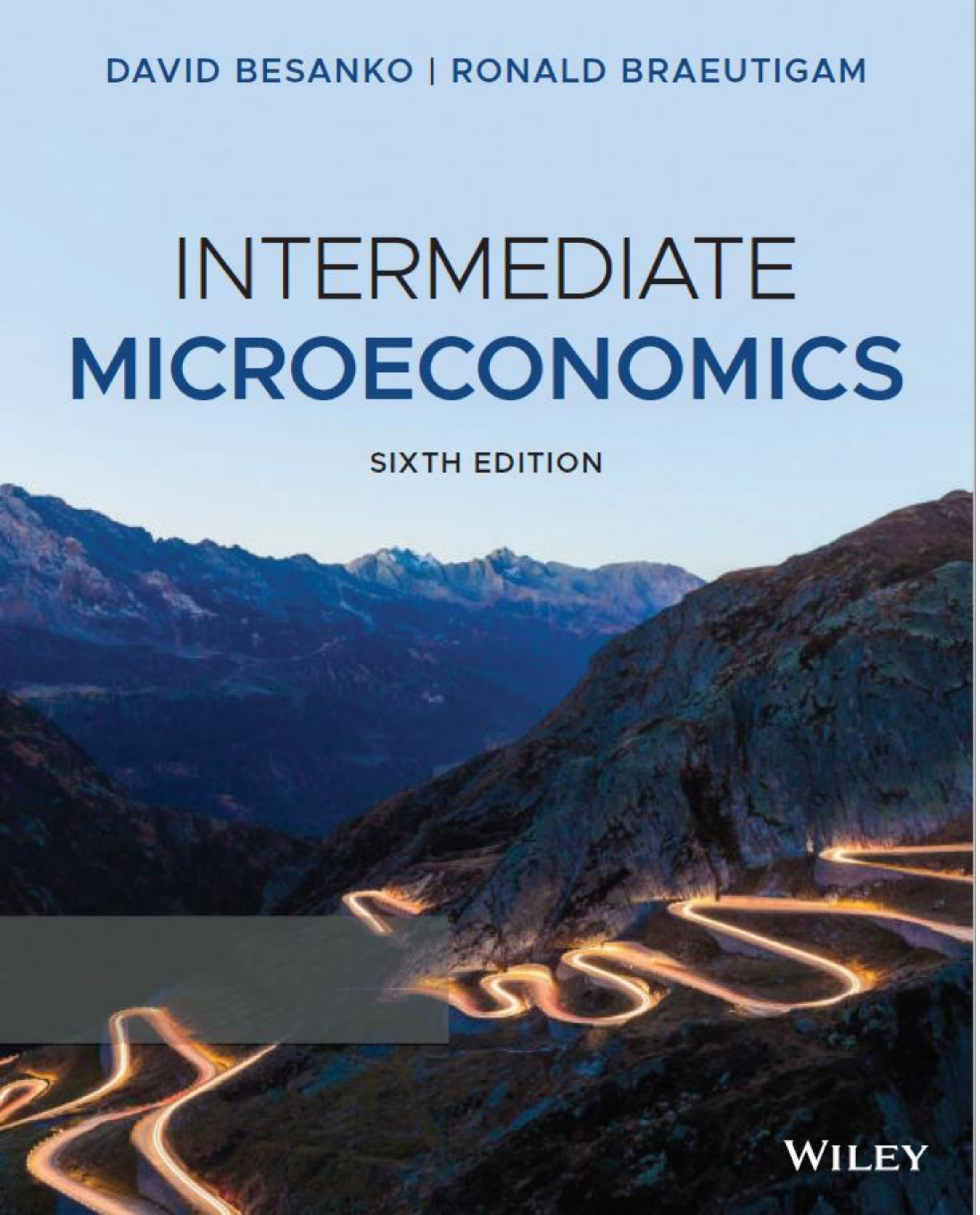


Chapter 6

Inputs and Production Functions

Chapter Six Overview

- Introduction to Inputs and Production Functions
- Production Functions with a Single Input
- Production Functions with More than one Input
- Substituabili9ty Among Inputs
- Returns to Scale
- Technological Progress
- The Elasticity of Substitution for a Cobb-Douglas Production Function

Can They Do It Better and Cheaper?

- GM started adding robots to their assembly lines in the 1990s
  - To save labor costs
- Producers of semiconductor chips use robots in factories that cost up to $14 billion to construct
  - Must be 1,000 times cleaner than a hospital operating room
  - Easier with robots than humans
- Robots that perform sophisticated tasks are not cheap
- Companies face a trade-off: is the added investment in robots worth the production cost savings

Key Concepts

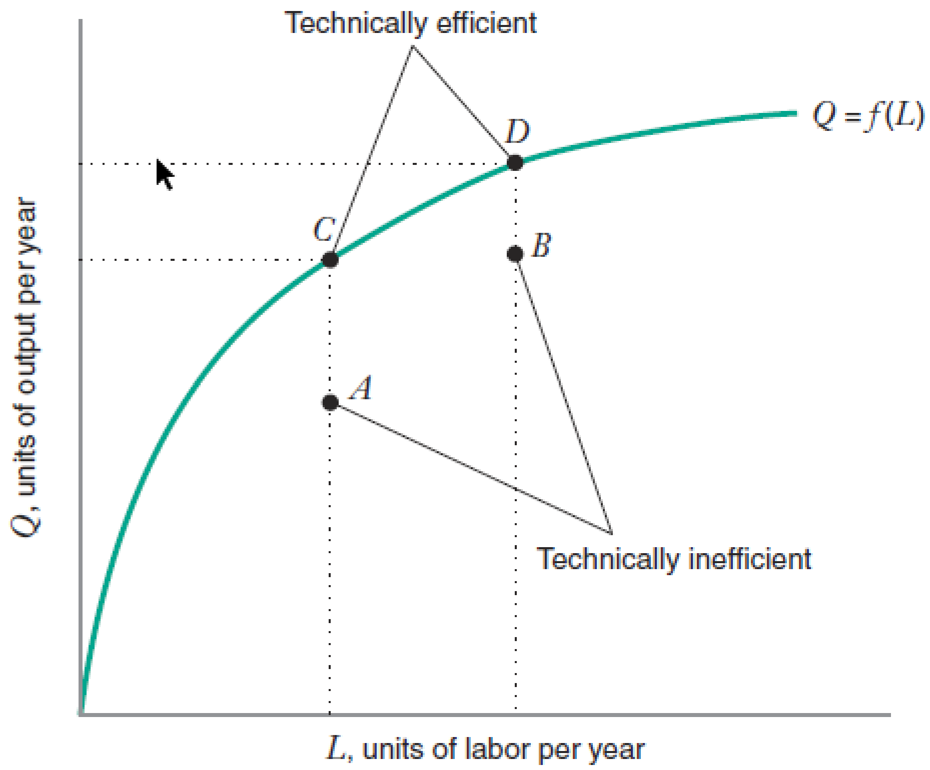
- Productive resources, such as labor and capital equipment, that firms use to manufacture goods and services are called inputs or factors of production
- The amount of goods and services produces by the firm is the firm's output
- Production transforms a set of inputs into a set of outputs
- Technology determines the quantity of output that is feasible to attain for a given set of inputs

Production Function

- The production function tells us the maximum possible output that can be attained by the firm for any given quantity of inputs
- Production Function $Q = f(L, K)$
  - Q = Output
  - K = Capital
  - L = Labor
- The production set is a set of technically feasible combinations of inputs and outputs

Technical Efficiency and Inefficiency

- Technically efficient: sets of points in the production function that maximizes output given input (labor)
  - $Q = f(L, K)$
- Technically inefficient: sets of points that produces less output than possible for a given set of input (labor)
  - $Q < f(L, K)$



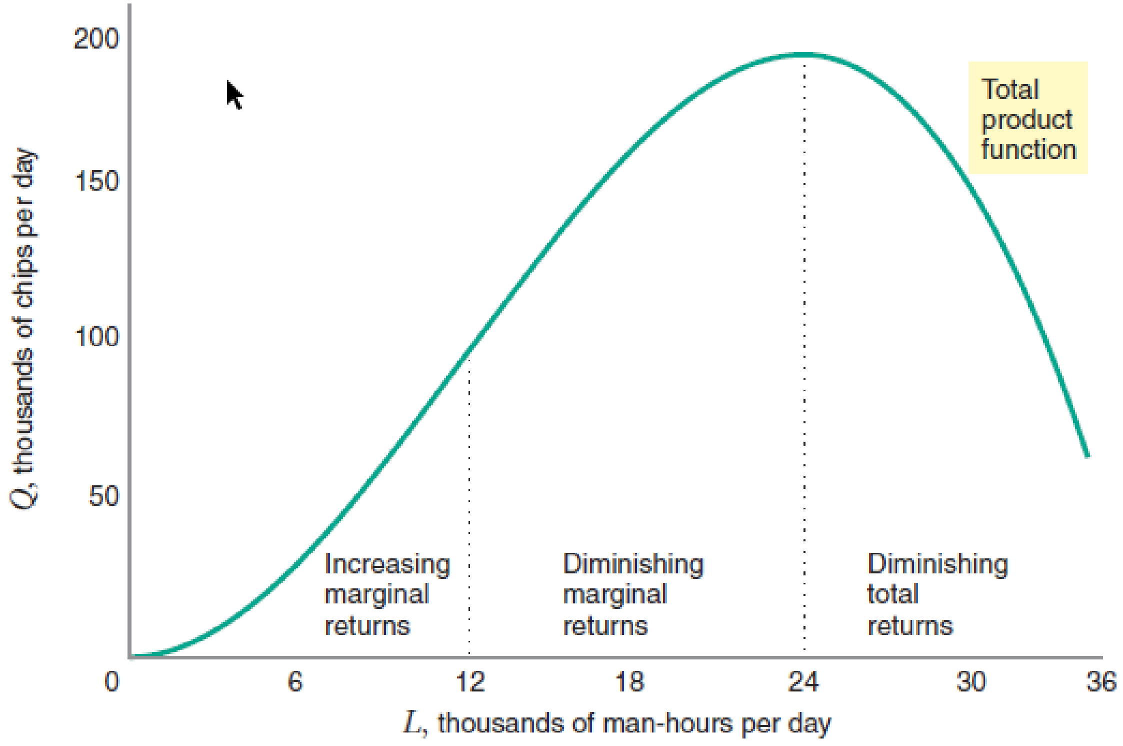
Labor Requirements Function

- Labor requirements function: $L = g(Q)$
- Example: $L = Q^2$
- For production function: $Q = \sqrt{L}$

Total Product

- Total product function: A single-input production function
  - It shows how total output depends on the level of the input
- Increasing marginal returns to labor: An increase in the quantity of labor increases total output at an increasing rate
- Diminishing marginal returns to labor: An increase in the quantity of labor increases total output but at a decreasing rate
- Diminishing total returns to labor: An increase in the quantity of labor decreases total output

Total Product Function



L*	Q
0	0
6	30
12	96
18	162
24	192
30	150

The Marginal Product

- The marginal product of an input is the change in output that results from a small change in an input holding the levels of all other inputs constant
- MPL = $\Delta Q/\Delta L$
  - (holding constant all other inputs)
- MPK = $\Delta Q/\Delta K$
  - (holding constant all other inputs)
- Example: $Q = K^{1/2}L^{1/2}$
  - $MP_L = (1/2)L^{-1/2}K^{1/2}$
  - $MP_K = (1/2)K^{-1/2}L^{1/2}$

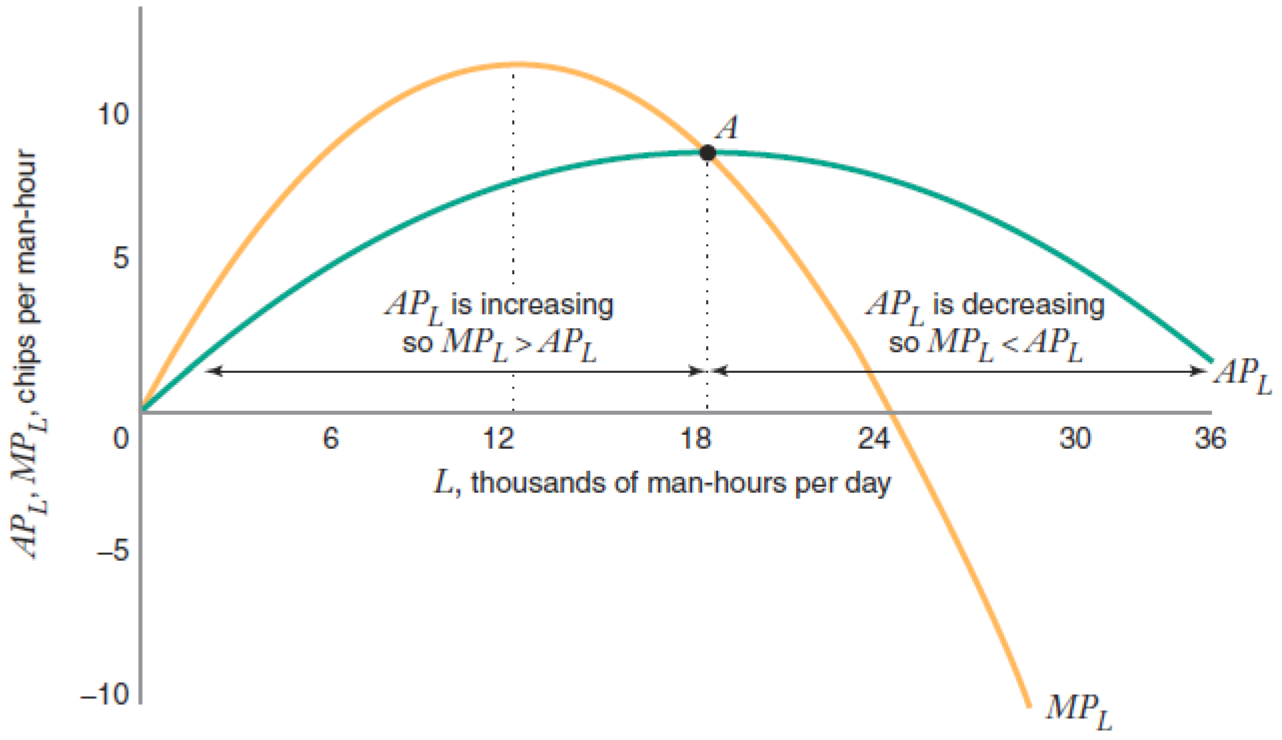
The Average Product & Diminishing Returns

- The average product of an input is equal to the total output that is to be produced divided by the quantity of the input that is used in its production
  - $AP_L = {}^{Q}/_{L}$
  - $AP_K = {}^{Q}/_{K}$
- Example:
  - $AP_L = {}^{[K^{1/2}L^{1/2}]}/_{L} = K^{1/2}L^{-1/2}$
  - $AP_K = {}^{[K^{1/2}L^{1/2}]}/_{K} = L^{1/2}K^{-1/2}$
- The law of diminishing marginal returns states that marginal products (eventually) decline as the quantity used of a single input increases

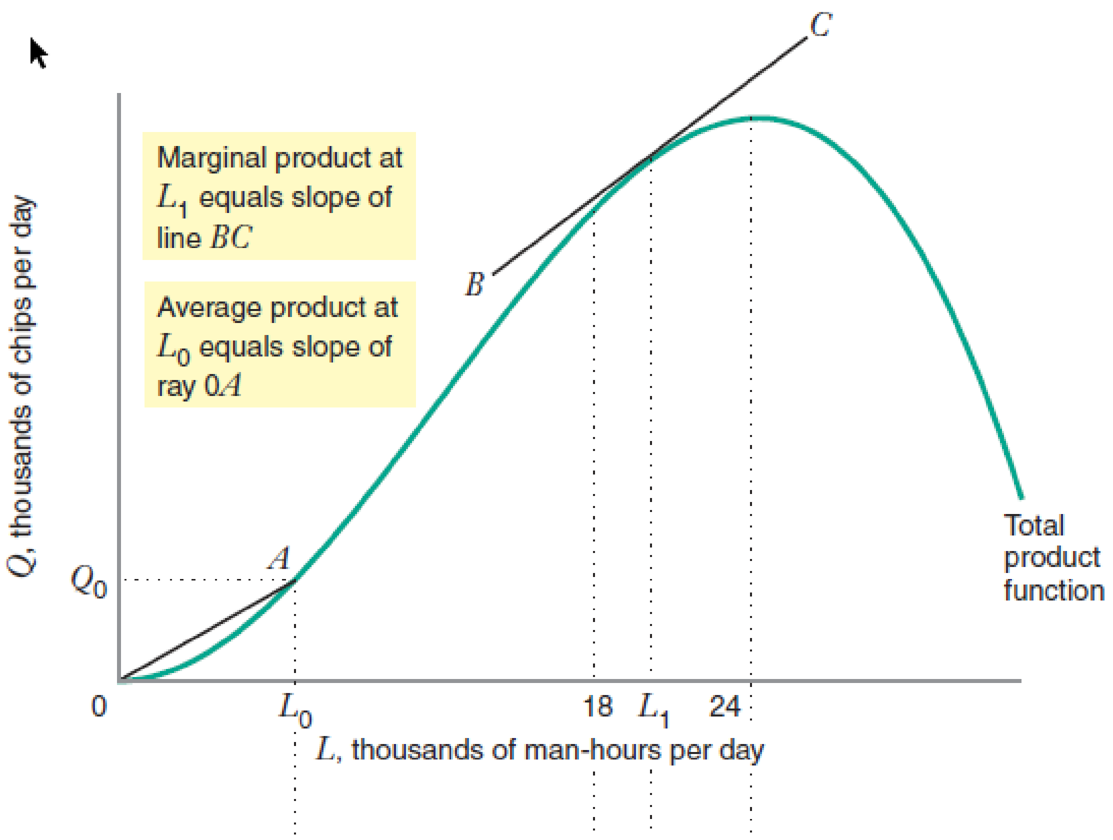
Total, Average, and Marginal Products

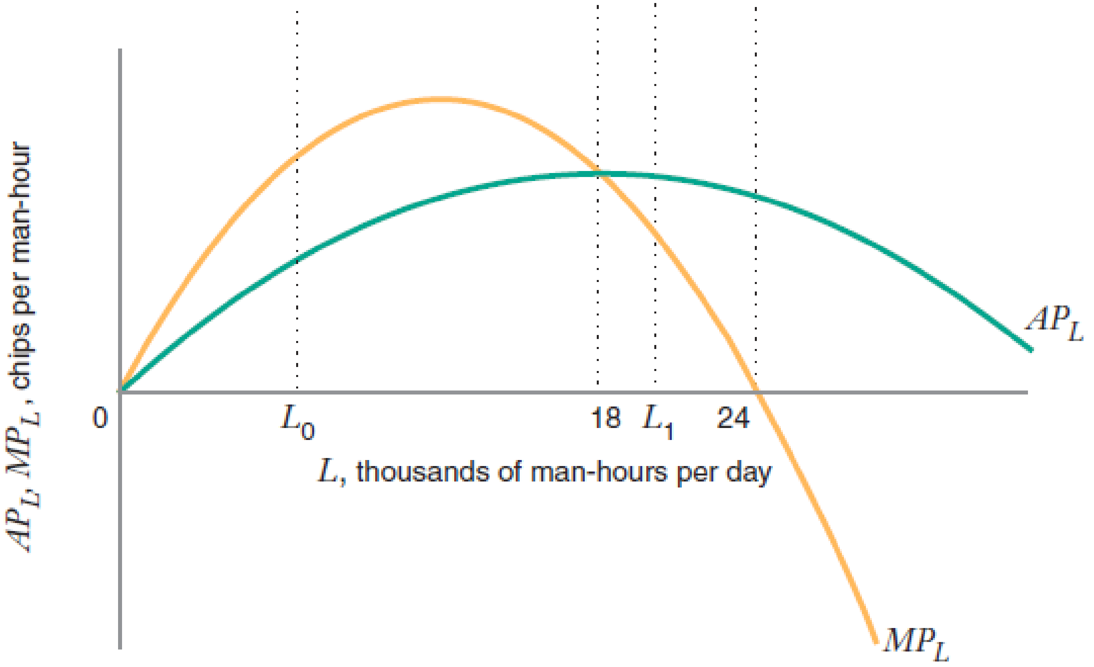
L	Q	APL	MPL
6	30	5	-
12	96	8	11
18	162	9	11
24	192	8	5
30	150	5	-7

Average and Marginal Product Functions



Relationship among Total, Average, and Marginal Product Functions



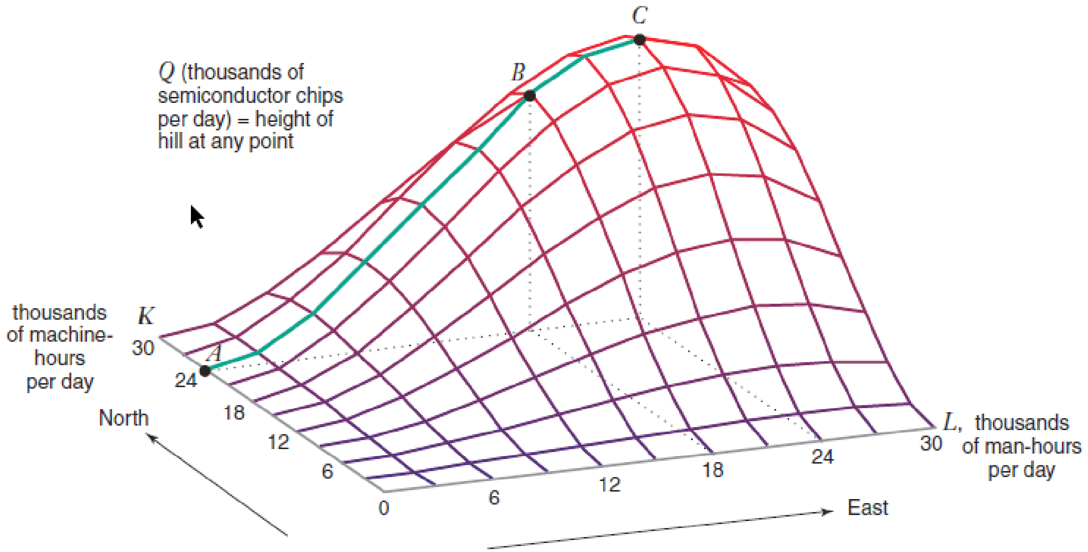


Production Functions with Two Inputs

- Marginal product: change in total product holding other inputs fixed
- $MP_L = \frac{Change\ in\ the\ quantity\ of\ output, Q}{Change\ in\ the\ quantity\ of\ labor, L}$ where K is constant
- $MP_L = \frac{\Delta Q}{\Delta L}$ where K is held constant
- $MP_K = \frac{\Delta Q}{\Delta K}$ where L is held constant

Production Function and Total Product Hill for Semiconductors

		K**					
		0	6	12	18	24	30
	0	0	0	0	0	0	0
	6	0	5	15	25	30	23
	12	0	15	48	81	96	75
L**	18	0	25	81	137	162	127
	24	0	30	96	162	192	150
	30	0	23	75	127	150	117

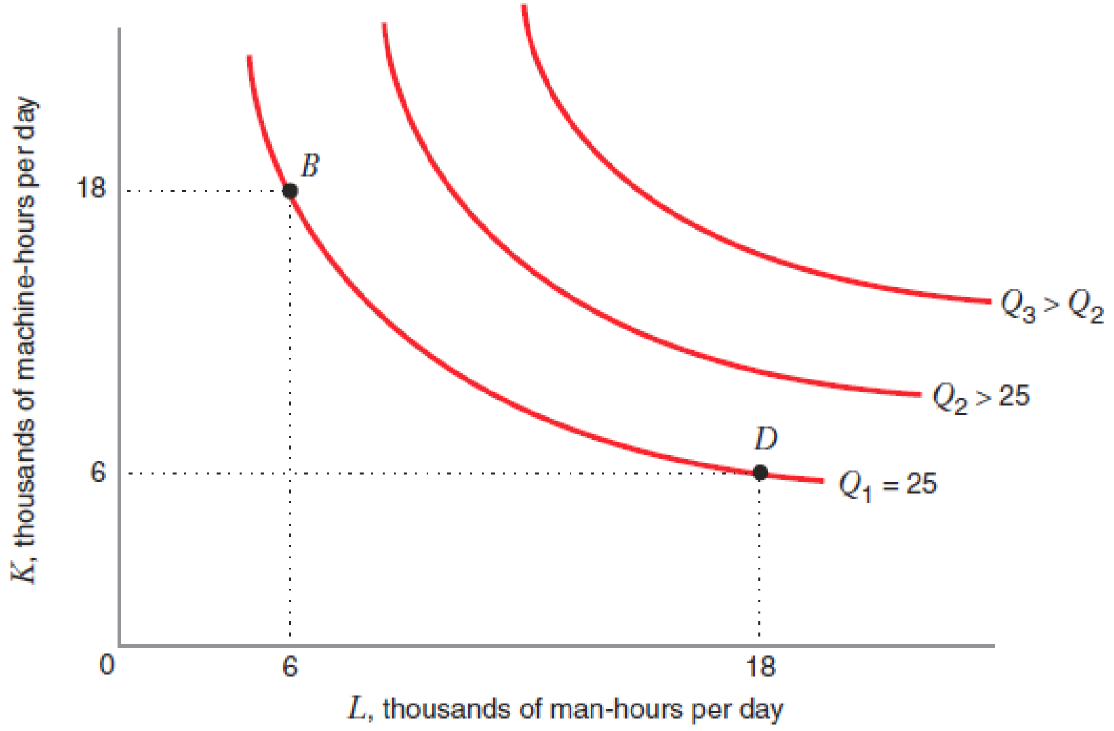


Isoquants

- An isoquant traces out all the combinations of inputs (labor and capital) that allow that firm to produce the same quantity of output
- Example: when $Q = K^{1/2}L^{1/2}$, what is the equation of the isoquant for Q=20?
  - $Q = 20 = K^{1/2}L^{1/2} \rightarrow 400 = KL \rightarrow K = {}^{400}/_{L}$

Isoquants Graphically

		K**					
		0	6	12	18	24	30
L**	0	0	0	0	0	0	0
	6	0	5	15	25	30	23
	12	0	15	48	81	96	75
	18	0	25	81	137	162	127
	24	0	30	96	162	192	150
	30	0	23	75	127	150	117



Marginal Rate of Technical Substitution

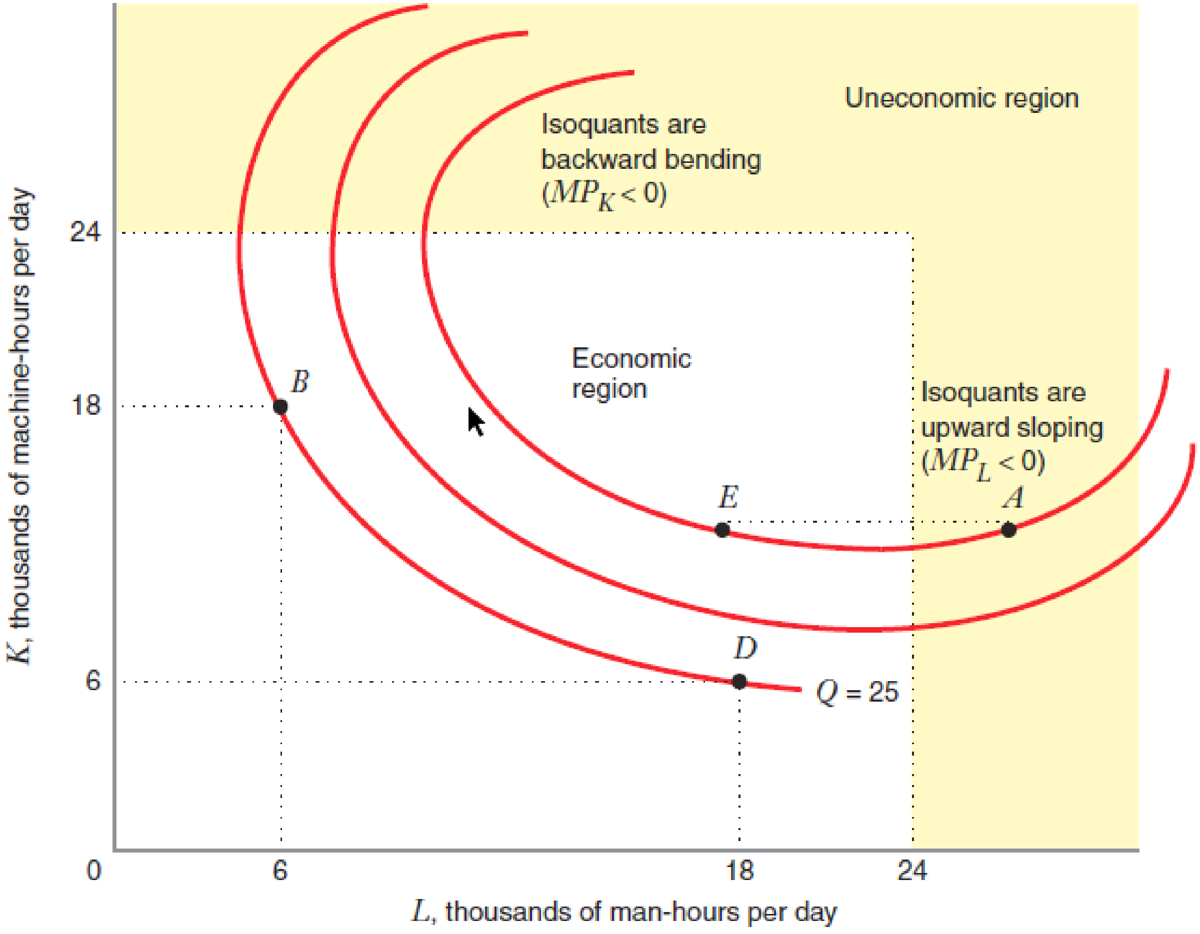
- The marginal rate of technical substitution measures the amount of an input, L, the firm would require in exchange for using a little less of another input, K, in order to just be able to produce the same output as before
- $MRTS_{L,K} = -\Delta K/\Delta L$ (for a constant level of output)
- Marginal products and the MRTS are related:
  - $MP_L(\Delta L) + MP_K(\Delta K) = 0 \Rightarrow$
  - $MP_L/MP_K = -\Delta K/\Delta L = MRTS_{L,K}$

Marginal Rate of Technical Substitution

Continued

- The rate at which the quantity of capital that can be decreased for every unit of increase in the quantity of labor, holding the quantity of output constant, or
- The rate at which the quantity of capital that can be increased for every unit of decrease in the quantity of labor, holding the quantity of output constant
- If both marginal products are positive, the slope of the isoquant is negative
- If we have diminishing marginal returns, we also have a diminishing marginal rate of technical substitution isoquants are convex to the origin
  - The marginal rate of technical substitution of labor for capital diminishes as the quantity of labor increases, along an isoquant
  - For many production functions, marginal products eventually become negative

Economic and Uneconomic Regions of Production

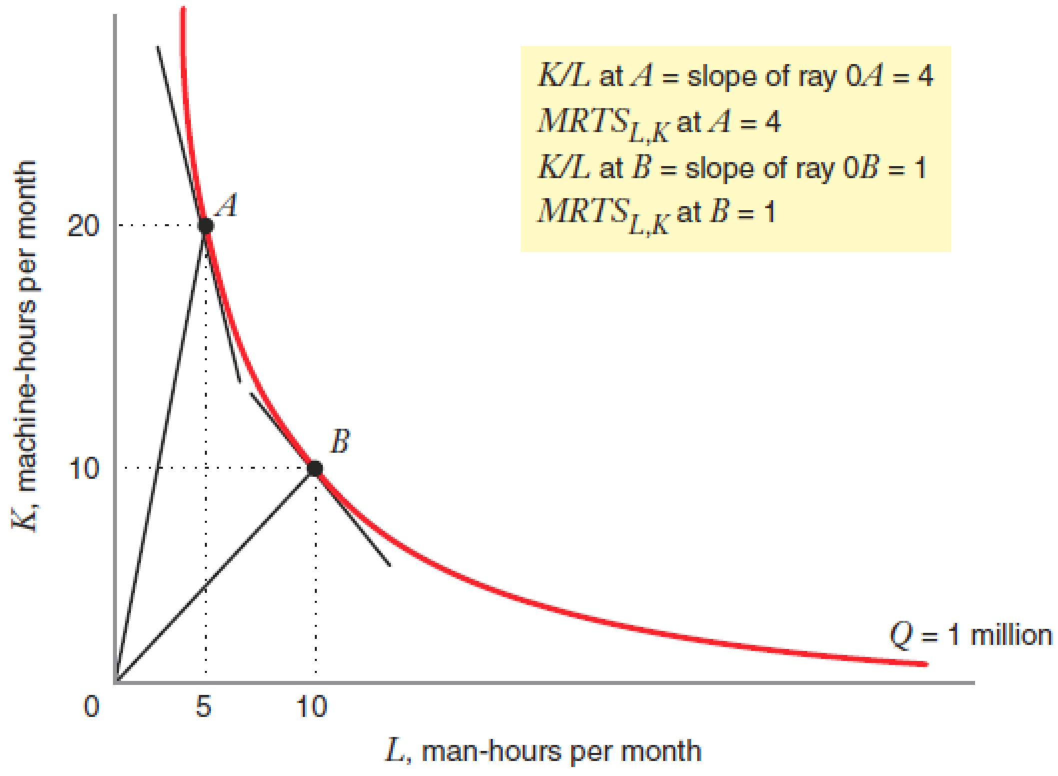


Elasticity of Substitution

- A measure of how easy is it for a firm to substitute labor for capital
- It is the percentage change in the capital-labor ratio for every one percent change in the $MRTS_{L,K}$ along an isoquant
- The elasticity of substitution, σ, measures how the capital-labor ratio, K/L, changes relative to the change in the $MRTS_{L,K}$
- $\sigma = \frac{Percentage\ change\ in\ capital - labor\ ratio}{Percentage\ change\ in\ MRTS_{L,K}} = \frac{\%\Delta\left({}^{K}/_{L}\right)}{\%\Delta\left(MRTS_{L,K}\right)}$

Example of Elasticity of Substitution of Labor for Capital

- $MRTS^A_{L,K} = 4, {}^{K^A}/_{L^A} = 4$
- $MRTS^B_{L,K} = 1, {}^{K^A}/_{L^A} = 1$
- $\sigma = \left[{}^{\Delta\left({}^{K}/_{L}\right)}/_{\Delta MRTS_{L,K}}\right]\left[{}^{MRTS_{L,K}}/_{\left({}^{K}/_{L}\right)}\right]$
- $\sigma = \left({}^{-3}/_{-3}\right)\left({}^{4}/_{4}\right)$=1

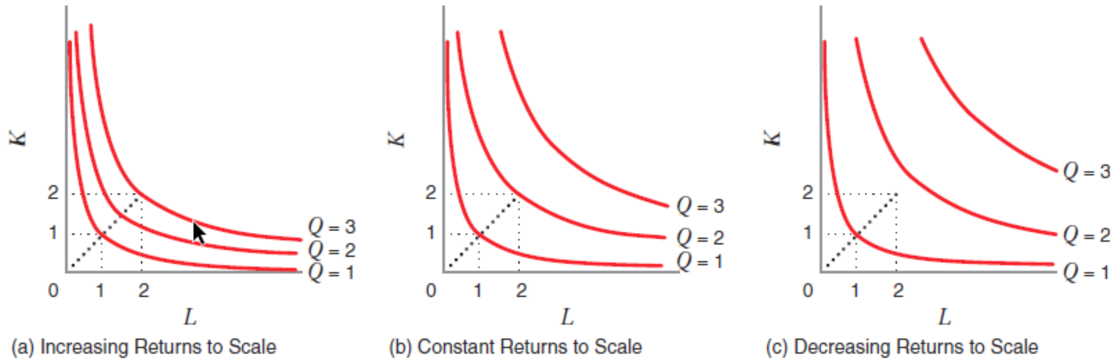


Returns to Scale

- How much will output increase when all inputs increase by a particular amount
- $Returns\ to\ scale = \dfrac{\%\Delta(quantity\ of\ output)}{\%\mathrm{A}(quantity\ of\ all\ inputs)}$
- Let λ be the amount by which both inputs, labor and capital, increase
  - $Q = f(\lambda L, \lambda K)\ for\ \lambda > 1$
- Let Φ represent the resulting proportionate increase in output, Q
  - Increasing returns: $\varphi > \lambda$
  - Decreasing returns: $\varphi < \lambda$
  - Constant returns: $\varphi = \lambda$

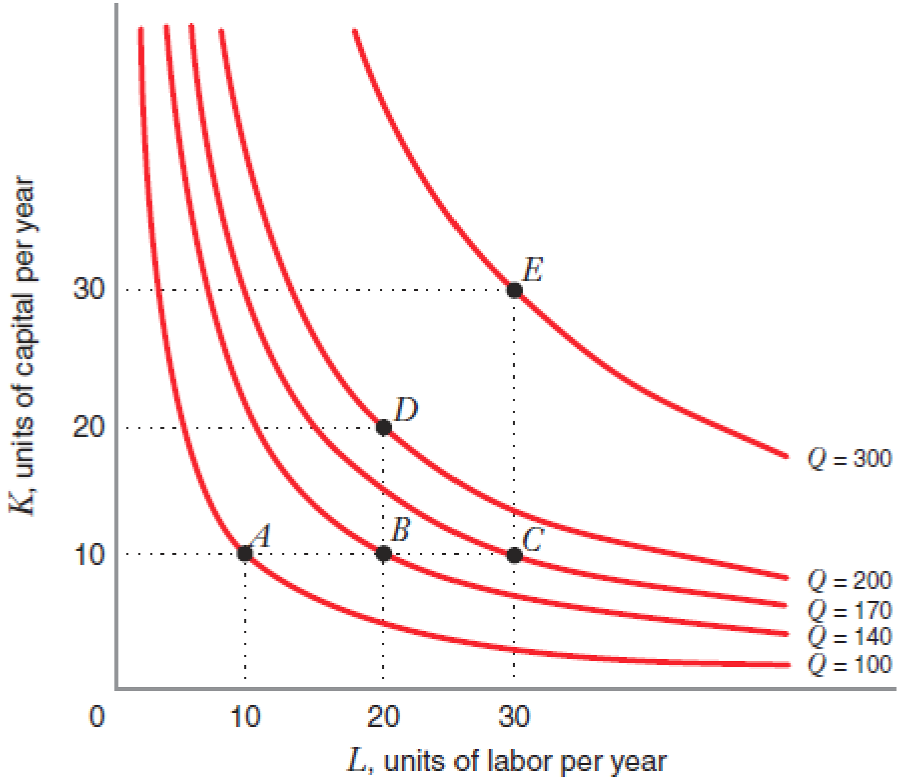
Increasing, Constant, and Decreasing Returns to Scale

- If a 1% increase in all inputs results in a greater than 1% increase in output, then the production function exhibits increasing returns to scale
- If a 1% increase in all inputs results in exactly a 1% increase in output, then the production function exhibits constant returns to scale
- If a 1% increase in all inputs results in a less than 1% increase in output, then the production function exhibits decreasing returns to scale



Returns to Scale versus Marginal Returns

- Returns to scale: all inputs are increased simultaneously
- Marginal returns: increase in the quantity of a single input holding all others constant
- The marginal product of a single factor may diminish while the returns to scale do not
- Returns to scale need not be the same at different levels of production

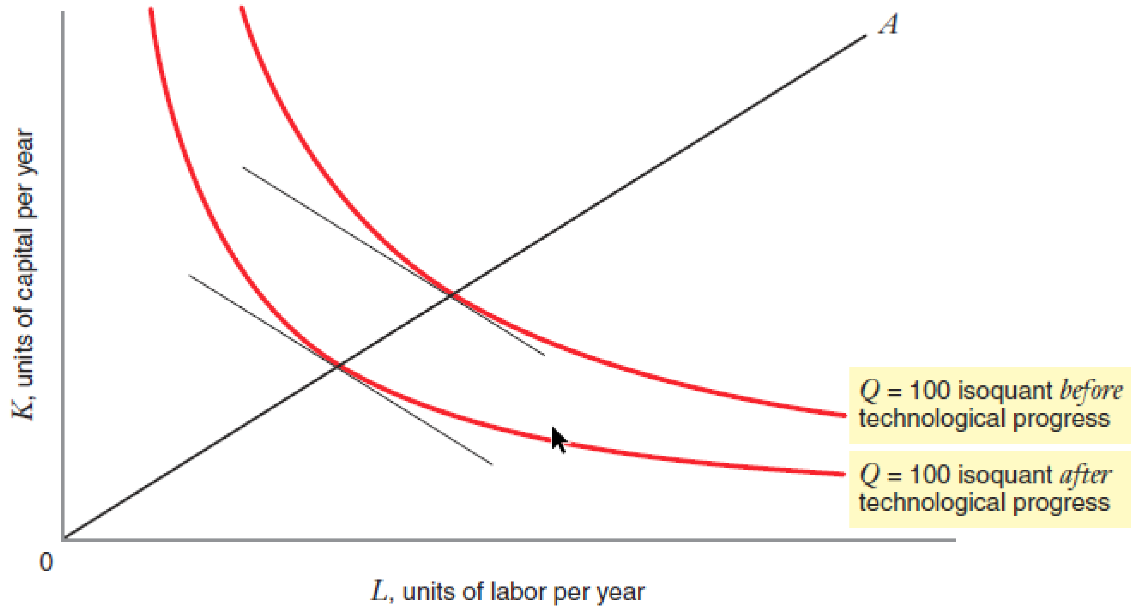


Technological Progress

- Technological progress (or invention) shifts the production function by allowing the firm to achieve more output from a given combination of inputs
  - Or the same output with fewer inputs
- Labor saving technological progress results in a fall in the $MRTS_{L,K}$ along any ray from the origin
- Capital saving technological progress results in a rise in the $MRTS_{L,K}$ along any ray from the origin

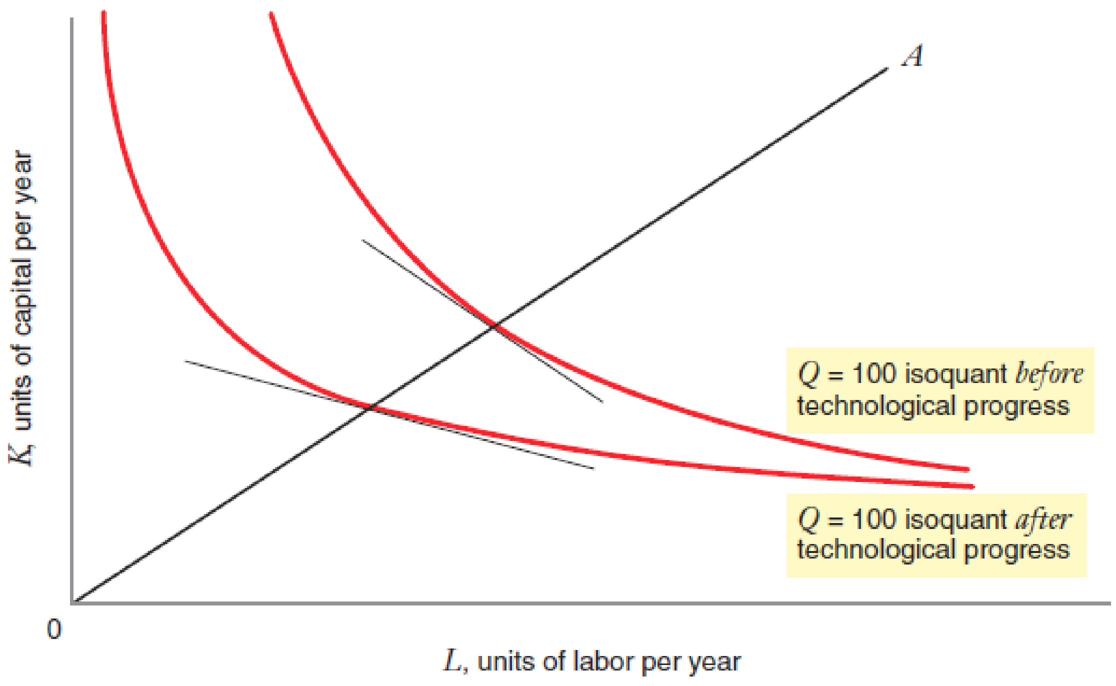
Neutral Technological Progress

- Technological progress that decreases the amounts of labor and capital needed to produce a given output
- $MRTS_{K,L}$ remains the same
- An isoquant corresponding to any particular level of output shifts inward, but the $MRTS_{L,K}$ along any ray from the origin, such as 0A, remains the same



Labor Saving Technological Progress

- Technological progress that causes the marginal product of capital to increase relative to the marginal product of labor
- An isoquant corresponding to any particular level of output shifts inward
- $MRTS_{L,K}$ along any ray from the origin goes down



Capital Saving Technological Progress

- Technological progress that causes the marginal product of labor to increase relative to the marginal product of capital
- An isoquant corresponding to any particular level of output shifts inward
- $MRTS_{L,K}$ along any ray from the origin goes up