

Take Home Final Exam
EE426
Semester 2/2020

There are five questions - answer all questions.
Each question is worth 20 points.

1. From the data set "Final_q1-1_no.dta":

Model for the degree of acceptance is stated as follows:

$$Prob(y_i=j|X) = f(x_1, x_2, x_3, x_4) \quad (1)$$

where: y_i is categorical data = 1 for disagree, = 2 for neutral, and = 3 for agree.
 x_k is independent variable k . $k = 1, 2, 3, 4$.

- (a) Estimate the model using multinomial logit model using $y_i=0$ is the base outcome. Make interpretation of your estimated results (sign & meaning (in term of rrr), overall test, goodness of fit, and individual test). Perform IIA test whether it is appropriated to apply multinomial logit in this case? Give explanation why IIA is important for multinomial logit. Determine whether multinomial logit is appropriated for this case? Why? (5 points).

```
. mlogit y x*, b(0)
Iteration 0:   log likelihood = -149.55771
. . .
Iteration 5:   log likelihood = -101.82217
```

Multinomial logistic regression	Number of obs	=	170
	LR chi2(8)	=	95.47
	Prob > chi2	=	0.0000
Log likelihood = -101.82217	Pseudo R2	=	0.3192

```
-----+-----
            y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
0            | (base outcome)
-----+-----
1            |
      x1 |  -1.132993   .6280843    -1.80   0.071   -2.364015    .09803
      x2 |  -.7710116   .7633309    -1.01   0.312   -2.267113    .7250896
      x3 |  -.6175655   .6301033    -0.98   0.327   -1.852545    .6174143
```



```

2          |
          x1 |   .3698816   .2578889   -1.43   0.154   .0943165   1.450566
          x2 |   .217033   .1729809   -1.92   0.055   .0455089   1.035036
          x3 |   1.857544   1.252089    0.92   0.358   .4956574   6.961404
          x4 |   6.591981   2.073033    6.00   0.000   3.559031   12.20956
        _cons |   1.42e-12   6.67e-12   -5.79   0.000   1.39e-16   1.45e-08
-----

```

```
. est store mlogit
```

```
. fitstat
```

Measures of Fit for mlogit of y

Log-Lik Intercept Only:	-149.558	Log-Lik Full Model:	-101.822
D(155):	203.644	LR(8):	95.471
		Prob > LR:	0.000
McFadden's R2:	0.319	McFadden's Adj R2:	0.219
ML (Cox-Snell) R2:	0.430	Cragg-Uhler(Nagelkerke) R2:	0.519
Count R2:	0.735	Adj Count R2:	0.262
AIC:	1.374	AIC*n:	233.644
BIC:	-592.404	BIC':	-54.385
BIC used by Stata:	255.002	AIC used by Stata:	223.644

According to the estimated results, comparing with the base outcome (y=0), x4 has a positive effect on the choice y=1 and y=2 (it seems like x4 has a larger (positive) effect on choice y = 2 since (RRR > 1) while x3 has the positive effect only on choice y=2. The other x (x1 x2) have negative effects on both choices since RRR < 1.

From the overall test, the null hypothesis is rejected since p-value = 0.000 < 0.05; thereby, all independent variables(x1 x2 x3 x4) altogether significantly explain the dependent variable (y).

In term of Goodness-of-fit, the pseudo R-squared is 0.319 (there are rooms for improvements) and counted R-squared = 0.735 meaning that this model can correctly predict the outcome 73.5%

The individual test suggests that only x4 is significant, and the rest are not.

IIA Test

```
. qui mlogit y x* if y!=1, b(0)
. est store mlogit1
. qui mlogit y x* if y!=2, b(0)
. est store mlogit2
. hausman mlogit1 mlogit,alleqs constant
```

---- Coefficients ----				
	(b)	(B)	(b-B)	sqrt(diag(V_b-V_B))
	mlogit1	mlogit	Difference	S.E.
x1	-1.510394	-.9945724	-.5158214	.5647965
x2	-2.032272	-1.527706	-.504566	.6287449
x3	.5052412	.6192554	-.1140142	.3330594
x4	2.119195	1.885854	.2333412	.3529509
_cons	-30.33564	-27.28322	-3.052417	5.065843

b = consistent under Ho and Ha; obtained from mlogit

B = inconsistent under Ha, efficient under Ho; obtained from mlogit

Test: Ho: difference in coefficients not systematic

$$\begin{aligned} \text{chi2}(5) &= (b-B)'[(V_b-V_B)^{-1}](b-B) \\ &= 12.45 \\ \text{Prob}>\text{chi2} &= 0.0291 \end{aligned}$$

```
. hausman mlogit mlogit2,alleqs constant
```

---- Coefficients ----				
	(b)	(B)	(b-B)	sqrt(diag(V_b-V_B))
	mlogit	mlogit2	Difference	S.E.
x1	-1.132993	-.9574483	-.1755444	.1691332
x2	-.7710116	-.5193718	-.2516398	.1573762

```

x3 |   -.6175655   -.4413165   -.1762489   .
x4 |    .6370348    .5667076    .0703272   .0475769
_cons |  -7.436673   -6.747037   -.6896369   .

```

b = consistent under Ho and Ha; obtained from mlogit

B = inconsistent under Ha, efficient under Ho; obtained from mlogit

Test: Ho: difference in coefficients not systematic

```

chi2(5) = (b-B)'[(V_b-V_B)^(-1)](b-B)
        =          1.20
Prob>chi2 =          0.9446
(V_b-V_B is not positive definite)

```

From the Hausman test, the IIA does not be held since y not equal to 1 case rejects the null hypothesis. Therefore, it could lead to a Red-Bus Blue-Bus Problem causing the estimated results to be biased.

IIA property is important since, as mentioned above, it could lead to a biasness in the estimated result. Moreover, Multinomial Logit, the result, compares with the base case. That is why IIA must be held.

Since IIA cannot be held, Multinomial Logit is not appropriated in this case.

- (b) Estimate the model using order probit model of y_i . Make interpretation of your estimated results (sign & meaning, overall test, goodness of fit, and individual test). Perform the test to determine whether order probit is appropriated in this case? Give explanation why? (5 points).

```

. oprobit y x*
Iteration 0:   log likelihood = -149.55771
. . .
Iteration 4:   log likelihood = -106.87931

```

Ordered probit regression	Number of obs	=	170
	LR chi2(4)	=	85.36
	Prob > chi2	=	0.0000
Log likelihood = -106.87931	Pseudo R2	=	0.2854

	y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	x1	-.2072486	.2327348	-0.89	0.373	-.6634004	.2489032
	x2	-.4271198	.239373	-1.78	0.074	-.8962822	.0420427
	x3	.5026001	.2192698	2.29	0.022	.0728392	.9323611
	x4	.7067322	.0937676	7.54	0.000	.522951	.8905133
	/cut1	9.831223	1.423997			7.04024	12.62221
	/cut2	11.08689	1.480594			8.184983	13.9888

```
. fitstat
```

Measures of Fit for oprobit of y

Log-Lik Intercept Only:	-149.558	Log-Lik Full Model:	-106.879
D(164):	213.759	LR(4):	85.357
		Prob > LR:	0.000
McFadden's R2:	0.285	McFadden's Adj R2:	0.245
ML (Cox-Snell) R2:	0.395	Cragg-Uhler(Nagelkerke) R2:	0.477
McKelvey & Zavoina's R2:	0.519		
Variance of y*:	2.080	Variance of error:	1.000
Count R2:	0.735	Adj Count R2:	0.262
AIC:	1.328	AIC*n:	225.759
BIC:	-628.512	BIC':	-64.814
BIC used by Stata:	244.573	AIC used by Stata:	225.759

From the estimated results, x1 and x2 have the negative effects on y, but x3 and x4 have the positive effects on the dependent variable.

From the overall test, it suggests that all independent variables can significantly explain the dependent variable since the null hypothesis is rejected p-value = 0.00 < 0.05

From the Pseudo R-squared = 0.285, it is quite low, therefore there exists rooms for improvements. The Counted R-squared = 0.735 meaning that this model can correctly predict the outcome 73.5%.

The individual test suggests that x3, x4, cut1, and cut2 are individually significant, but x1 and x2 are not.

probit12_y12							
x1		-.0859675	.2917692	-0.29	0.768	-.6578247	.4858897
x2		-.4665854	.3010255	-1.55	0.121	-1.056584	.1234137
x3		.6657033	.2613174	2.55	0.011	.1535307	1.177876
x4		.7898094	.1308796	6.03	0.000	.5332902	1.046329
_cons		-12.5982	2.071075	-6.08	0.000	-16.65743	-8.538965

```
. test [probit01_y01]x1 = [probit12_y12]x1

( 1)  [probit01_y01]x1 - [probit12_y12]x1 = 0

      chi2( 1) =      1.53
      Prob > chi2 =      0.2162

. test [probit01_y01]x2 = [probit12_y12]x2

( 1)  [probit01_y01]x2 - [probit12_y12]x2 = 0

      chi2( 1) =      0.05
      Prob > chi2 =      0.8162

. test [probit01_y01]x3 = [probit12_y12]x3

( 1)  [probit01_y01]x3 - [probit12_y12]x3 = 0

      chi2( 1) =      3.00
      Prob > chi2 =      0.0835

. test [probit01_y01]x4 = [probit12_y12]x4

( 1)  [probit01_y01]x4 - [probit12_y12]x4 = 0

      chi2( 1) =      0.72
      Prob > chi2 =      0.3954
```

From the test, all null hypotheses are failed to reject. So, Ordered Probit is appropriated.

From the data set "Final_q1-2_no.dta":

In the study of decision to subscribe mobile phone services, three choices have been studied including AIS, DTAC, and TRUE. The probit model can be stated as:

$$I_{ji} = \beta_{j0} + \beta_{j1}x_{1i} + \beta_{j2}x_{2i} + \beta_{j3}x_{3i} + \beta_{j4}x_{4i} + u_{ji} \quad (2)$$

and

$$\Pr(Y_{ji} = 1) = \Phi(I_{ji})$$

where: I_{ji} is index variables.

Y_{ji} is decision to choose financial choice J , value equals to 1 if choosing choice J or 0 if not. $J = 1$ for AIS, 2 for DTAC, 3 for TRUE.

x_{ki} is independent variable k .

$\Phi(\cdot)$ is multivariate normal probability distribution function.

u_{ji} is disturbance term.

(c) Estimate models for Y_{1i} , Y_{2i} , and Y_{3i} assuming that the probability functions follow separate normal distribution function. Should we use these three separate probit models? Why? (4 points)

```
. probit y1 x*, nolog
```

Probit regression	Number of obs	=	550
	LR chi2(4)	=	33.18
	Prob > chi2	=	0.0000
Log likelihood = -362.36305	Pseudo R2	=	0.0438

```
-----+-----
```

y1	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	1.394585	.4001617	3.49	0.000	.6102824 2.178887
x2	-.8054803	.3810398	-2.11	0.035	-1.552305 -.058656
x3	.6878752	.1775705	3.87	0.000	.3398435 1.035907
x4	.2237994	.1847999	1.21	0.226	-.1384017 .5860006
_cons	-.8169487	.3025723	-2.70	0.007	-1.40998 -.2239178

```
-----+-----
```

```
. probit y2 x*, nolog
```

Probit regression	Number of obs	=	550
	LR chi2(4)	=	14.83
	Prob > chi2	=	0.0051
Log likelihood = -360.17388	Pseudo R2	=	0.0202

y2	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
x1	-.739561	.3784533	-1.95	0.051	-1.481316	.0021938
x2	-.5164994	.3686219	-1.40	0.161	-1.238985	.2059862
x3	-.2704721	.1752924	-1.54	0.123	-.6140389	.0730947
x4	.6061985	.1902349	3.19	0.001	.233345	.979052
_cons	.3278563	.3006511	1.09	0.275	-.2614089	.9171216

. probit y3 x*, nolog

Probit regression	Number of obs	=	550
	LR chi2(4)	=	11.92
	Prob > chi2	=	0.0180
Log likelihood = -327.43565	Pseudo R2	=	0.0179

y3	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
x1	-.5188997	.3896565	-1.33	0.183	-1.282612	.244813
x2	.7601524	.383251	1.98	0.047	.0089943	1.511311
x3	-.3898287	.1970154	-1.98	0.048	-.7759717	-.0036856
x4	-.3178066	.1831805	-1.73	0.083	-.6768338	.0412206
_cons	-.471871	.3152587	-1.50	0.134	-1.089767	.1460246

To determine whether these separate Probit models are appropriated, we should be able to know that there is no any correlation among the error terms, i.e. Tetrachoric Correlation between the error term ε_i and ε_j are equal to zero. If not, then we should not apply the separate models. To clarify, the test is required.

(d) Estimate models for Y_{1i} , Y_{2i} , and Y_{3i} assuming that the probability functions follow multivariate normal probability distribution function (MV Probit models). Determine whether MVProbit is appropriated. Why? How does the MV Probit models differ from three separate probit models? (6 points)

```
. mvprobit (y1 x*) (y2 x*) (y3 x*)
```

```
Iteration 0:   log likelihood = -1049.9726
```

```
. . .
```

```
Iteration 5:   log likelihood =   -929.435
```

```
Multivariate probit (MSL, # draws = 5)           Number of obs   =           550
                                                    Wald chi2(12)    =           44.62
Log likelihood =   -929.435                      Prob > chi2      =           0.0000
```

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----							
y1							
	x1	1.242071	.3903115	3.18	0.001	.477074	2.007067
	x2	-.6719538	.3750584	-1.79	0.073	-1.407055	.0631472
	x3	.6696143	.1786955	3.75	0.000	.3193775	1.019851
	x4	.204277	.1839031	1.11	0.267	-.1561664	.5647204
	_cons	-.7530149	.3046773	-2.47	0.013	-1.350172	-.1558583
-----+-----							
y2							
	x1	-.8113389	.3748761	-2.16	0.030	-1.546083	-.0765952
	x2	-.4461635	.3665143	-1.22	0.223	-1.164518	.2721913
	x3	-.2892671	.1757756	-1.65	0.100	-.633781	.0552469
	x4	.5873489	.1894209	3.10	0.002	.2160909	.958607
	_cons	.3688367	.2992854	1.23	0.218	-.2177519	.9554253
-----+-----							
y3							
	x1	-.4616851	.3829211	-1.21	0.228	-1.212197	.2888264
	x2	.5866839	.3783605	1.55	0.121	-.1548892	1.328257

x3		-.2848337	.1854174	-1.54	0.124	-.6482451	.0785778
x4		-.275354	.1869964	-1.47	0.141	-.6418603	.0911522
_cons		-.418268	.317282	-1.32	0.187	-1.040129	.2035933
-----+-----							
/atrho21		-.5774284	.0738682	-7.82	0.000	-.7222074	-.4326494
-----+-----							
/atrho31		-.4353952	.0750685	-5.80	0.000	-.5825268	-.2882636
-----+-----							
/atrho32		-.3384142	.071239	-4.75	0.000	-.47804	-.1987883
-----+-----							
rho21		-.5207938	.0538332	-9.67	0.000	-.6182747	-.407533
-----+-----							
rho31		-.4098203	.0624606	-6.56	0.000	-.5244996	-.2805358
-----+-----							
rho32		-.3260609	.0636652	-5.12	0.000	-.4446726	-.1962105

Likelihood ratio test of rho21 = rho31 = rho32 = 0:

chi2(3) = 241.075 Prob > chi2 = 0.0000

From the LR-test, the null hypothesis that is all $\rho(\rho)$ are equal to zero, i.e. there is no correlations among the error terms, is rejected. Therefore, MV Probit is more appropriated than the separate ones. The main difference between the separate Probit and MV Probit is that MV Probit is like the model is run as a system (run together) because there exists Tetrachoric Correlation between the error term ε_i and ε_j

2. From the data set "Final_q2_no.dta":

According to the following model,

$$y_{ki} = \beta_{k0} + \beta_{k1}x_i + u_{ki} \quad (3)$$

where: y_{ki} is Dependent variable, $k=1, 2, 3$.

x_i is Independent variable

u_i is Stochastic disturbance term

(a) Plot histogram of y_{1i}, y_{2i}, y_{3i} , compute descriptive statistics of these three variables. Determine limitations of these three dependent variables. (5 points)

```
. sum y*
```

Variable	Obs	Mean	Std. Dev.	Min	Max
-----+-----					
y1	368	525.8013	301.8327	50.21759	1578.51
y2	450	430.993	339.0019	0	1578.51
y3	450	3343.313	14544.82	-466.0042	98951.63

```
. histogram y1
```

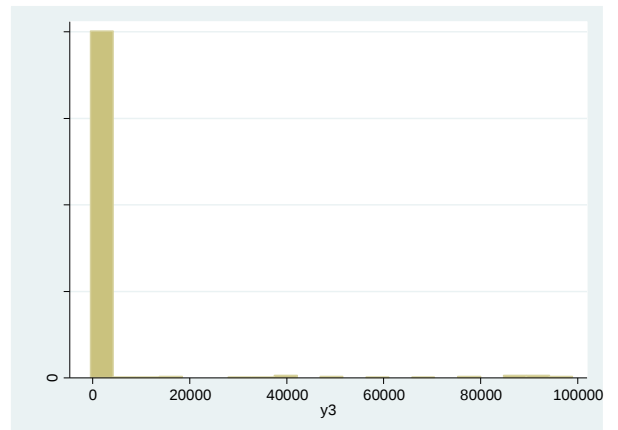
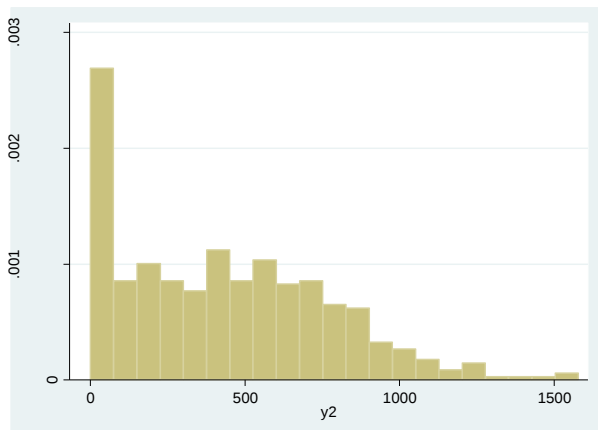
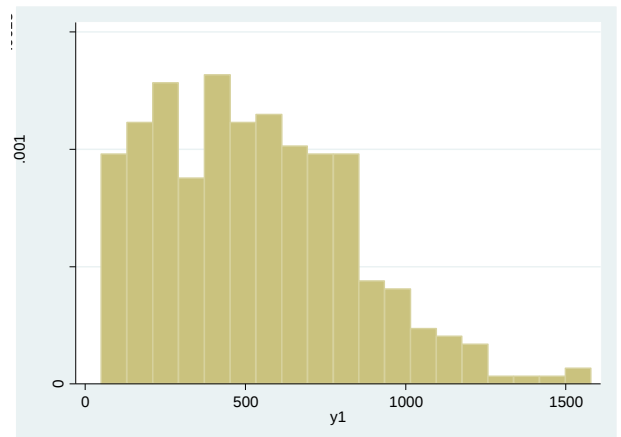
```
(bin=19, start=50.217594, width=80.436449)
```

```
. histogram y2
```

```
(bin=21, start=0, width=75.167149)
```

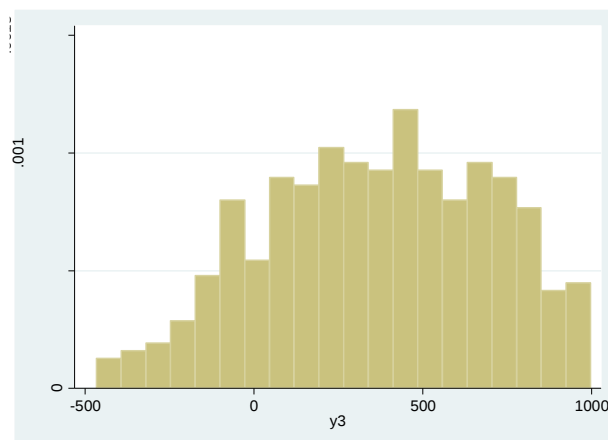
```
. histogram y3
```

```
(bin=21, start=-466.00415, width=4734.1732)
```



```
. histogram y3 if y3 < 4000
(bin=20, start=-466.00415, width=73.209753)
```

According to the summarized statistics table and histograms, it might be the case that there exists the truncated problem for y_1 (the value of y below 50 cannot be observed). For y_2 , we can clearly observe that the dependent variable can be fully observed but the value equal to zero has significantly high density. Therefore, this can be the case that y_2 has a censored data problem (censored at 0). For y_3 , it is highly likely to be the case of outlier problem.



- (b) Estimate the model (3) for y_{li} using OLS and truncated regression model. Determine the most appropriated model in this case. Why? What is the major problem in this case? What will happen if we ignore the problem? (5 points)

Run y_1 with OLS

```
. reg y1 x
```

Source	SS	df	MS	Number of obs	=	368
Model	9137792.8	1	9137792.8	F(1, 366)	=	137.65
Residual	24297002.7	366	66385.2534	Prob > F	=	0.0000
				R-squared	=	0.2733
				Adj R-squared	=	0.2713
Total	33434795.5	367	91102.9851	Root MSE	=	257.65

y_1	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x	165.724	14.1254	11.73	0.000	137.9469	193.5011
_cons	-9.272247	47.5433	-0.20	0.845	-102.7646	84.22007

```
. est store y1_ols
```

Run y1 with Truncated Regression

```
. truncreg y1 x, ll(50)
(note: 0 obs. truncated)

Fitting full model:

. . .

Iteration 4:   log likelihood = -2529.8335

Truncated regression
Limit:   lower =          50           Number of obs   =          368
         upper =         +inf           Wald chi2(1)    =          104.94
Log likelihood = -2529.8335           Prob > chi2      =          0.0000

-----+-----
          y1 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
          x |   230.7558   22.52558    10.24   0.000    186.6065    274.9052
        _cons | -297.5609   85.92795    -3.46   0.001   -465.9766   -129.1453
-----+-----
        /sigma |   305.8521   16.81074    18.19   0.000    272.9036    338.8005
-----+-----

. est store y1_trunc
```

Test whether Truncated Regression is appropriated

```
. lrtest y1_ols y1_trunc, force

Likelihood-ratio test                                LR chi2(1)   =          68.66
(Assumption: y1_ols nested in y1_trunc)              Prob > chi2 =    0.0000
```

Since the LR-test rejects the null hypothesis, the Truncated Regression is appropriated for y1. The major problem is that some observations are not available to observe; however, those observation exists. Therefore, the sample is selected based on the value of the dependent variable (y), i.e. Sample Selection Bias. If the problem is ignored and continue to use the traditional OLS, the estimated results is biased.

(c) Estimate the model (3) for y_{2i} using OLS and Tobit model. Determine the most appropriated model in this case. Why? What is the major problem in this case? What will happen if we ignore the problem? (5 points)

Run y2 with OLS

. reg y2 x

Source		SS		df		MS		Number of obs	=	450
-----+-----										
								F(1, 448)	=	264.88
Model		19172494.6		1		19172494.6		Prob > F	=	0.0000
Residual		32427611.1		448		72383.0604		R-squared	=	0.3716
-----+-----										
								Adj R-squared	=	0.3702
Total		51600105.7		449		114922.284		Root MSE	=	269.04

y2		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----+-----							
x		199.1064	12.23388	16.27	0.000	175.0635	223.1493
_cons		-169.2913	39.00344	-4.34	0.000	-245.9437	-92.63885

. est store y2_ols

Run y2 with Tobit Regression

. tobit y2 x, ll(0)

Tobit regression		Number of obs	=	450
		LR chi2(1)	=	220.85
		Prob > chi2	=	0.0000
Log likelihood = -2793.8497		Pseudo R2	=	0.0380

y2		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----+-----							
x		236.3091	14.58958	16.20	0.000	207.6367	264.9814
_cons		-311.5844	47.33943	-6.58	0.000	-404.6188	-218.5501
-----+-----							
/sigma		300.5597	11.10914			278.7274	322.3921

```
-----
        66  left-censored observations at y2 <= 0
        384      uncensored observations
           0 right-censored observations
. est store y2_tobit
```

Test whether Tobit Regression is appropriated

```
. lrtest y2_ols y2_tobit, force
```

```
Likelihood-ratio test                                LR chi2(1)  =    722.72
(Assumption: y2_ols nested in y2_tobit)              Prob > chi2 =    0.0000
```

Since the LR-test rejects the null hypothesis, the Tobit Regression is more appropriated for y2. In this case, all observations can be observed. However, the major problem is that some of the dependent variable (y) are censored at 0. That is the real values might be lower than 0, but according to what is observable, they are all 0. If the problem is ignored and OLS is applied, the estimated result is biased because of the censored data.

- (d) Estimate the model (3) for y_{3i} using OLS and Tobit model. Determine the most appropriated model in this case. Why? What is the major problem in this case? What will happen if we ignore the problem? (5 points)

Run y3 with OLS

```
. reg y3 x
```

Source	SS	df	MS	Number of obs	=	450
-----+-----				F(1, 448)	=	36.36
Model	7.1301e+09	1	7.1301e+09	Prob > F	=	0.0000
Residual	8.7857e+10	448	196108662	R-squared	=	0.0751
-----+-----				Adj R-squared	=	0.0730
Total	9.4987e+10	449	211551878	Root MSE	=	14004

```
-----
        y3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
```

x	3839.672	636.7872	6.03	0.000	2588.211	5091.133
_cons	-8232.881	2030.172	-4.06	0.000	-12222.72	-4243.038

. est store y3_ols

Run y3 with Truncated Regression

. tobit y3 x, ul(4000)

Tobit regression	Number of obs	=	450
	LR chi2(1)	=	123.88
	Prob > chi2	=	0.0000
Log likelihood = -3495.9514	Pseudo R2	=	0.0174

y3	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
-----+-----					
x	429.1709	36.0279	11.91	0.000	358.3667 499.9752
_cons	-733.6497	114.6576	-6.40	0.000	-958.9819 -508.3176
-----+-----					
/sigma	788.7746	27.59472			734.5438 843.0055

0 left-censored observations

427 uncensored observations

23 right-censored observations at y3 >= 4000

. est store y3_tobit

Test whether Tobit Regression is appropriated

. lrtest y3_ols y3_tobit, force

Likelihood-ratio test	LR chi2(1)	=	2875.52
(Assumption: y3_ols nested in y3_tobit)	Prob > chi2	=	0.0000

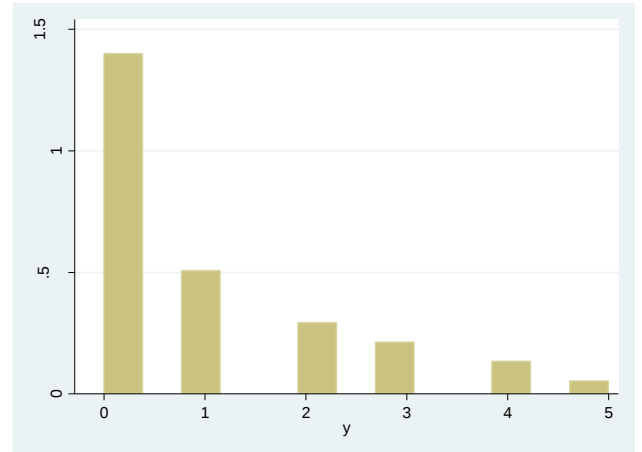
Since the LR-test rejects the null hypothesis, the Tobit Regression is more appropriated for y3. In this case all observations can be fully observed, but there exists the outlier problem on the dependent variable (y). Of course, if OLS is used, the estimated result is biased because of those extreme values. So, they should be removed by setting up the upper limit such that the outliers are eliminated. This is likely to the censored data problem in which the traditional OLS leads to a biasness. Therefore, Tobit should be applied instead.

Source		SS		df		MS		Number of obs	=	195		
-----+							F(4, 190)				=	5.61
Model		36.4072626		4		9.10181564		Prob > F	=	0.0003		
Residual		308.464532		190		1.62349754		R-squared	=	0.1056		
-----+							Adj R-squared				=	0.0867
Total		344.871795		194		1.77768966		Root MSE	=	1.2742		

y		Coef.		Std. Err.		t		P> t		[95% Conf. Interval]		
-----+												
x1		.0847709		.0439189		1.93		0.055		-.0018604 .1714022		
x2		.1428018		.046958		3.04		0.003		.0501758 .2354279		
x4		-.0365707		.0475964		-0.77		0.443		-.130456 .0573146		
x3		.1901753		.0668986		2.84		0.005		.0582159 .3221347		
_cons		.9012497		.1080894		8.34		0.000		.6880403 1.114459		

```
. histogram y
(bin=13, start=0, width=.38461538)
```

There exists a limitation. It is obvious that the dependent variable is not normally distributed. It looks like Poisson distribution instead. Moreover, it seems to have the excess zero problem.



- (b) Estimate models for Y_i assuming that the probability functions follow Poisson probability distribution. Perform GOF test and determine whether Poisson is appropriated in this case. Interpret the estimated result (sign and meaning (in term of incidence-rate ratios), overall test, individual test, pseudo R^2 , marginal effects). (5 points)

Poisson Regression

```
. poisson y x*
Iteration 0:   log likelihood = -270.78875
Iteration 1:   log likelihood = -270.78874
```

Poisson regression	Number of obs	=	195
	LR chi2(4)	=	37.99
	Prob > chi2	=	0.0000
Log likelihood = -270.78874	Pseudo R2	=	0.0655

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----						
x1	.0876226	.0344264	2.55	0.011	.0201482	.155097
x2	.1470317	.036948	3.98	0.000	.0746151	.2194484
x4	-.0378213	.0376442	-1.00	0.315	-.1116026	.03596
x3	.2078186	.0552623	3.76	0.000	.0995065	.3161308
cons	-.2000234	.0955572	-2.09	0.036	-.3873121	-.0127347

variable	dy/dx	Std. Err.	z	P> z	[	95% C.I.	]	X
x1	.0770753	.02985	2.58	0.010	.018573	.135577	-.269304	
x2	.1293333	.03141	4.12	0.000	.067781	.190886	.865611	
x4	-.0332687	.03303	-1.01	0.314	-.098006	.031469	-.778541	
x3	.1828032	.04685	3.90	0.000	.090985	.274622	-.295223	

```
. estat gof
```

```
Deviance goodness-of-fit = 313.6265
```

```
Prob > chi2(190) = 0.0000
```

```
Pearson goodness-of-fit = 332.5726
```

```
Prob > chi2(190) = 0.0000
```

From the estimated result, it suggests that each independent variables has a positive effect on the dependent variable (Marginal effect > 0 and IRR > 1) except x4 that negatively affects the dependent variable (Marginal effect < 0 and IRR < 1).

The overall test shows that all independent variables can jointly explain the dependent variable.

The pseudo R-squared = 0.0655 is quite low. Therefore, it can be improved.

The individual test reveals that each of the independent variables is significant except x4.

However, from the goodness of fit test, the null hypothesis is rejected (P-value < 0.05). It means that **Poisson Regression should not be applied.**

- (c) Estimate models for Y_i assuming that the probability functions follow Negative Binomial probability distribution. Determine whether Negative Binomial regression model is appropriated in this case. Interpret your estimated result (sign and meaning (in term of incidence-rate ratios), overall test, individual test, pseudo R^2 , marginal

Negative Binomial Probability Distribution

```
. nbreg y x*, nolog
```

```
Negative binomial regression
```

```
Number of obs = 195
```

```
LR chi2(4) = 20.53
```

```
Dispersion = mean
```

```
Prob > chi2 = 0.0004
```

```
Log likelihood = -256.37137
```

```
Pseudo R2 = 0.0385
```

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		
-----+-----							
x1	.1157622	.0526251	2.20	0.028	.0126188	.2189055	
x2	.1653749	.0528556	3.13	0.002	.0617799	.2689699	
x4	-.0395441	.050271	-0.79	0.432	-.1380735	.0589852	
x3	.1980496	.0707578	2.80	0.005	.0593669	.3367323	

_cons		-.2205923	.1249006	-1.77	0.077	-.4653931	.0242084
-----+-----							
/lnalpha		-.1768216	.2946078			-.7542424	.4005991
-----+-----							
alpha		.8379292	.2468605			.4703668	1.492719

Likelihood-ratio test of alpha=0: chibar2(01) = 28.83 Prob>=chibar2 = 0.000

. nbreg y x*, ir nolog

Negative binomial regression	Number of obs	=	195
	LR chi2(4)	=	20.53
Dispersion = mean	Prob > chi2	=	0.0004
Log likelihood = -256.37137	Pseudo R2	=	0.0385

y	IRR	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+						
x1	1.122729	.0590837	2.20	0.028	1.012699	1.244714
x2	1.179835	.0623609	3.13	0.002	1.063728	1.308616
x4	.9612275	.0483219	-0.79	0.432	.8710347	1.06076
x3	1.219023	.0862553	2.80	0.005	1.061164	1.400364
_cons	.8020436	.1001758	-1.77	0.077	.6278882	1.024504
-----+						
/lnalpha	-.1768216	.2946078			-.7542424	.4005991
-----+						
alpha	.8379292	.2468605			.4703668	1.492719

Likelihood-ratio test of alpha=0: chibar2(01) = 28.83 Prob>=chibar2 = 0.000

. mfx

Marginal effects after nbreg

y = Predicted number of events (predict)
= .87258142

variable	dy/dx	Std. Err.	z	P> z	[	95% C.I.	]	X
x1	.1010119	.04575	2.21	0.027	.011349	.190675	-.269304	
x2	.144303	.04587	3.15	0.002	.054393	.234213	.865611	
x4	-.0345055	.04386	-0.79	0.431	-.120469	.051458	-.778541	
x3	.1728144	.06142	2.81	0.005	.05243	.293199	-.295223	

The overdispersion test rejects the null hypothesis that is $\alpha = 0$ meaning that the Negative Binomial Model is more appropriated compared with the Poisson Model.

From the estimated result, it suggests that each independent variables has a positive effect on the dependent variable (Marginal effect > 0 and IRR > 1) except x4 that negatively affects the dependent variable (Marginal effect < 0 and IRR < 1).

The overall test shows that all independent variables can jointly explain the dependent variable since p-value = 0.0004 < 0.05.

The pseudo R-squared = 0.0385 is quite low. Therefore, it can be improved.

The individual test reveals that each of the independent variables is significant except x4.

- (d) Estimate models for Y_i assuming that the model is Zero Inflated Poisson (x_{1i} , x_{2i} , and x_{3i} are independent variables in Poisson model and x_{4i} is independent variable in Inflated (Logit) model). Interpret your estimated result. Determine which model (Linear regression model, Poisson, Negative Binomial, or ZIP) is the most appropriated model in this case? Why? (provide the tests) (7 points)

Zero Inflated Poisson

```
. zip y x1 x2 x3, inflate(x4) vuong nolog
```

Zero-inflated Poisson regression	Number of obs	=	195
	Nonzero obs	=	90
	Zero obs	=	105

Inflation model = logit

Log likelihood = -255.2978

LR chi2(3)	=	15.67
Prob > chi2	=	0.0013

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
y					

x1		.1029852	.0431638	2.39	0.017	.0183857	.1875848
x2		.1310613	.0411813	3.18	0.001	.0503474	.2117753
x3		.1464086	.0561832	2.61	0.009	.0362916	.2565256
_cons		.289415	.1204087	2.40	0.016	.0534183	.5254117
-----+-----							
inflate							
x4		.041464	.1015959	0.41	0.683	-.1576603	.2405883
_cons		-.5217272	.2645918	-1.97	0.049	-1.040318	-.0031368

Vuong test of zip vs. standard Poisson:	z =	2.46	Pr>z = 0.0070
---	-----	------	---------------

. mfx

Marginal effects after zip

y = Predicted number of events (predict)
= .88504026

variable		dy/dx	Std. Err.	z	P> z	[95% C.I.]	X
-----+-----							
x1		.0911461	.03794	2.40	0.016	.016781 .165511	-.269304
x2		.1159945	.03564	3.25	0.001	.046136 .185853	.865611
x3		.1295775	.04864	2.66	0.008	.034248 .224907	-.295223
x4		-.0133921	.03302	-0.41	0.685	-.078107 .051323	-.778541

The Vuong test rejects the null hypothesis ($p\text{-value} = 0.007 < 0.05$) meaning that the Zero-Inflated Poisson Model is more appropriated compared with the Poisson Model.

From the estimated result, it suggests that each independent variables has a positive effect on the dependent variable (Marginal effect > 0) except x4 that negatively affects the dependent variable (Marginal effect < 0).

The overall test shows that all independent variables can jointly explain the dependent variable.

The individual test reveals that each of the independent variables is significant except x4.

To determine which model is the most appropriated, Firstly, it cannot be the Poisson Regression since the Goodness of Fit Test (GOF Test) rejects the null hypothesis (implying that the Poisson is inappropriate).

Next, the Negative Binomial Model seems to be good because the Overdispersion also confirms that the Poisson Model is inappropriate; however, there exists an insignificant variable in this model.

Lastly, look at the Zero-Inflated Poisson Model, the Vuong test shows that this model is appropriated; however, x_4 is still insignificant as same as in the Negative Binomial case.

In conclusion, the Zero-Inflated Poisson Model is the most appropriated model because of the reason provided above; plus, from the histogram at the beginning, it illustrates that the dependent variable follows zero inflated distribution. Therefore, Zero Inflated Poisson Regression model should be applied in this case.

4. From the data set "Final_q4_no.dta":

- (a) Test whether the series y_t and x_t are stationary series and determine their order of integration. Give explanation of the process of the test. Why is testing equation important? Why is stationary property important to time series analysis? (5 points)

Test whether y and x are stationary by Augmented Dickey-Fuller Test

. dfuller y, trend lag(1) regress

Augmented Dickey-Fuller test for unit root Number of obs = 498

----- Interpolated Dickey-Fuller -----				
	Test	1% Critical	5% Critical	10% Critical
	Statistic	Value	Value	Value

Z(t)	-15.058	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = 0.0000

D.y		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
-----+-----						
y						
L1.		-.9760353	.0648188	-15.06	0.000	-1.10339 -.8486808
LD.		-.0594166	.0449897	-1.32	0.187	-.1478114 .0289782
_trend		.0016656	.0021092	0.79	0.430	-.0024786 .0058098
_cons		.8305426	.6112434	1.36	0.175	-.3704148 2.0315

. dfuller x, trend lag(1) regress

Augmented Dickey-Fuller test for unit root Number of obs = 498

----- Interpolated Dickey-Fuller -----				
	Test	1% Critical	5% Critical	10% Critical
	Statistic	Value	Value	Value

Z(t)	-15.212	-3.980	-3.420	-3.130

MacKinnon approximate p-value for $Z(t) = 0.0000$

D.x		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
	x					
	L1.	-.9874688	.0649129	-15.21	0.000	-1.115008 - .8599295
	LD.	-.0501877	.0449965	-1.12	0.265	-.1385958 .0382203
	_trend	.0026561	.0030122	0.88	0.378	-.0032622 .0085743
	_cons	.4140897	.8693345	0.48	0.634	-1.293959 2.122139

According to Augmented Dickey-Fuller Test, it suggests that both y and x are stationary since the null hypothesis that is $\beta = 1$ is rejected ($p\text{-value} = 0.00 < 0.05$).

Therefore, y and x are stationary at level, i.e. y and x are integrated series of order 0.

The process of the test is that, from,

$$\Delta y_t = \alpha_0 + \alpha_1 t + \gamma y_{t-1} + \sum \beta_i \Delta y_{t-1} + \varepsilon_t \text{ where } \gamma = \beta - 1$$

We test whether $H_0: \gamma=0$ ($\beta=1$), if it is rejected, then there is no Unit Root i.e. Stationary.

However, if the null hypothesis is failed to reject, the significance of testing equation comes to play a role, the null hypothesis cannot be rejected might be because we have an excess variable in the model. So, we have to look at other coefficients (α_1 and α_0 which are trend and constant respectively), if it is insignificant, drop it and test again.

Moreover, Stationary is very important for time series because if it is Nonstationary, it will lead to a Spurious Problem (False Relationship)

- (b) Estimate Autoregressive Integrated Moving Average (ARIMA(p,d,q)) model for y_t – determine the most appropriated order for p , d , and q using SBIC given the maximum lag equals 4. Make dynamic forecast for period time = 501 to 505. (5 points)

According to (a), parameter "d" = 0 since they are integrated order 0.

```
. qui arima y, arima(1,0,1)
. est store arima101
. qui arima y, arima(1,0,2)
. est store arima102
. qui arima y, arima(1,0,3)
. est store arima103
```

```
. qui arima y, arima(1,0,4)
. est store arima104
. qui arima y, arima(2,0,1)
. est store arima201
. qui arima y, arima(2,0,2)
. est store arima202
. qui arima y, arima(2,0,3)
. est store arima203
. qui arima y, arima(2,0,4)
. est store arima204
. qui arima y, arima(3,0,1)
. est store arima301
. qui arima y, arima(3,0,2)
. est store arima302
. qui arima y, arima(3,0,3)
. est store arima303
. qui arima y, arima(3,0,4)
. est store arima304
. qui arima y, arima(4,0,1)
. est store arima401
. qui arima y, arima(4,0,2)
. est store arima402
. qui arima y, arima(4,0,3)
. est store arima403
. qui arima y, arima(4,0,4)
. est store arima404
. est table arima10*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)
```

Variable	arima101	arima102	arima103	arima104
-----+-----				
y				
_cons	1.2822737***	1.2852717***	1.2830207***	1.2834182***
-----+-----				

```

ARMA      |
      ar |
L1. | -.85017938***      .81704179***      -.85050658***      -.80668976***
      |
      ma |
L1. | .80583931***      -.85594029***      .81624866***      .77253015***
L2. |                      .07309734      .02708951      .02664907
L3. |                      .01783289      .04904404
L4. |                      .04879605
-----+-----
sigma      |
      _cons | 6.7312771***      6.7316582***      6.7295342***      6.7221303***
-----+-----
Statistics  |
      N |          500          500          500          500
      ll | -1662.8601      -1662.8819      -1662.7286      -1662.1835
      chi2 | 49.8428      22.748584      49.803311      40.869619
      aic | 3333.7202      3335.7639      3337.4571      3338.3669
      bic | 3350.5786      3356.8369      3362.7448      3367.8692
-----+-----

```

legend: * p<.1; ** p<.05; *** p<.01

```
. est table arima20*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)
```

```

-----+-----
Variable |      arima201      arima202      arima203      arima204
-----+-----
y          |
      _cons | 1.2852742***      1.2849413***      1.2850517***      1.2848488***
-----+-----
ARMA      |
      ar |
L1. | .72718848***      .03434887      -.0071867      -.01900249
L2. | .07767849      .72045376***      .72060757***      .67835837**

```

ma				
L1.		-.7684233***	-.05809605	-.0297673
L2.			-.65119293**	-.6531575**
L3.				.02414018
L4.				.02572472
-----+-----				
sigma				
_cons		6.7307631***	6.7199691***	6.7183825***
-----+-----				
Statistics				
N		500	500	500
ll		-1662.8217	-1662.0242	-1661.9107
chi2		19.588959	20.181824	21.261155
aic		3335.6434	3336.0484	3337.8214
bic		3356.7165	3361.3361	3367.3236
-----+-----				

legend: * p<.1; ** p<.05; *** p<.01

. est table arima30*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

-----+-----				
Variable		arima301	arima302	arima303
-----+-----				
y				
_cons		1.2829597***	1.2851221***	1.3010793***
-----+-----				
ARMA				
ar				
L1.		-.84116634**	-.03809733	1.074127***
L2.		.0303744	.72200193***	.63826377**
L3.		.01991732	.02474792	-.82783544***
ma				

L1.		.80847988**	.00208541	-1.133384	-1.1333014
L2.			-.65521465**	-.53766529	-.53847758
L3.				.79668804	.79801641
L4.					-.00064372
-----+-----					
sigma					
_cons		6.7293725***	6.7185376***	6.635098	6.6350719
-----+-----					
Statistics					
N		500	500	500	500
ll		-1662.7154	-1661.919	-1657.7457	-1657.7457
chi2		50.95581	21.823841	50151.443	35434.925
aic		3337.4309	3337.8381	3329.4915	3331.4913
bic		3362.7185	3367.3403	3358.9937	3365.2082

legend: * p<.1; ** p<.05; *** p<.01

. est table arima40*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

Variable		arima401	arima402	arima403	arima404
-----+-----					
y					
_cons		1.283736***	1.2851361***	1.3010538***	1.2857328***
-----+-----					
ARMA					
ar					
L1.		-.75902834**	-.05431084	1.072809***	.08134479
L2.		.03140494	.64109765*	.64018188	-.23331753
L3.		.05089759	.02507798	-.82789454***	-.04574194
L4.		.05080793	.02801024	-.0006621	.69819982***
ma					
L1.		.72522412**	.01787627	-1.132502	-.11980729

```

L2. |          -.58863871      -.53937619      .31872652
L3. |                  .79757316      .04504616
L4. |                  -.64787648**

-----+-----
sigma |
      _cons |  6.7218422***      6.716615***      6.635093      6.6766313***

-----+-----

Statistics |
          N |          500          500          500          500
          ll | -1662.1618      -1661.7838      -1657.7457      -1659.0454
        chi2 |  33.841275      19.156218      35372.689      1213.8068
         aic |  3338.3237      3339.5677      3331.4913      3338.0908
         bic |  3367.8259      3373.2845      3365.2082      3380.2368

-----+-----

                                legend: * p<.1; ** p<.05; *** p<.01

```

From the result, the best ARIMA model (ARIMA(p,d,q)) for y is ARIMA(1,0,1) since it gives the lowest value of BIC (SBIC) among any other combinations of (p,0,q), $1 \leq p, q \leq$

```
. arima y, arima(1,0,1)
```

```
(setting optimization to BHHH)
```

```
. . .
```

```
Iteration 7:   log likelihood = -1662.8601
```

```
ARIMA regression
```

```

Sample:   1 - 500                                Number of obs   =           500
                                                Wald chi2(2)         =           49.84
Log likelihood = -1662.86                      Prob > chi2          =           0.0000

```

```

-----+-----
          |                      OPG
          y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
          y |

```

```

      _cons |   1.282274   .2957515    4.34   0.000    .7026113    1.861936
-----+-----
ARMA      |
      ar |
      L1. |  -1.8501794   .1677986   -5.07   0.000   -1.179059   -1.5213001
      |
      ma |
      L1. |   .8058393   .189501    4.25   0.000   .4344241    1.177255
-----+-----
      /sigma |   6.731277   .2150805   31.30   0.000   6.309727    7.152827
-----+-----

```

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.

```

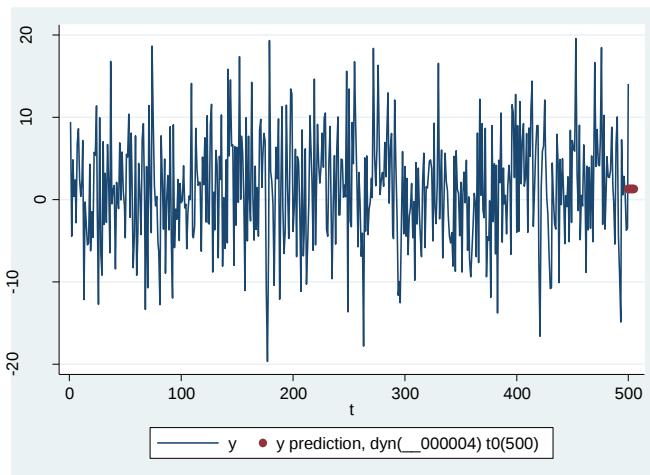
. set obs 505
number of observations (_N) was 500, now 505

. replace t = _n
(5 real changes made)

. predict yhat, y dynamic(.) t0(500)
Note: beginning dynamic predictions in period
(499 missing values generated)

. twoway (line y t, sort) (scatter yhat t, sort)

```



From the following GARCH model:

Mean Equation: $y_t = \alpha + \beta x_t + \varepsilon_t$ (5)

Variance Equation: $\sigma_t^2 = \alpha_0 + \sum_{j=1}^p \delta_j \sigma_{t-j}^2 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$ (6)

where: y_t is Dependent variable.
 x_t is Independent variable
 ε_t is Stochastic disturbance term
 σ_t^2 is Conditional variance of stochastic disturbance term.

(c) Estimate model (5) using OLS by employing y_t as dependent variable and x_t as explanatory variable, determine whether ARCH-effect significantly occurs, and state null hypothesis of the test. Why the test is classified as LM test? (3 points)

```
. reg y x
```

Source	SS	df	MS	Number of obs	=	500
-----+-----						
Model	22617.2475	1	22617.2475	F(1, 498)	=	57380.10
Residual	196.294343	498	.394165348	Prob > F	=	0.0000
-----+-----						
Total	22813.5419	499	45.7185208	R-squared	=	0.9914
-----+-----						
				Adj R-squared	=	0.9914
				Root MSE	=	.62783

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
y					
-----+-----					
x	.6976501	.0029124	239.54	0.000	.6919279 .7033723
_cons	.5173437	.028259	18.31	0.000	.4618221 .5728653


```
. estat archlm
```

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags (p)	chi2	df	Prob > chi2
-----+-----			
1	38.724	1	0.0000

H0: no ARCH effects vs. H1: ARCH(p) disturbance

From the test, there exists a significant ARCH effect since p-value of the test = 0.00 < 0.05 (The null hypothesis of the test is that $H_0: \alpha_1 = \alpha_2 = \alpha_q = 0$). This test uses LM test since uses only 1 restricted model which is the OLS one.

- (d) Estimate GARCH(p,q) for y_t using x_t as explanatory variable for mean equation (model (5) and (6)) – determine the most appropriated order p and q for variance equation using SBIC given the maximum lag equals to 2. Then, predict the variance of y_t using the estimated result of GARCH(p,q) model with the most appropriated lag. (4 points)

```
. qui arch y x, garch(1/1) arch(1/1)
. est store garch11
. qui arch y x, garch(1/1) arch(1/2)
. est store garch12
. qui arch y x, garch(1/2) arch(1/1)
. est store garch21
. qui arch y x, garch(1/2) arch(1/2)
. est store garch22
. estimates table garch11 garch12 garch21 garch22, star(0.1 0.05 0.01) stat(aic bic
11)
```

Variable		garch11	garch12	garch21	garch22
-----+-----					
y					
x		.69776959***	.6977543***	.69775887***	.69763896***
_cons		.51862389***	.51858003***	.51860028***	.51612708***
-----+-----					
ARCH					
arch					
L1.		.36335397***	.36174929***	.36211559***	.37334452***
L2.			.01489233		.30932813**
garch					
L1.		.31446618***	.28733881	.32471553*	-.39406428
L2.				-.00925651	.10779269
_cons		.12902835***	.13455126*	.12912145***	.24581943***
-----+-----					

Statistics |

aic		897.80341	899.79061	899.79388	900.15062
bic		918.87645	925.07826	925.08153	929.65288
ll		-443.9017	-443.8953	-443.89694	-443.07531

 legend: * p<.1; ** p<.05; *** p<.01

According to BIC(SBIC), the most appropriated lags order in this case is GARCH(1,1)

. arch y x, garch(1/1) arch(1/1) nolog

ARCH family regression

Sample: 1 - 500	Number of obs	=	500
Distribution: Gaussian	Wald chi2(1)	=	72809.92
Log likelihood = -443.9017	Prob > chi2	=	0.0000

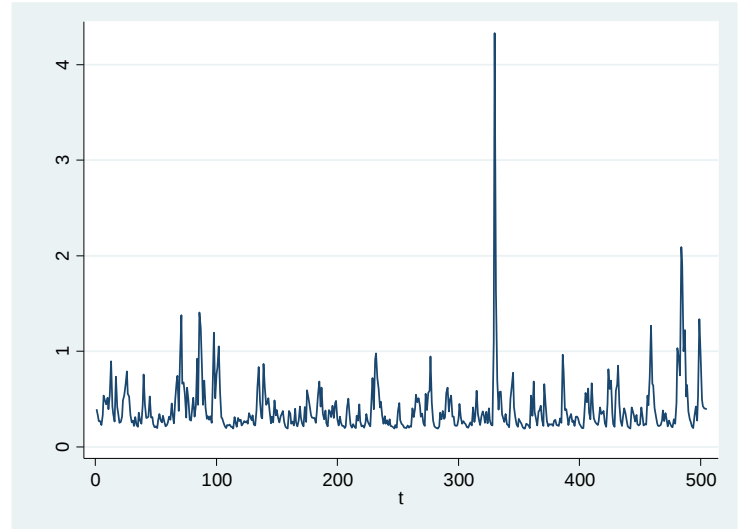
		OPG				
	y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
-----+-----						
y						
	x	.6977696	.0025859	269.83	0.000	.6927013 .7028379
	_cons	.5186239	.0239338	21.67	0.000	.4717145 .5655333

-----+-----

ARCH

arch						
L1.		.363354	.0747846	4.86	0.000	.2167788 .5099291
garch						
L1.		.3144662	.120527	2.61	0.009	.0782376 .5506948
_cons		.1290283	.0366217	3.52	0.000	.0572511 .2008056

```
. predict sigmahat, v
. line sigmahat t
```



(e) Among these three models, ARCH(1), GARCH(1,1), or EGARCH(1,1,1), which model is the most appropriated in this case? Why? What are the differences among ARCH, GARCH, and EGARCH? (3 points)

The differences among ARCH, GARCH, and EGARCH are that

Autoregressive Conditional Heteroskedasticity, or ARCH, is a method that explicitly models the change in variance over time in a time series.

Generalized Autoregressive Conditional Heteroskedasticity, or GARCH, is an extension of the ARCH model that incorporates a moving average component together with the autoregressive component.

While, the Exponential GARCH (EGARCH) Model is a subset of GARCH model in which the conditional variance process are done in exponential form.

```
. qui arch y x, arch(1) nolog
. est store arch1
. qui arch y x, arch(1) garch(1) nolog
. est store archgarch
. qui arch y x, arch(1) garch(1) egarch(1) nolog
. est store archgarchegarch

. estimates table arch1 archgarch archgarchegarch , star(0.1 0.05 0.01) stat(bic)
```

Variable	arch1	archgarch	archgarcheg~h
-----+-----			
y			
x	.69766735***	.69776959***	.69793118***

_cons		.51465166***	.51862389***	.51184646***
-----+-----				
ARCH				
arch				
L1.		.38285474***	.36335397***	.67472926***
garch				
L1.			.31446618***	-.00592512
egarch				
L1.				.36128496***
_cons		.2414657***	.12902835***	-.92456597***
-----+-----				
Statistics				
bic		922.68979	918.87645	932.02784
-----+-----				

legend: * p<.1; ** p<.05; *** p<.01

Based on BIC, GARCH(1,1) is the most appropriated one since it provides the lowest BIC.

5. From the data set "Final_q5_no.dta":

The following VARs models:

$$Y_t = A_0 + A_1 Y_{t-1} + \epsilon_t \quad (7)$$

where: $Y_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}$, $A_0 = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix}$, $A_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $\epsilon_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$

(a) Estimate VARs models using y_t and x_t as endogenous variables and determine the most appropriated lags models using SBIC. (5 points).

```
. varsoc y x
      Selection-order criteria
      Sample: 5 - 500                                Number of obs   =          496
+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|lag |      LL      LR      df      p      FPE      AIC      HQIC      SBIC  |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 0 | -1363.83                .845032   5.50737   5.51403   5.52434 |
| 1 | -1317.45  92.757*      4  0.000   .712297*  5.33649*  5.35647*  5.38738* |
| 2 | -1316.57   1.7619      4  0.779   .721313   5.34907   5.38236   5.43388 |
| 3 | -1315.39   2.3605      4  0.670   .729562   5.36044   5.40705   5.47917 |
| 4 | -1313.09   4.5974      4  0.331   .734588   5.3673    5.42722   5.51996 |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+
Endogenous:  y x
Exogenous:   _cons
```

Based on SBIC, the most appropriated lag order is 1 since it gives the lowest SBIC.

(b) Perform stability test and Granger exogeneity test. Determine whether assumptions of VARs are satisfied, explain your evaluation criteria. If the stability assumption is unsatisfied, what will happen? (5 points)

```
. var y x, lag(1/1)
```

Vector autoregression

Sample: 2 - 500		Number of obs	=	499
Log likelihood =	-1327.573	AIC	=	5.344982
FPE	= .7183688	HQIC	=	5.364859
Det(Sigma ml)	= .7012993	SBIC	=	5.395634

Equation	Parms	RMSE	R-sq	chi2	P>chi2
y	3	.990399	0.0132	6.670976	0.0356
x	3	.997289	0.0899	49.27269	0.0000

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----							
y							
	y						
	L1.	.1292078	.0501774	2.58	0.010	.0308619	.2275536
	x						
	L1.	.0483716	.0478178	1.01	0.312	-.0453495	.1420928
	_cons	.3388104	.0542276	6.25	0.000	.2325263	.4450946
-----+-----							
x							
	y						
	L1.	.3475582	.0505264	6.88	0.000	.2485282	.4465882
	x						
	L1.	.2127216	.0481504	4.42	0.000	.1183485	.3070947
	_cons	.1226802	.0546048	2.25	0.025	.0156567	.2297037

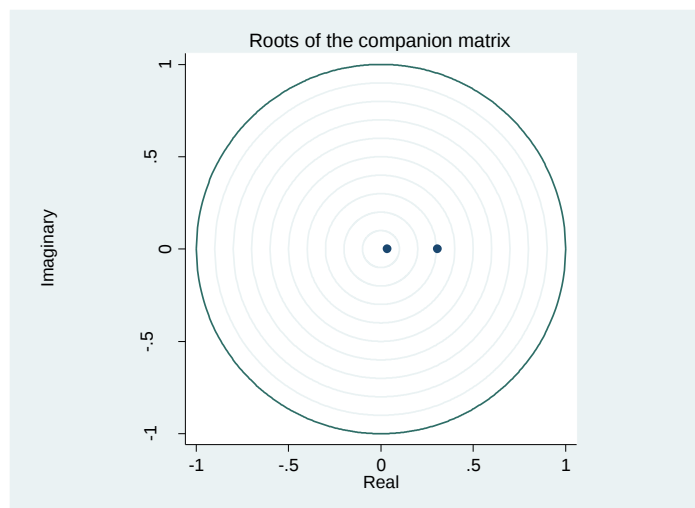
```
. varstable, graph
```

Eigenvalue stability condition

+-----+		
Eigenvalue	Modulus	
+-----+		
.3071836	.307184	
.03474578	.034746	
+-----+		

All the eigenvalues lie inside the unit circle.

VAR satisfies stability condition.



```
. vargranger
```

Granger causality Wald tests

+-----+					
Equation	Excluded	chi2	df	Prob > chi2	
+-----+					
y	x	1.0233	1	0.312	
y	ALL	1.0233	1	0.312	
+-----+					
x	y	47.317	1	0.000	
x	ALL	47.317	1	0.000	
+-----+					

According to the stability test, the system is stable because all the eigenvalues lie inside the unit circle.

According to the Granger exogeneity test, y is endogenous variable since the tests are significant while x seems not to be endogenous.

The stability is required because the this is the dynamic system. The effect of this period will affect the next period, i.e. it should be convergent. If this assumption is violated, the effect will diverge (and not die down).

- (c) Perform Impulse response function analysis (irf), Orthogonal impulse response function analysis (oirf), Cumulative impulse response function analysis (coirf), make interpretation of the analysis, and determine which variable has more impact (using Cholesky order – y_t x_t). (5 points)

```
. irf create order1, o(y x) step(10) set(irf001)
```

```
(file irf001.irf created)
```

```
(file irf001.irf now active)
```

```
(file irf001.irf updated)
```

```
. irf table irf, impulse(y x) response(y x)
```

Results from order1

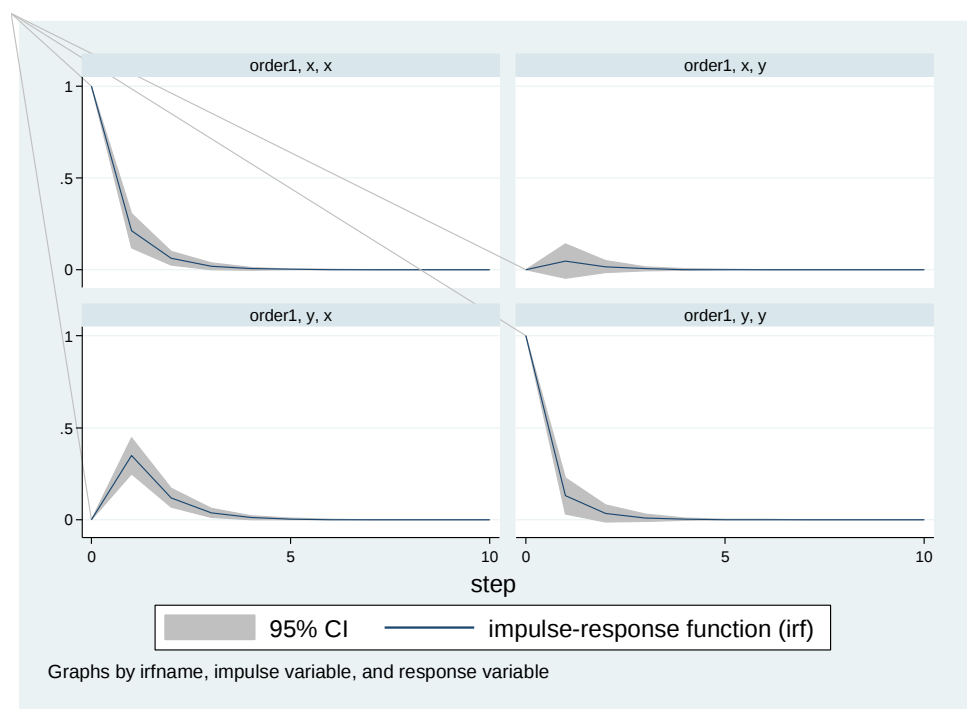
	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(3)	(4)	(4)	(4)	
step	irf	Lower	Upper	irf	Lower	Upper	irf	Lower	Upper	irf	Lower	Upper	

0	1	1	1	0	0	0	0	0	0	1	1	1	
1	.129208	.030862	.227554	.347558	.248528	.446588	.048372	-.04535	.142093	.212722	.118349	.307095	
2	.033507	-.014372	.081385	.11884	.067187	.170494	.01654	-.015834	.048914	.062062	.024204	.099921	
3	.010078	-.008978	.029133	.036925	.012175	.061676	.005139	-.006431	.016709	.018951	.001157	.036744	
4	.003088	-.003914	.01009	.011357	.000172	.022543	.001581	-.002539	.0057	.005817	-.001731	.013366	
5	.000948	-.00155	.003447	.003489	-.001173	.008152	.000486	-.000959	.00193	.001787	-.001192	.004766	
6	.000291	-.000584	.001167	.001072	-.000753	.002896	.000149	-.00035	.000649	.000549	-.000572	.00167	
7	.000089	-.000213	.000392	.000329	-.000354	.001013	.000046	-.000125	.000216	.000169	-.000239	.000577	
8	.000027	-.000076	.000131	.000101	-.000147	.000349	.000014	-.000044	.000072	.000052	-.000093	.000197	
9	8.4e-06	-.000026	.000043	.000031	-.000057	.000119	4.3e-06	-.000015	.000024	.000016	-.000035	.000067	
10	2.6e-06	-9.1e-06	.000014	9.5e-06	-.000021	.00004	1.3e-06	-5.1e-06	7.8e-06	4.9e-06	-.000013	.000022	

95% lower and upper bounds reported

- (1) irfname = order1, impulse = y, and response = y
- (2) irfname = order1, impulse = y, and response = x
- (3) irfname = order1, impulse = x, and response = y
- (4) irfname = order1, impulse = x, and response = x

```
irf graph irf, impulse(y x) response(y x)
```



```
. irf table oirf, impulse(y x) response(y x)
```

Results from order1												
	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(3)	(4)	(4)	(4)
step	oirf	Lower	Upper	oirf	Lower	Upper	oirf	Lower	Upper	oirf	Lower	Upper
0	.987418	.926157	1.04868	-.51896	-.60004	-.437881	0	0	0	.848107	.795489	.900725
1	.102479	.015961	.188998	.232791	.143376	.322206	.041024	-.038502	.120551	.180411	.099593	.261228
2	.024502	-.00738	.056384	.085137	.045814	.12446	.014027	-.013443	.041498	.052636	.020362	.084909
3	.007284	-.005717	.020285	.026626	.009405	.043848	.004359	-.005458	.014175	.016072	.000948	.031196
4	.002229	-.00259	.007048	.008196	.000669	.015722	.001341	-.002154	.004835	.004934	-.001476	.011343
5	.000684	-.001044	.002413	.002518	-.000624	.00566	.000412	-.000814	.001637	.001515	-.001013	.004043
6	.00021	-.000398	.000818	.000774	-.000466	.002013	.000127	-.000297	.00055	.000466	-.000485	.001417
7	.000065	-.000146	.000275	.000238	-.00023	.000705	.000039	-.000106	.000184	.000143	-.000203	.000489
8	.00002	-.000052	.000092	.000073	-.000098	.000244	.000012	-.000037	.000061	.000044	-.000079	.000167
9	6.1e-06	-.000018	.000031	.000022	-.000038	.000083	3.7e-06	-.000013	.00002	.000013	-.000029	.000056
10	1.9e-06	-6.3e-06	.00001	6.9e-06	-.000014	.000028	1.1e-06	-4.3e-06	6.6e-06	4.1e-06	-.000011	.000019

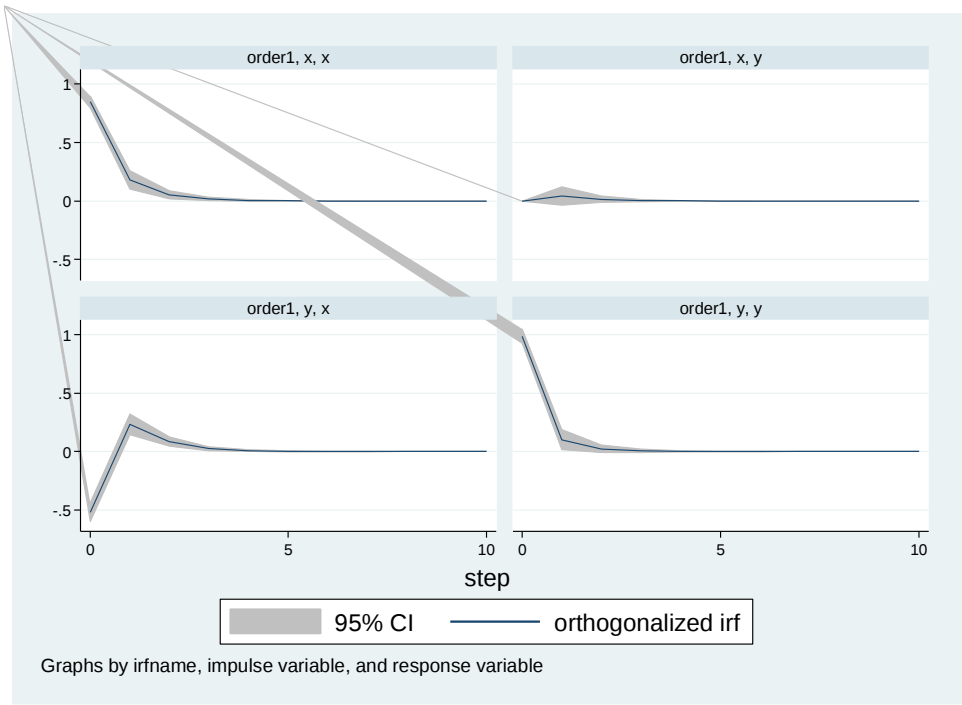
95% lower and upper bounds reported

(1) irfname = order1, impulse = y, and response = y

(2) irfname = order1, impulse = y, and response = x

- (3) irfname = order1, impulse = x, and response = y
- (4) irfname = order1, impulse = x, and response = x

```
. irf graph oirf, impulse(y x) response(y x)
```



```
. irf table coirf, impulse(y x) response(y x)
```

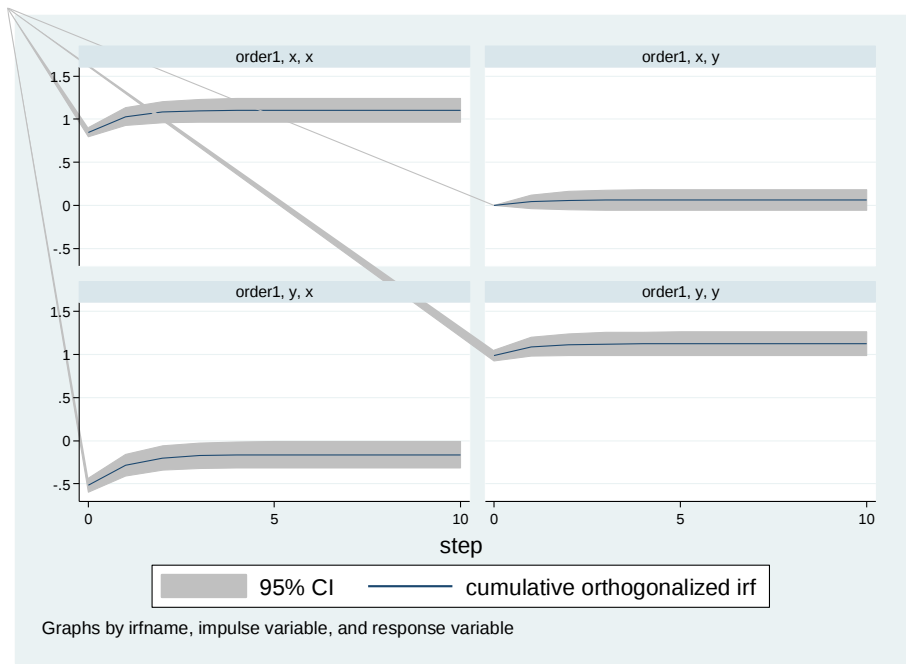
Results from order1

	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(3)	(4)	(4)	(4)
step	coirf	Lower	Upper	coirf	Lower	Upper	coirf	Lower	Upper	coirf	Lower	Upper
0	.987418	.926157	1.04868	-.51896	-.60004	-.437881	0	0	0	.848107	.795489	.900725
1	1.0899	.980273	1.19952	-.286169	-.412639	-.159699	.041024	-.038502	.120551	1.02852	.926156	1.13088
2	1.1144	.987392	1.2414	-.201032	-.342581	-.059483	.055052	-.051621	.161725	1.08115	.957139	1.20517
3	1.12168	.987583	1.25578	-.174406	-.322962	-.02585	.05941	-.056878	.175699	1.09723	.964314	1.23014
4	1.12391	.987135	1.26069	-.16621	-.317625	-.014796	.060751	-.058934	.180436	1.10216	.965731	1.23859
5	1.1246	.986867	1.26232	-.163692	-.31619	-.011195	.061163	-.059704	.18203	1.10367	.965939	1.24141
6	1.12481	.986749	1.26286	-.162919	-.315806	-.010031	.061289	-.059983	.182562	1.10414	.965937	1.24234
7	1.12487	.986702	1.26304	-.162681	-.315705	-.009657	.061328	-.060082	.182738	1.10428	.965918	1.24265
8	1.12489	.986684	1.2631	-.162608	-.315678	-.009538	.06134	-.060116	.182796	1.10433	.965906	1.24275
9	1.1249	.986678	1.26312	-.162586	-.315672	-.009499	.061344	-.060128	.182816	1.10434	.965901	1.24278
10	1.1249	.986676	1.26312	-.162579	-.31567	-.009487	.061345	-.060132	.182822	1.10434	.965899	1.24279

95% lower and upper bounds reported

- (1) irfname = order1, impulse = y, and response = y
- (2) irfname = order1, impulse = y, and response = x
- (3) irfname = order1, impulse = x, and response = y
- (4) irfname = order1, impulse = x, and response = x

```
. irf graph coirf, impulse(y x) response(y x)
```



According to IRF Analysis, y has a positive effect on x with a greater magnitude compared with what x does on y. Moreover, the impact of y on x exists longer (longer duration) until it dies down.

In conclusion, y has more impact on x.

(d) Perform Forecast error variance decomposition (fevd) and determine variable that has more impact on each endogenous variable. (3 points)

```
. irf table fevd, impulse(y x) response(y x)
```

Results from order1

	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(3)	(4)	(4)	(4)
step	fevd	Lower	Upper	fevd	Lower	Upper	fevd	Lower	Upper	fevd	Lower	Upper

10	0	0	0	0	0	0	0	0	0	0	0	0	
11	1	1	1	.272424	.205785	.339063	0	0	0	.727576	.660937	.794215	
12	.998295	.991692	1.0049	.300844	.236036	.365652	.001705	-.004898	.008308	.699156	.634348	.763964	
13	.998097	.990736	1.00546	.304746	.240486	.369005	.001903	-.005459	.009264	.695254	.630995	.759514	
14	.998078	.990633	1.00552	.305127	.240929	.369324	.001922	-.005523	.009367	.694873	.630676	.759071	
15	.998076	.990623	1.00553	.305163	.240971	.369354	.001924	-.00553	.009377	.694837	.630646	.759029	
16	.998076	.990622	1.00553	.305166	.240975	.369357	.001924	-.005531	.009378	.694834	.630643	.759025	
17	.998076	.990621	1.00553	.305167	.240976	.369358	.001924	-.005531	.009379	.694833	.630642	.759024	
18	.998076	.990621	1.00553	.305167	.240976	.369358	.001924	-.005531	.009379	.694833	.630642	.759024	
19	.998076	.990621	1.00553	.305167	.240976	.369358	.001924	-.005531	.009379	.694833	.630642	.759024	
110	.998076	.990621	1.00553	.305167	.240976	.369358	.001924	-.005531	.009379	.694833	.630642	.759024	

95% lower and upper bounds reported

- (1) irfname = order1, impulse = y, and response = y
- (2) irfname = order1, impulse = y, and response = x
- (3) irfname = order1, impulse = x, and response = y
- (4) irfname = order1, impulse = x, and response = x

According to the Forecast error variance decomposition, y has more impact on x.

(e) Determine whether changing Cholesky order from – “ y_t x_t ” to “ x_t y_t ” will change the results of irf, oirf, coirf, and fevd. Why? or why not? (2 points)

. irf table irf, impulse(x y) response(x y)

Results from order1													

		(1)	(1)	(1)		(2)	(2)	(2)		(3)	(3)	(3)	
	step	irf	Lower	Upper		irf	Lower	Upper		irf	Lower	Upper	

10	1	1	1	0	0	0	0	0	0	1	1	1	
11	.212722	.118349	.307095	.048372	-.04535	.142093	.347558	.248528	.446588	.129208	.030862	.227554	
12	.062062	.024204	.099921	.01654	-.015834	.048914	.11884	.067187	.170494	.033507	-.014372	.081385	
13	.018951	.001157	.036744	.005139	-.006431	.016709	.036925	.012175	.061676	.010078	-.008978	.029133	
14	.005817	-.001731	.013366	.001581	-.002539	.0057	.011357	.000172	.022543	.003088	-.003914	.01009	
15	.001787	-.001192	.004766	.000486	-.000959	.00193	.003489	-.001173	.008152	.000948	-.00155	.003447	
16	.000549	-.000572	.00167	.000149	-.00035	.000649	.001072	-.000753	.002896	.000291	-.000584	.001167	
17	.000169	-.000239	.000577	.000046	-.000125	.000216	.000329	-.000354	.001013	.000089	-.000213	.000392	
18	.000052	-.000093	.000197	.000014	-.000044	.000072	.000101	-.000147	.000349	.000027	-.000076	.000131	
19	.000016	-.000035	.000067	4.3e-06	-.000015	.000024	.000031	-.000057	.000119	8.4e-06	-.000026	.000043	
110	4.9e-06	-.000013	.000022	1.3e-06	-5.1e-06	7.8e-06	9.5e-06	-.000021	.00004	2.6e-06	-9.1e-06	.000014	

95% lower and upper bounds reported

```
(1) irfname = order1, impulse = x, and response = x
(2) irfname = order1, impulse = x, and response = y
(3) irfname = order1, impulse = y, and response = x
(4) irfname = order1, impulse = y, and response = y
. irf table oirf, impulse(x y) response(x y)
```

Results from order1

	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(3)	(4)	(4)	(4)
step	oirf	Lower	Upper	oirf	Lower	Upper	oirf	Lower	Upper	oirf	Lower	Upper
0	.994287	.932599	1.05597	-.515375	-.595895	-.434856	0	0	0	.842248	.789994	.894503
1	.032383	-.055074	.119841	-.018495	-.10205	.06506	.29273	.207368	.378093	.108825	.025719	.191931
2	.00046	-.026231	.027152	-.000823	-.012926	.01128	.100093	.056147	.144039	.028221	-.012143	.068584
3	-.000188	-.009805	.009429	-.000084	-.002898	.00273	.0311	.010165	.052035	.008488	-.00757	.024546
4	-.000069	-.003091	.002953	-.00002	-.000837	.000797	.009566	.000126	.019006	.002601	-.003298	.008501
5	-.000022	-.000948	.000905	-5.9e-06	-.000253	.000242	.002939	-.000992	.00687	.000799	-.001306	.002904
6	-6.7e-06	-.00029	.000277	-1.8e-06	-.000077	.000074	.000903	-.000635	.002441	.000245	-.000492	.000983
7	-2.0e-06	-.000089	.000084	-5.6e-07	-.000024	.000022	.000277	-.000298	.000853	.000075	-.000179	.00033
8	-6.3e-07	-.000027	.000026	-1.7e-07	-7.2e-06	6.9e-06	.000085	-.000124	.000294	.000023	-.000064	.00011
9	-1.9e-07	-8.3e-06	7.9e-06	-5.3e-08	-2.2e-06	2.1e-06	.000026	-.000048	.0001	7.1e-06	-.000022	.000036
10	-5.9e-08	-2.5e-06	2.4e-06	-1.6e-08	-6.7e-07	6.4e-07	8.0e-06	-.000018	.000034	2.2e-06	-7.7e-06	.000012

95% lower and upper bounds reported

```
(1) irfname = order1, impulse = x, and response = x
(2) irfname = order1, impulse = x, and response = y
(3) irfname = order1, impulse = y, and response = x
(4) irfname = order1, impulse = y, and response = y
. irf table coirf, impulse(x y) response(x y)
```

Results from order1

	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(3)	(4)	(4)	(4)
step	coirf	Lower	Upper	coirf	Lower	Upper	coirf	Lower	Upper	coirf	Lower	Upper
0	.994287	.932599	1.05597	-.515375	-.595895	-.434856	0	0	0	.842248	.789994	.894503
1	1.02667	.918494	1.13485	-.533871	-.656139	-.411603	.29273	.207368	.378093	.951073	.849374	1.05277
2	1.02713	.909734	1.14453	-.534694	-.666579	-.402808	.392823	.275251	.510396	.979294	.8471	1.11149
3	1.02694	.906338	1.14755	-.534778	-.668901	-.400655	.423924	.292377	.555471	.987782	.842865	1.1327
4	1.02687	.905186	1.14856	-.534798	-.669568	-.400028	.43349	.296281	.570698	.990383	.840743	1.14002
5	1.02685	.904821	1.14888	-.534804	-.66977	-.399838	.436428	.297057	.575799	.991182	.839866	1.1425
6	1.02684	.904709	1.14898	-.534806	-.669831	-.39978	.437331	.297172	.577491	.991427	.83953	1.14333

7		1.02684	.904674	1.14901		-.534806	-.66985	-.399762		.437609	.297171	.578046		.991503	.839406	1.1436	
8		1.02684	.904664	1.14902		-.534806	-.669856	-.399757		.437694	.29716	.578228		.991526	.839362	1.14369	
9		1.02684	.90466	1.14902		-.534806	-.669858	-.399755		.43772	.297153	.578287		.991533	.839347	1.14372	
10		1.02684	.904659	1.14902		-.534806	-.669858	-.399755		.437728	.29715	.578306		.991535	.839342	1.14373	

95% lower and upper bounds reported

(1) irfname = order1, impulse = x, and response = x

(2) irfname = order1, impulse = x, and response = y

(3) irfname = order1, impulse = y, and response = x

(4) irfname = order1, impulse = y, and response = y

. irf table fevd, impulse(x y) response(x y)

Results from order1

		(1)	(1)	(1)		(2)	(2)	(2)		(3)	(3)	(3)		(4)	(4)	(4)	
	step	fevd	Lower	Upper		fevd	Lower	Upper		fevd	Lower	Upper		fevd	Lower	Upper	
0		0	0	0		0	0	0		0	0	0		0	0	0	
1		1	1	1		.272424	.205785	.339063		0	0	0		.727576	.660937	.794215	
2		.920313	.876547	.964079		.269408	.202867	.335949		.079687	.035921	.123453		.730592	.664051	.797133	
3		.911818	.863879	.959757		.269191	.202631	.335751		.088182	.040243	.136121		.730809	.664249	.797369	
4		.911006	.862601	.959411		.269172	.202608	.335735		.088994	.040589	.137399		.730828	.664265	.797392	
5		.910929	.862476	.959383		.26917	.202606	.335734		.089071	.040617	.137524		.73083	.664266	.797394	
6		.910922	.862464	.959381		.26917	.202606	.335733		.089078	.040619	.137536		.73083	.664267	.797394	
7		.910921	.862462	.959381		.26917	.202606	.335733		.089079	.040619	.137538		.73083	.664267	.797394	
8		.910921	.862462	.959381		.26917	.202606	.335733		.089079	.040619	.137538		.73083	.664267	.797394	
9		.910921	.862462	.959381		.26917	.202606	.335733		.089079	.040619	.137538		.73083	.664267	.797394	
10		.910921	.862462	.959381		.26917	.202606	.335733		.089079	.040619	.137538		.73083	.664267	.797394	

95% lower and upper bounds reported

(1) irfname = order1, impulse = x, and response = x

(2) irfname = order1, impulse = x, and response = y

(3) irfname = order1, impulse = y, and response = x

(4) irfname = order1, impulse = y, and response = y

Now, the result changes. With the order (y x), we have that y has more impact on x. However, when to order changes to (x y), the x, now, has more impact on x. Since we use the Cholesky Order, order matters.