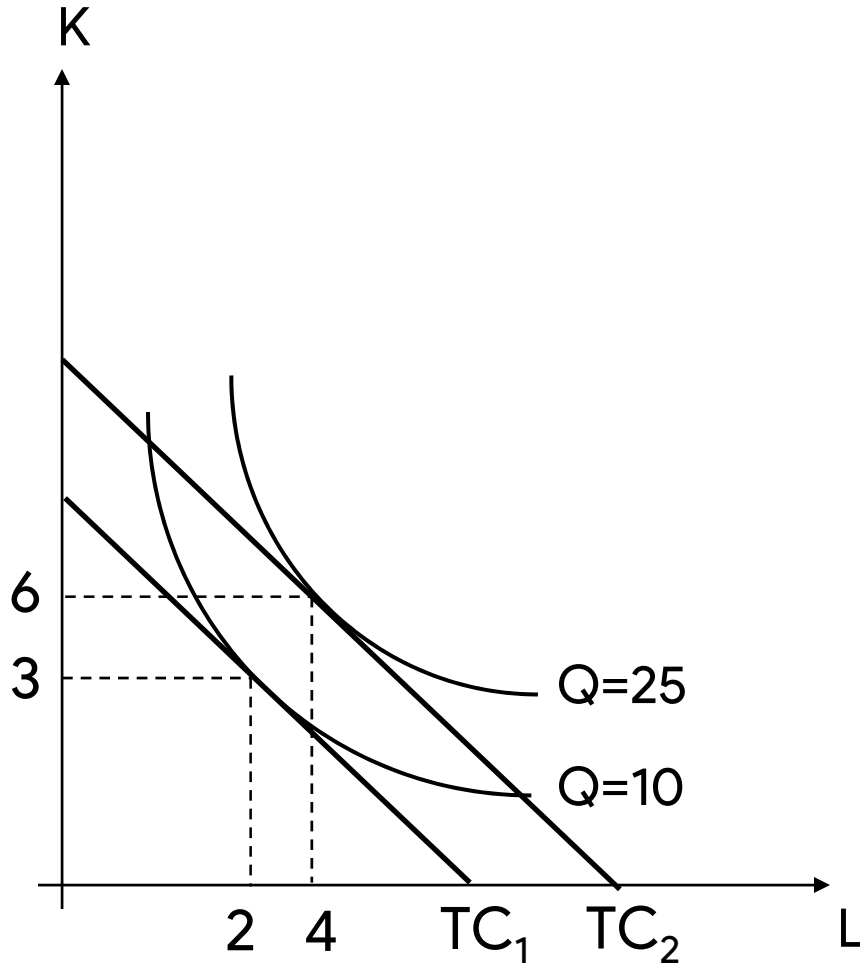
When a firm decides to increase all factors in the same ratio, the returns to scale of output can be one of these.

- Increasing returns to scale
- Constant returns to scale
- Decreasing returns to scale

4. Cost Minimization

4.4 Returns to scale

Increasing returns to scale



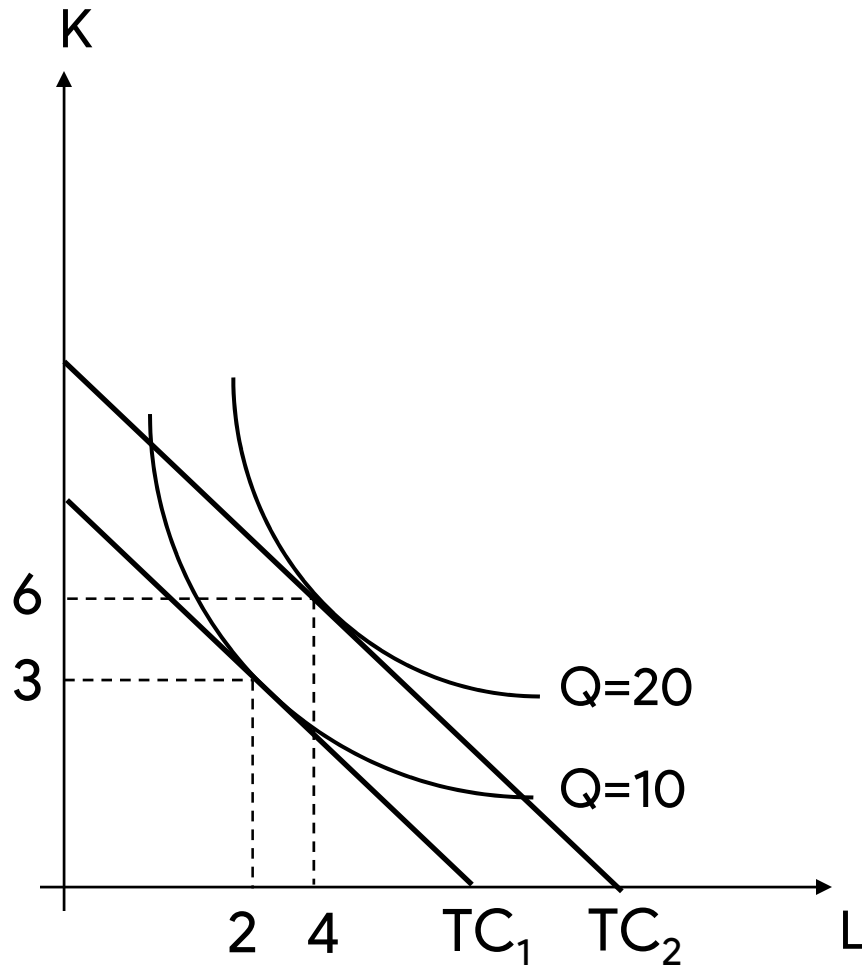
Note

4. Cost Minimization

4.4 Returns to scale

Constant returns to scale

Note

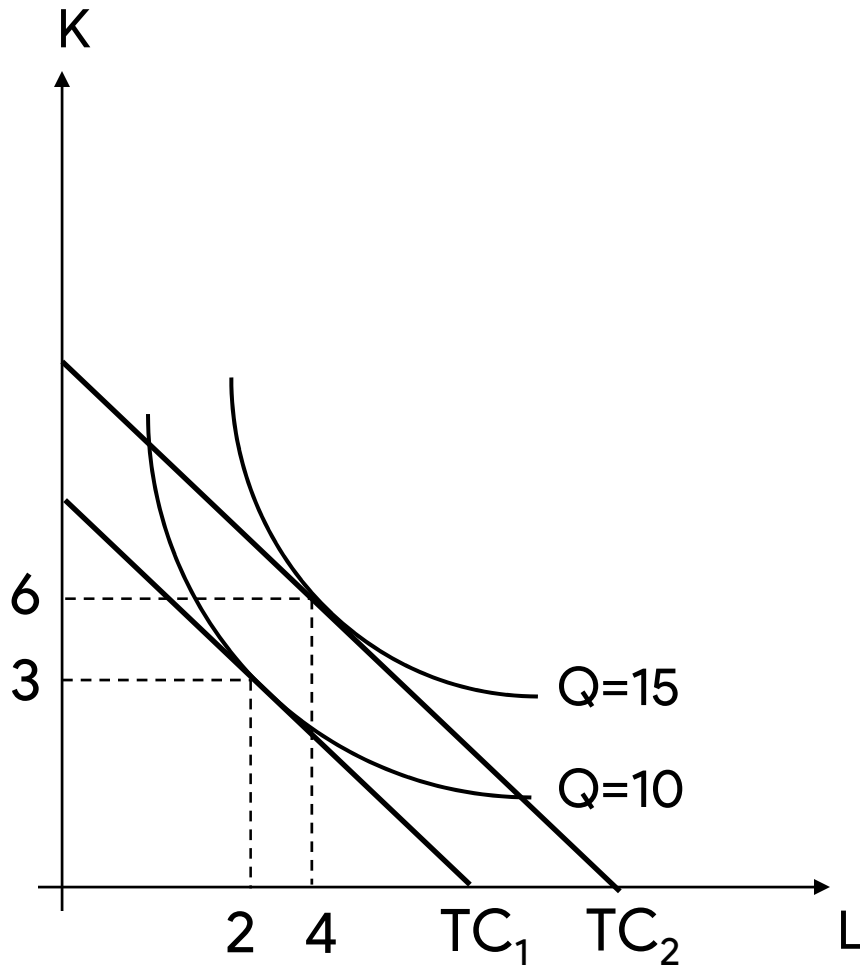


4. Cost Minimization

4.4 Returns to scale

Decreasing returns to scale

Note



4. Cost Minimization

4.4 Returns to scale

Mathematical expression

Comparing $\forall a > 0; F(aK, aL)$ and $aF(K, L)$

4. Cost Minimization

4.4 Returns to scale

Increasing returns to scale

is true when $\forall a > 0; F(aK, aL) > aF(K, L)$

Example: supposed that a firm has a Cobb-Douglas as follows

$$Q = AK^\alpha L^\beta = (2)K^1L^1$$

If this firm hires 10 labors and has 10 capital, later it doubles all the factors. What is the returns to scale?

- find $F(K, L)$:
- find $F(aK, aL)$:
- find $aF(K, L)$:

4. Cost Minimization

4.4 Returns to scale

Constant returns to scale

is true when $\forall a > 0; F(aK, aL) = aF(K, L)$

Example: supposed that a firm has a Cobb-Douglas as follows

$$Q = AK^{\alpha}L^{\beta} = (2)K^{0.5}L^{0.5}$$

If this firm hires 10 labors and has 10 capital, later it doubles all the factors. What is the returns to scale?

- find $F(K, L)$:
- find $F(aK, aL)$:
- find $aF(K, L)$:

4. Cost Minimization

4.4 Returns to scale

Decreasing returns to scale

is true when $\forall a > 0; F(aK, aL) = aF(K, L)$

Example: supposed that a firm has a Cobb-Douglas as follows

$$Q = AK^{\alpha}L^{\beta} = (2)K^{0.2}L^{0.2}$$

If this firm hires 10 labors and has 10 capital, later it doubles all the factors. What is the returns to scale?

- find $F(K, L)$:
- find $F(aK, aL)$:
- find $aF(K, L)$:

5. Long-Run Cost Curves

5.1 Types of long-run cost

5.7

Definition – Long-Run Total Cost (LTC)

Total amount of cost in the long-run.

5.8

Definition – Long-Run Average Cost (LAC)

Average amount of long-run cost per output.

$$LAC = \frac{LTC}{Q}$$

5.9

Definition – Long-Run Marginal Cost (LMC)

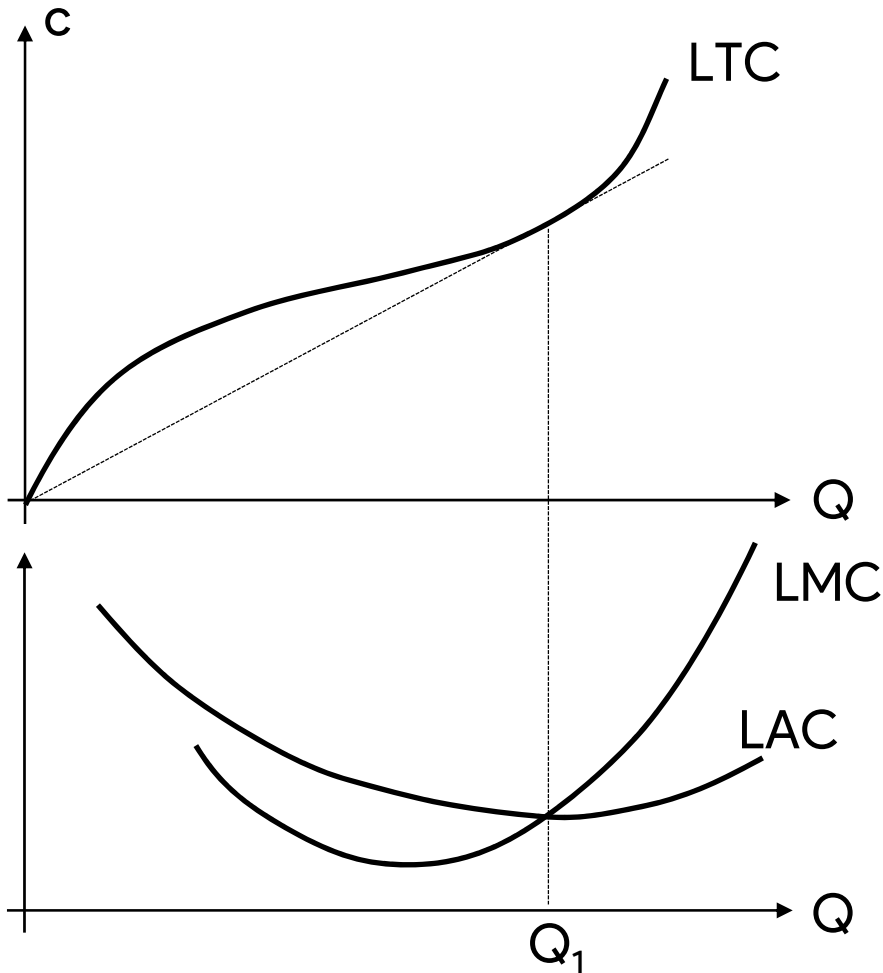
Additional cost producing one more unit of output in the long-run.

$$LMC = \frac{\Delta LTC}{\Delta Q}$$

5. Long-Run Cost Curves

5.2 Cost curves

Characteristics



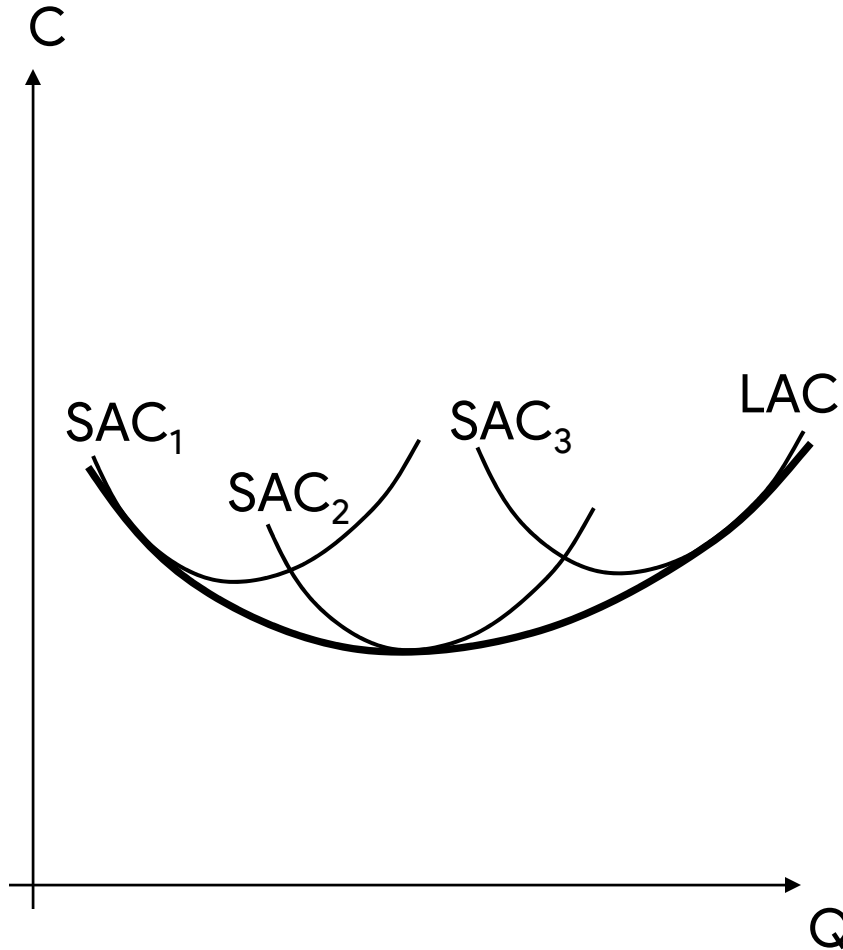
Note

- LTC starts from the origin
- LMC intersects with LAC at its lowest, also the same point when LTC touches a tangent line.

5. Long-Run Cost Curves

5.2 Cost curves

Factory sizes



Note

- Limited choice on factory size.
- Unlimited choice on factory size.

5. Long-Run Cost Curves

5.3 Economies of scale

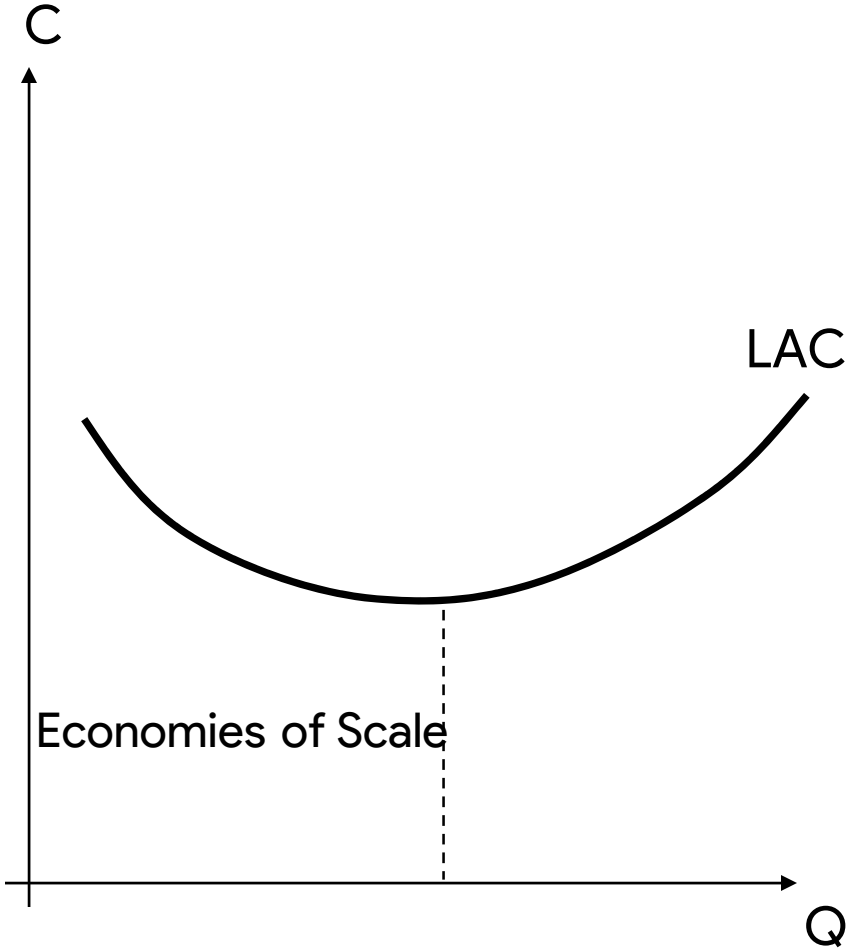
5.10

Definition – Economies of Scale

As a firm increases its output, this firm is considered having economies of scale if average cost is being lower, and vice versa if the average cost is being rising.

5. Long-Run Cost Curves

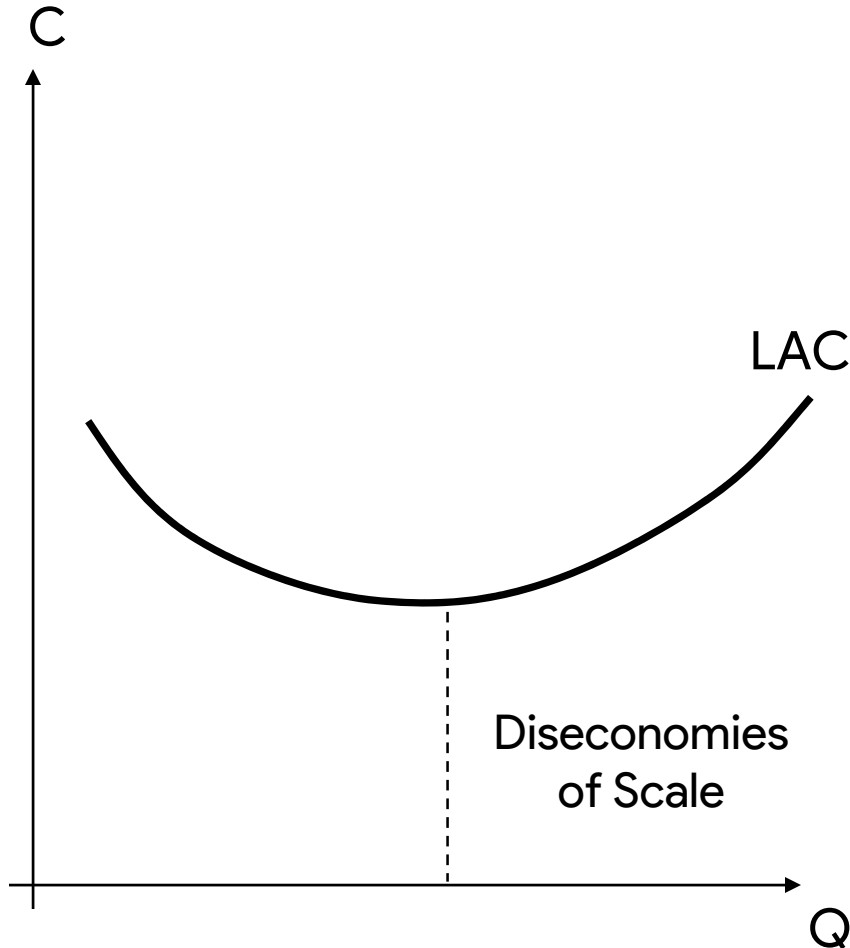
5.3 Economies of scale

Economies of scale	Note
 The graph illustrates the Long-Run Average Cost (LAC) curve. The vertical axis represents Cost (C) and the horizontal axis represents Quantity (Q). The LAC curve is U-shaped, indicating that costs initially decrease as quantity increases (economies of scale) and then eventually increase (diseconomies of scale). A vertical dashed line marks the minimum point of the curve, and the region to the left of this line is labeled 'Economies of Scale'.	Internal economies of scale - Division of labor - Bulk order External economies of scale - Research and development - Search cost for material and factors - Infrastructure setup

5. Long-Run Cost Curves

5.3 Economies of scale

Diseconomies of scale



Note

- Management complications
- Monitoring cost
- Rising search cost for material and labor
- Inventory management

Conclusion

5.1

Proposition – Long-Run Production

- When all factors can become variable, the production is considered to be long-run.
- Least cost combination is where the isoquant touches with isocost.
- Expansion path is a curve joining the least cost combinations altogether.
- Returns to scale compares between the increasing ratio of factors and output.
- There is no long-run fixed cost and curve. All logics in the short-run also apply.
- Economies of scale, which is different from returns to scale, occur when producing more leads to lower average cost.