



# Introductory Financial Econometrics

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Road Map of this class:





## 1. Financial Time Series and Their Characteristics

Financial time series (FTS) analysis

Financial time series (FTS) analysis is concerned with theory and practice of asset valuation over time.

What is the difference, if any, from traditional time series analysis?

Two topics are highly related, but FTS has added uncertainty, because it must deal with the ever-changing business & economic environment and the fact that volatility is not directly observed.

### 1.1 The Objectives of this chapter

1. to access financial data online and to process the embedded information
2. to provide basic knowledge of FTS data such as skewness, heavy tails, and measure of dependence between asset returns
3. to introduce statistical tools econometric models useful for analyzing these series.
4. to gain experience in analyzing FTS

### 1.2 Examples of financial time series

1. Daily log returns of Apple stock: 2007 to 2018 (12 years). Data downloaded using quantmod

2. The VIX index.
3. CDS spreads: Daily 3-year CDS spreads of JP Morgan from July 20, 2004 to September 19, 2018.
4. Quarterly earnings of Coca-Cola Company: 1983-2009 Seasonal time series useful in
  - earning forecasts
  - pricing weather related derivatives (e.g. energy) • modeling intraday behavior of asset returns
5. US monthly interest rates (3m & 6m Treasury bills)  
Relations between the two asset returns? Term structure of interest rates.
6. Exchange rate between US Dollar vs Euro Fixed income, hedging, carry trade.
7. Size of insurance claims.
8. High-frequency financial data:  
Tick-by-tick data of Caterpillars stock: January 04, 2010.

### 1.3 Asset Returns

Let  $P_t$  be the price of an asset at time  $t$ , and assume no dividend. One-period simple return: Gross return

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

One-Period Simple Net Return or Simple Return:

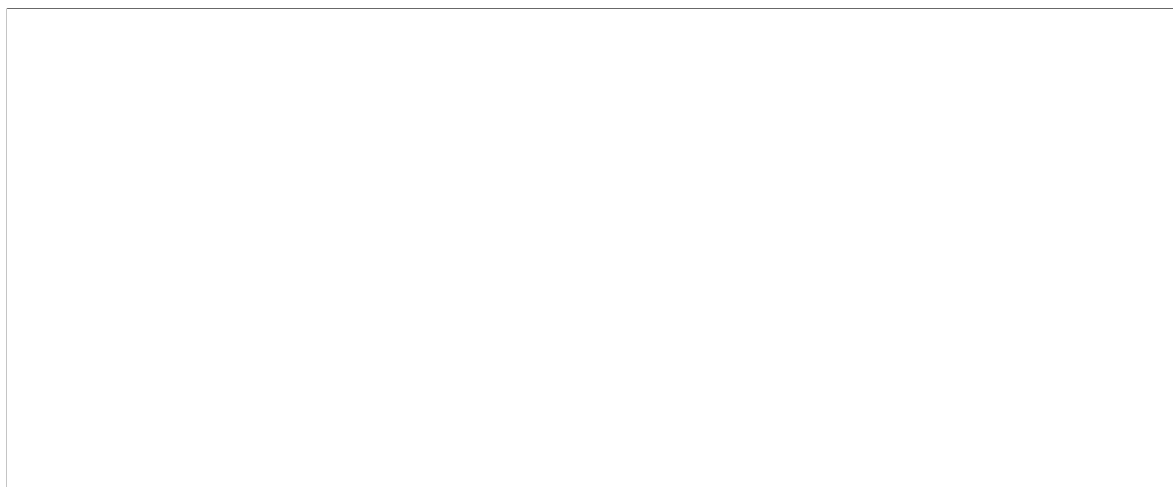
Multiperiod simple return: Gross return)

Example: Table below gives six daily (adjusted) closing prices of Apple stock in December 2015.

Date	Price
12/23	108.02
12/24	107.45
12/28	106.24
12/29	108.15
12/30	106.74
12/31	104.69

what is one-day gross return of holding the stock from 12/28 to 12/29 and the daily simple return?

Time interval is important! Default is one year. Annualized (average) return:



Besides the simple return, we can also compute the continuously compounding interest rate where  $r$  is the interest rate per annum,  $C$  is the initial capital,  $n$  is the number of years, and  $\exp$  is the exponential function.

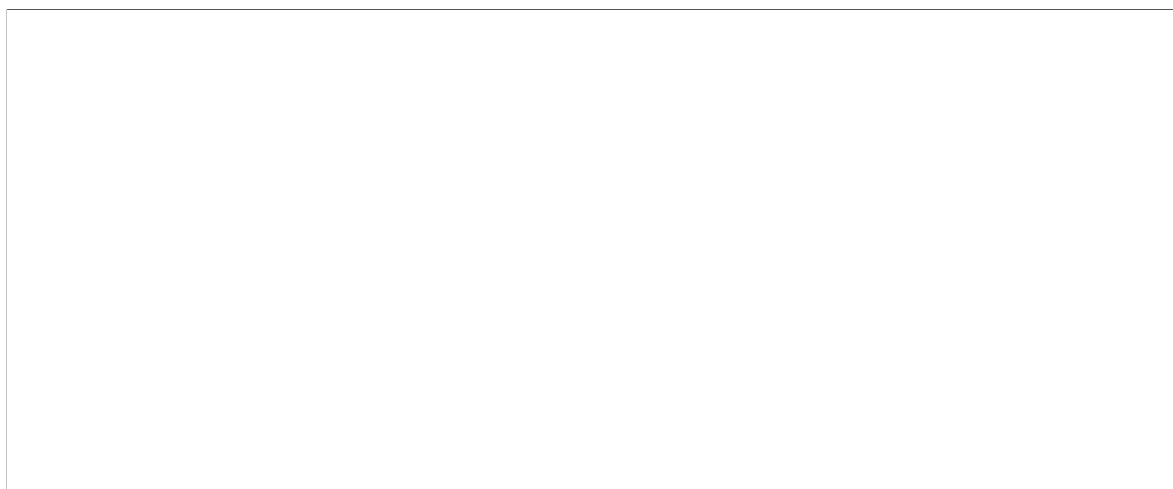
$$A = C \times \exp(r \times n)$$

Continuously compounded (or log) return

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1}$$

where  $p_t = \ln(P_t)$

Multiperiod log return:



Continuously compounding: Illustration of the power of compounding (int. rate 10 % per annum)

Type	#(payment)	Int.	Net
Annual	1	0.1	\$1.10000
Semi-Annual	2	0.05	\$1.10250
Quarterly	4	0.025	\$1.10381
Monthly	12	0.0083	\$1.10471
Weekly	52	$\frac{0.1}{52}$	\$1.10506
Daily	365	$\frac{0.1}{365}$	\$1.10516
Continuously	$\infty$		\$1.10517

Portfolio return: N assets

Dividend payment:

Excess Returns (adjusting for risk)

Example

1. What is the log return from 12/23 to 12/24?

Remarks:

A large, empty rectangular box with a thin black border, intended for the user to write their remarks. It occupies the majority of the page's vertical space below the 'Remarks:' label.

Example If the monthly log returns of an asset are 4.46 %, -7.34 % and 10.77 %, then what is the corresponding quarterly log return?

Example If the monthly simple returns of an asset are 4.46 %, -7.34 %, and 10.77 %, then what is the corresponding quarterly simple return?

## 1.4 Distributional Properties of Returns

What is the distribution of  $r_{it}$  where  $i = 1, \dots, N$ ; and  $t = 1, \dots, T$

Some theoretical properties:

Moments of a random variable  $X$  with density  $f(x)$ :  $l$ -th moment

$$m'_l = E(X^l) = \int_{-\infty}^{\infty} x^l f(x) dx$$

First Moment: mean or expectation of  $X$ .

$l$ -th central moment

$$m_l = E[(X - \mu_x)^l] = \int_{-\infty}^{\infty} (x - \mu_x)^l f(x) dx$$

2nd central moment.

Standard deviation: square-root of variance

Skewness (Symmetry)

$$S(x) = E \left[ \frac{(X - \mu_x)^3}{\sigma_x^3} \right]$$

Kurtosis (Fat-tails)

$$K(x) = E \left[ \frac{(X - \mu_x)^4}{\sigma_x^4} \right]$$

Q1: Why study the mean and variance of returns?

They are concerned with long-term return and risk, respectively.

Q2: Why is symmetry important?

Symmetry has important implications in holding short or long financial positions and in risk management.

Q3: Why is kurtosis important?

Related to volatility forecasting, efficiency in estimation and tests High kurtosis implies heavy (or long) tails in distribution.

Estimation

Sample mean, Sample Variance, Sample Skewness and Sample Kurtosis



**1.5 Hypothesis Testing**

A random variable under the normal distribution

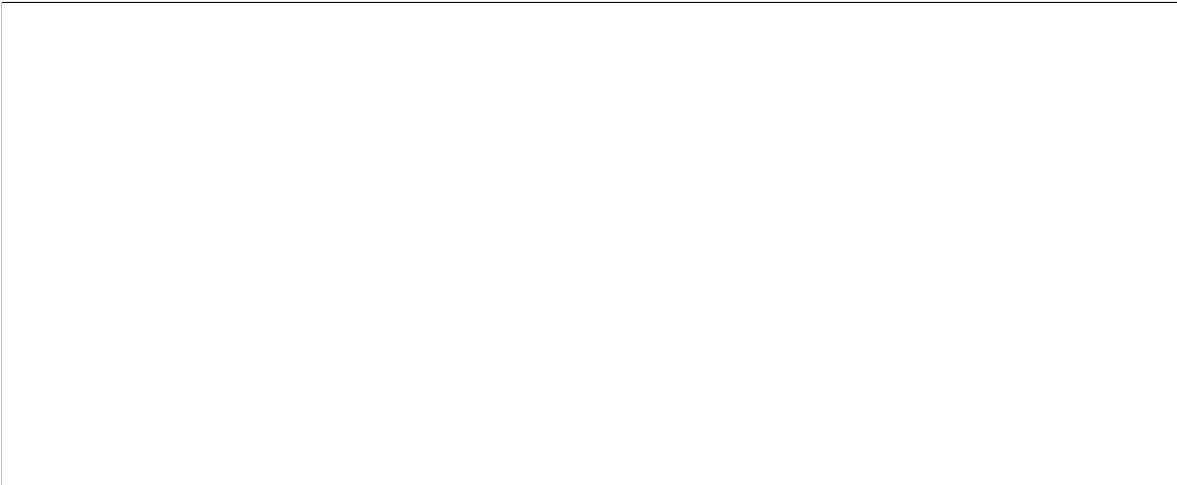
$$\widehat{S}(x) \sim N\left(0, \frac{6}{T}\right)$$

$$\widehat{K}(x) - 3 \sim N\left(0, \frac{24}{T}\right)$$

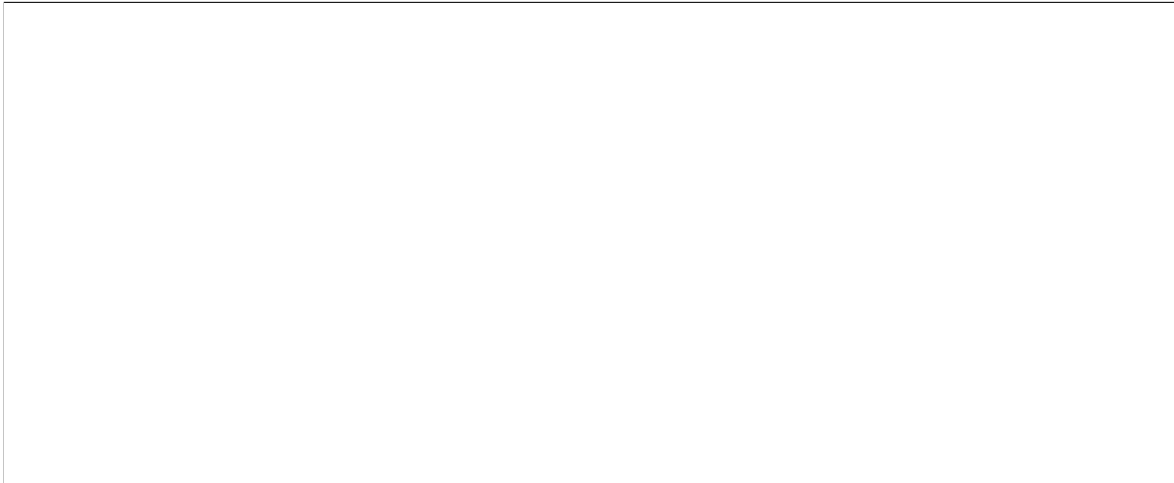
Test for symmetry



Test for tail thickness



Test for normality :(Jarque-Bera test)



## 1.6 Empirical work using R program

### FE Toolbox

```
#EE435 Wasin Siwasarit Lecture1 Spring/2018
setwd("/Users/wasinsiwasarit/Desktop/EE435")
cat(rep("\n",50)) #clear R Console
#install.packages("quantmod")
#install.packages("fBasics")
#install.packages("sn")
#install.packages("PerformanceAnalytics")
#install.packages("car")
#install.packages("tseries")
#install.packages("forecast")
library(quantmod)
library(fBasics)
library(sn)
library(PerformanceAnalytics)
library(car)
library(tseries)
library(forecast)

getSymbols("^GSPC",from="2000-01-03",to="2017-01-28")
dim(GSPC)
head(GSPC)
tail(GSPC)
da=GSPC
chartSeries(GSPC,theme="white")
price=da[,6]
plot(price,type='l')
```

```
logprice=log(price)
plot(logprice,type='l')
logreturn=diff(log(price))
simplereturn <-exp(logreturn)-1
#1 Plot the series of log return and simple return

par(mfrow=c(1,1))
plot(logreturn,type='l')
plot(simplereturn)

newlogreturn <- logreturn[2:nrow(logreturn),]
newsimplereturn <- simplereturn[2:nrow(logreturn),]

#2 Histogram and sample statistics
hist(logreturn, breaks=100, col="slateblue")
chart.Histogram(logreturn,methods = c("add.normal"))
table.Stats(logreturn)

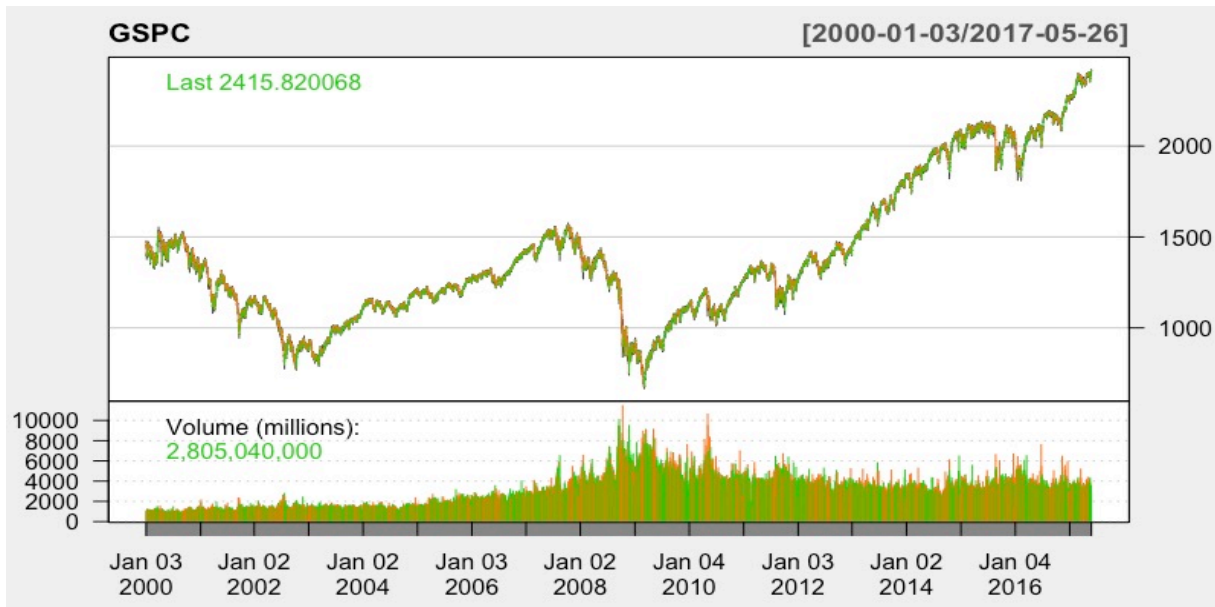
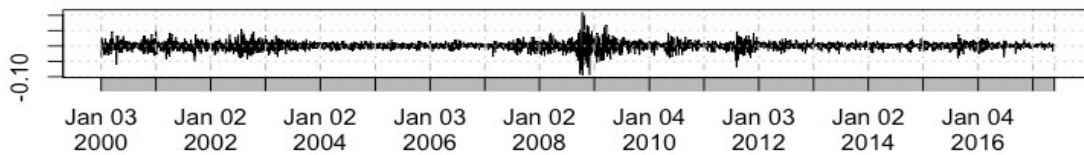
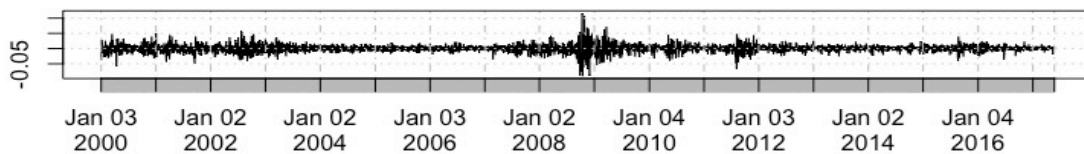
#3 QQ-plots and tests for normality
#
# use qqnorm function
par(mfrow=c(1,1))
qqnorm(newlogreturn)
qqline(newlogreturn, col = 2)
jarque.bera.test(newlogreturn)
```

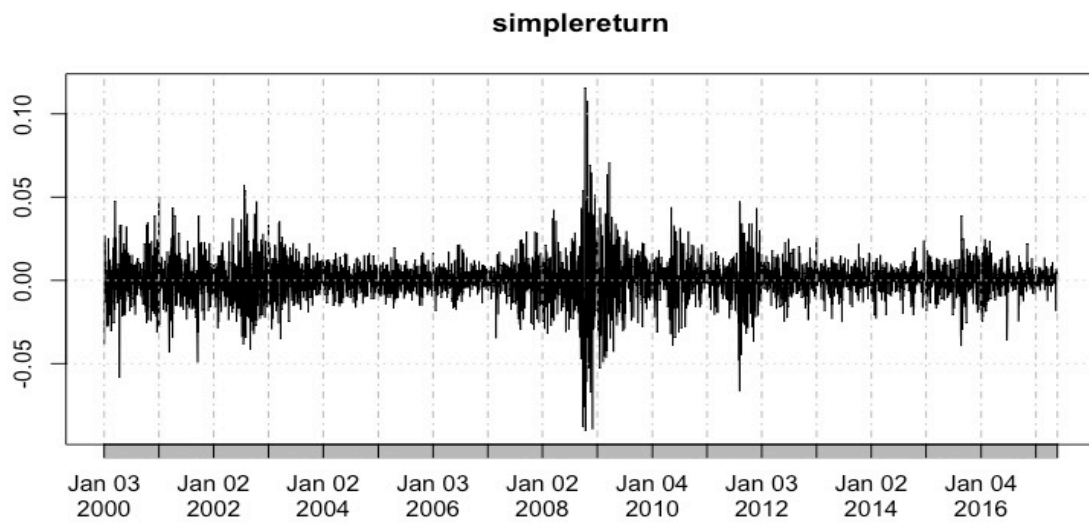
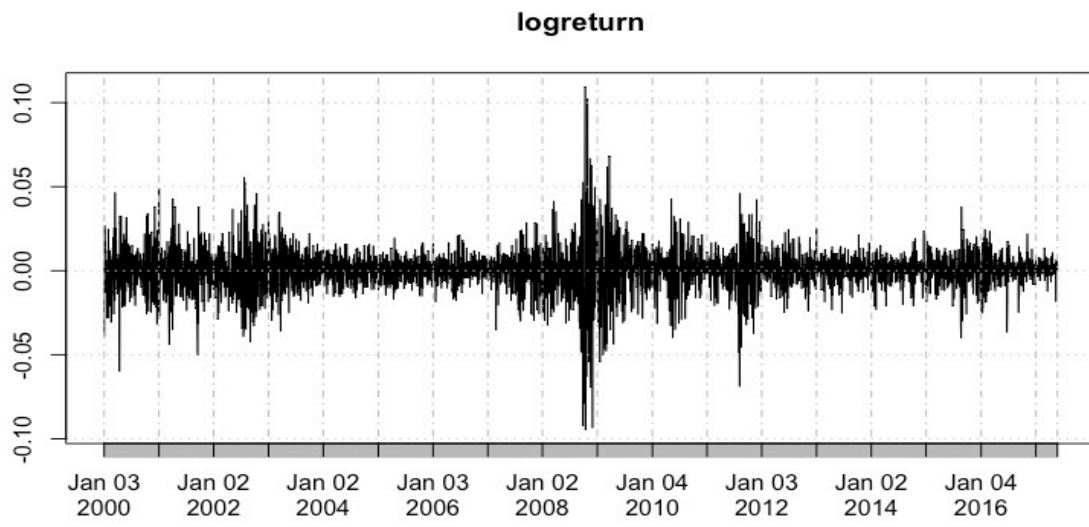
## FE Analysis

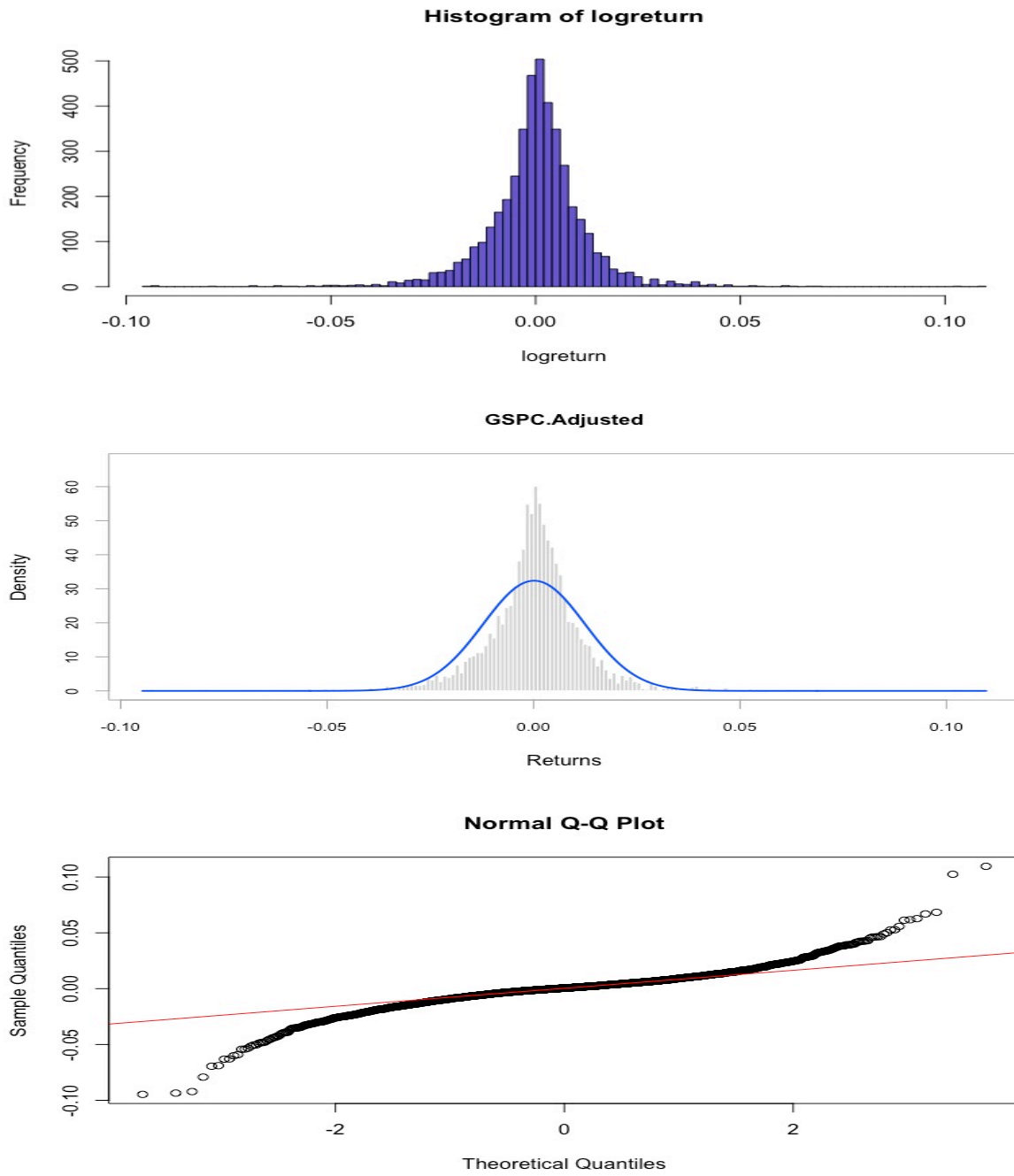
```
> table.Stats(logreturn)
              GSPC.Adjusted
Observations  4377.0000
NAs           1.0000
Minimum       -0.0947
Quartile 1    -0.0051
Median        0.0005
Arithmetic Mean 0.0001
Geometric Mean 0.0000
Quartile 3    0.0058
Maximum       0.1096
SE Mean       0.0002
LCL Mean (0.95) -0.0002
UCL Mean (0.95) 0.0005
Variance      0.0002
Stdev         0.0123
Skewness      -0.1989
Kurtosis      8.3611
> par(mfrow=c(1,1))
> qqnorm(newlogreturn)
> qqline(newlogreturn, col = 2)
> jarque.bera.test(newlogreturn)
```

Jarque Bera Test

```
data: newlogreturn
X-squared = 12778, df = 2, p-value < 2.2e-16
```

**logreturn****simplereturn**





## FE toolbox (Cont.)

```
#4 Test mean = 0
t.test(newlogreturn)

#5 Test Skewness = 0
T=length(newlogreturn)
s3=skewness(newlogreturn)
tst = s3/sqrt(6/T)
tst
pv = 2*pnorm(tst)
pv

#6 Test excess kurtosis =0
k4 = kurtosis(newlogreturn)
tst = k4/sqrt(24/T)
tst
pv = 2*(1-pnorm(tst))
pv
```

## FE Analysis (Cont.)

```
> t.test(newlogreturn)

      One Sample t-test

data:  newlogreturn
t = 0.62168, df = 4376, p-value = 0.5342
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.0002493954  0.0004810069
sample estimates:
 mean of x
0.0001158057

> T=length(newlogreturn)
> s3=skewness(newlogreturn)
> tst = s3/sqrt(6/T)
> tst
[1] -5.370912
> pv = 2*pnorm(tst)
> pv
[1] 7.833935e-08
> k4 = kurtosis(newlogreturn)
> tst = k4/sqrt(24/T)
> tst
```

```
[1] 112.913
> pv = 2*(1-pnorm(tst))
> pv
[1] 0
>
```



## 2. Linear Time Series (TS) Models

### 2.1 Basic Concepts

Financial TS: collection of a financial measurement over time.

Example: log return of apple  $r_t$

Data:  $\{r_1, r_2, \dots, r_T\}$

Purpose What is the information contained in series of  $r_t$

Definition: Stationarity

-Strict: Distributions are time-invariant

-Weak: First 2 moments are time-invariant

What does weak stationarity mean in practice?

Past: time plot of  $r_t$  varies around a fixed level within a finite range!

Future: the first 2 moments of future  $r_t$  are the same as those of the data so that meaningful inferences can be made.

Mean (or expectation) of returns

$$\mu = E(r_t)$$

Variance (variability) of returns

$$\text{Var}(r_t) = E[(r_t - \mu)^2]$$

Sample mean and Sample Variance are used to estimate the mean and variance of returns.

$$\bar{r} = \frac{1}{T} \sum_{t=1}^T r_t$$

$$\text{Var}(r_t) = \frac{1}{T-1} \sum_{t=1}^T (r_t - \bar{r})^2$$

testing the mean of  $r_t$  is different from zero or not

$$H_0 : \mu = 0$$

$$H_a : \mu \neq 0$$

$$t_{cal} =$$

Decision rule: Reject  $H_0$  if  $|t| > Z_{\frac{\alpha}{2}}$  or p-value is less than  $\alpha$

Lag-kk autocovariance:

$$\gamma_k = \text{Cov}(r_t, r_{t-k}) = E[(r_t - \mu)(r_{t-k} - \mu)]$$

Serial (or-auto) correlations:

$$\rho_l = \frac{\text{cov}(r_t, r_{t-l})}{\text{var}(r_t)}$$

Remark The existence of serial correlation in  $r_t$  implies that.....

Sample Autocorrelation function (AFC) can be computed by:

$$\hat{\rho}_l = \frac{\sum_{t=1}^{T-l} (r_t - \bar{r})(r_{t+l} - \bar{r})}{\sum_{t=1}^T (r_t - \bar{r})^2}$$

## Test Zero Serial Correlations (Market Efficiency)

## 1. Individual Test

$$H_0: \rho_1 = 0$$

$$H_a: \rho_1 \neq 0$$

$$t_{cal} =$$

Decision rule: Reject the null hypothesis when  $|t| > Z_{\frac{\alpha}{2}}$  or the p-value has the value less than  $\alpha$

## 2. Joint Test (Ljung-Box Statistics):

$$H_0: \rho_1 = \dots = \rho_m = 0$$

$$H_a: \rho_i \neq 0$$

$$Q(m) = T * (T + 2) \sum_{l=1}^m \frac{\rho_l^2}{T - l}$$

Decision rule: Reject the null hypothesis when  $Q(m) > \chi_m^2(\alpha)$  or the p-value has the value less than  $\alpha$

## FE toolbox 2

```
#EE435 Wasin Siwasarit
setwd("/Users/wasinsiwasarit/Desktop/EE435")
library(fBasics)
cat(rep("\n",50)) #clear R Console
da <- read.table("CRSP.txt")
log_return =da[,1]
par(mfcol=c(1,1))
length(log_return)
tdx = c(1:456)/12+1961
plot(tdx, log_return, xlab='year', ylab='log_return', type='l')
basicStats(log_return)
normalTest(log_return, method="jb")
t.test(log_return)
tt1 = skewness(log_return)/sqrt(6/546)
tt1
pv = 2*pnorm(tt1)
pv
tt2 = kurtosis(log_return)/sqrt(24/546)
tt2
```

```

pv = 2*(1-pnorm(tt2))
pv
m1=acf(log_return)
names(m1)
m1$acf
m2=pacf(log_return)
names(m2)
m2$acf
Box.test(log_return,lag=12,type='Ljung')

```

### FE Analysis 2

```

> da <- read.table("CRSP.txt")
> log_return =da[,1]
> par(mfcol=c(1,1))
> length(log_return)
[1] 456
> tdx = c(1:456)/12+1961
> plot(tdx,log_return,xlab='year',ylab='log_return',type='l')
> basicStats(log_return)

```

	log_return
nobs	456.000000
NAs	0.000000
Minimum	-31.588000
Maximum	26.175000
1. Quartile	-1.860000
3. Quartile	4.268250
Mean	1.059511
Median	1.494500
Sum	483.137000
SE Mean	0.262245
LCL Mean	0.544149
UCL Mean	1.574873
Variance	31.360270
Stdev	5.600024
Skewness	-0.673271
Kurtosis	4.122884

```

> normalTest(log_return,method="jb")

```

Title:

Jarque - Bera Normalality Test

Test Results:

STATISTIC:

X-squared: 362.5726

```

P VALUE:
  Asymptotic p Value: < 2.2e-16

Description:
  Sat Aug 19 22:51:45 2017 by user:

> t.test(log_return)

      One Sample t-test

data:  log_return
t = 4.0402, df = 455, p-value = 6.27e-05
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.544149 1.574873
sample estimates:
mean of x
 1.059511

> tt1 = skewness(log_return)/sqrt(6/546)
> tt1
[1] -6.443776
> pv = 2*pnorm(tt1)
> pv
[1] 1.165367e-10
> tt2 = kurtosis(log_return)/sqrt(24/546)
> tt2
[1] 19.81441
> pv = 2*(1-pnorm(tt2))
> pv
[1] 0
> m1=acf(log_return)
> names(m1)
[1] "acf"      "type"     "n.used"   "lag"      "series"   "snames"
> m1$acf
, , 1

      [,1]
[1,] 1.000000000
[2,] 0.226356832
[3,] -0.009975215
[4,] -0.038128697
[5,] -0.015760585
...
[26,] -0.012220663
[27,] 0.064944965

```

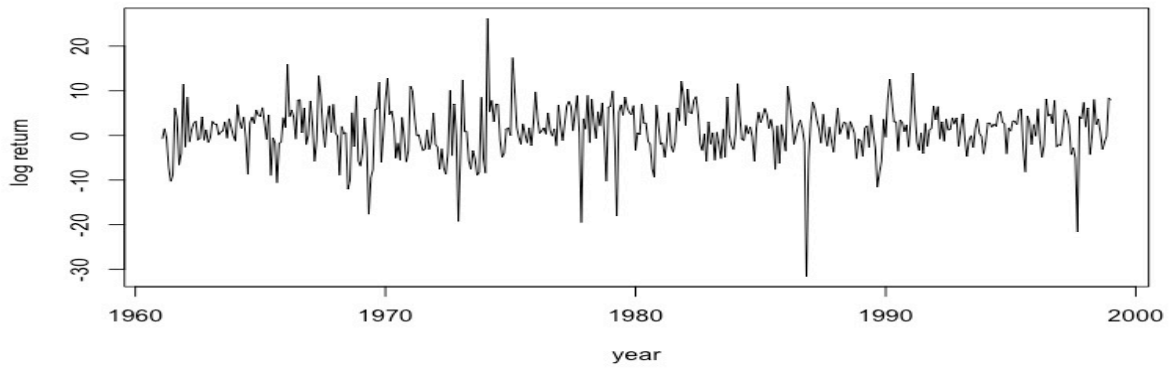
```
> m2=pacf(log_return)
> names(m2)
[1] "acf"      "type"     "n.used"  "lag"     "series"  "snames"
> m2$acf
, , 1

          [,1]
[1,]  0.2263568318
[2,] -0.0645183854
[3,] -0.0223545572
[4,] -0.0022849724
[5,]  0.0116224218
...
[25,] -0.0142841594
[26,]  0.0788941527

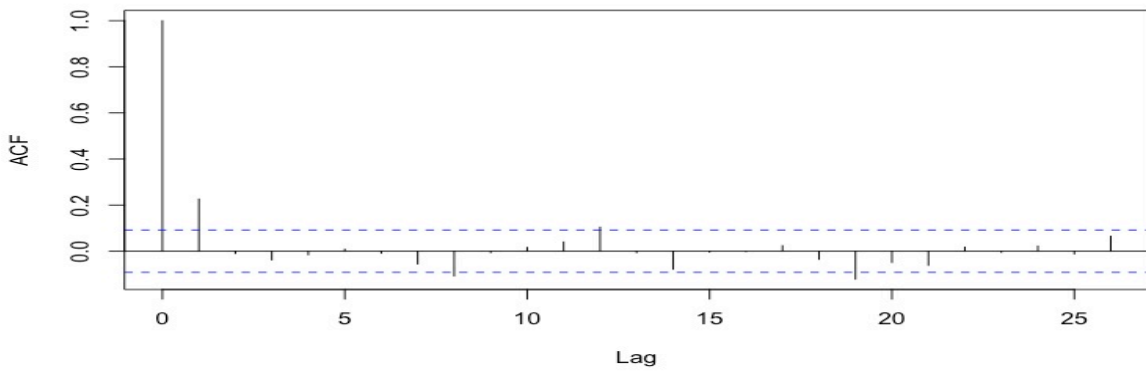
> Box.test(log_return,lag=12,type='Ljung')

      Box-Ljung test

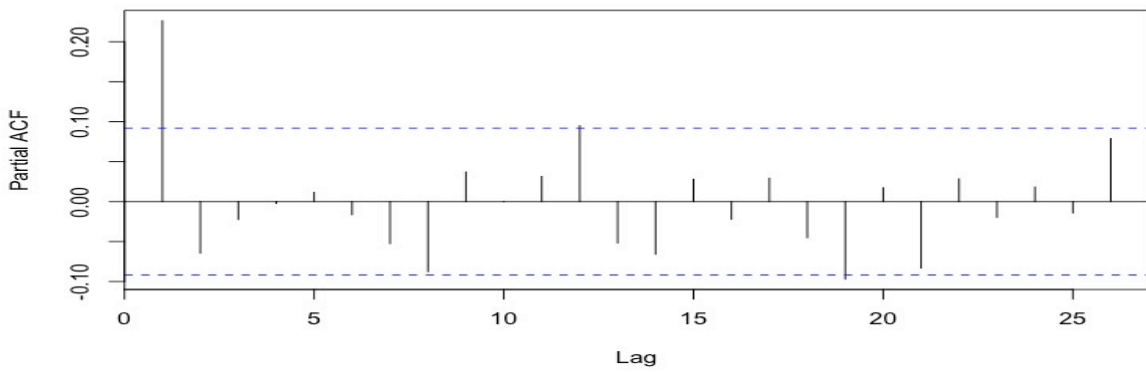
data:  log_return
X-squared = 37.302, df = 12, p-value = 0.0001995
```



Series log\_return



Series log\_return





## 2.2 Back-Shift (lag) operator

Definition  $Br_t = r_{t-1}$  or  $Lr_t = r_{t-1}$

$$B^2r_t = B(Br_t) = Br_{t-1} = r_{t-1}$$

B or L means Time Shift

For example  $Br_t$  is the value of the series at time  $t-1$

For example

The table of log return:

Date	$r_t$
1	0.025
2	0.013
3	-0.003
4	0.035

What is the following value of

$$r_2 =$$

$$Br_3 =$$

$$B^2r_5 =$$

A proper perspective: at a time point  $t$

Available data:  $\{r_1, r_2, \dots, r_{t-1}\} \equiv F_{t-1}$

The return is decomposed into two parts:

$$r_t = \text{predictable part} + \text{not predictable part}$$



Traditional TS modeling is concerned with  $\mu_t$ :

Model for  $\mu_t$  : mean equation

Volatility modeling concerns  $\sigma_t$

Model for  $\sigma_t^2$ : volatility equation

### 2.3 Linear Time Series

$r_t$  is linear if

- . the predictable part is a linear function of  $F_{t-1}$
- .  $\{a_t\}$  are independent and have the same distribution (iid)

Mathematically, it means  $r_t$  can be written as

$$r_t = \mu + \sum_{i=1}^{\infty} \psi_i a_{t-i}$$

where  $\mu$  is constant and  $\psi_0 = 1$  and  $a_t$  is an iid sequence with mean 0 and well-defined distribution.

In the economic literature  $a_t$  is the shock or innovation at time  $t$  and  $\psi_i$  are the impulse responses of  $r_t$ .

White noise: iid sequence (with finite variance), which is the building block of linear TS models. White noise is not predictable, but has zero mean and finite variance.

In EE435 we will study the (Univariate linear time series models) as follow:

1. autoregressive (AR) models
2. moving-average (MA) models
3. mixed ARMA models
4. seasonal models

Important properties of a model

- Stationarity condition
- Basic properties: mean, variance, serial dependence
- Empirical model building: specification, estimation, & checking
- Forecasting

## 2.4 Autoregressive (AR) model

If the series  $r_t$  and  $r_{t-1}$  are correlated, we might be able to use the series  $r_{t-1}$  in forecasting  $r_t$ . Thus the linear model can be:

$$r_t = \phi_0 + \phi_1 r_{t-1} + a_t$$

where  $a_t$  are white noise series with mean equal to 0 and variance equals to  $\sigma_a^2$

The above model is known as AR(1) since the variation of  $r_t$  can be explained by the variation of  $r_{t-1}$ . From this model we can calculate the conditional mean and conditional variance as the follow:



In general, we can write down the model of AR(p) as

$$r_t = \phi_0 + \phi_1 r_{t-1} + \dots + \phi_p r_{t-p} + a_t$$


**2.4.1 Properties of AR models**

AR (1) model

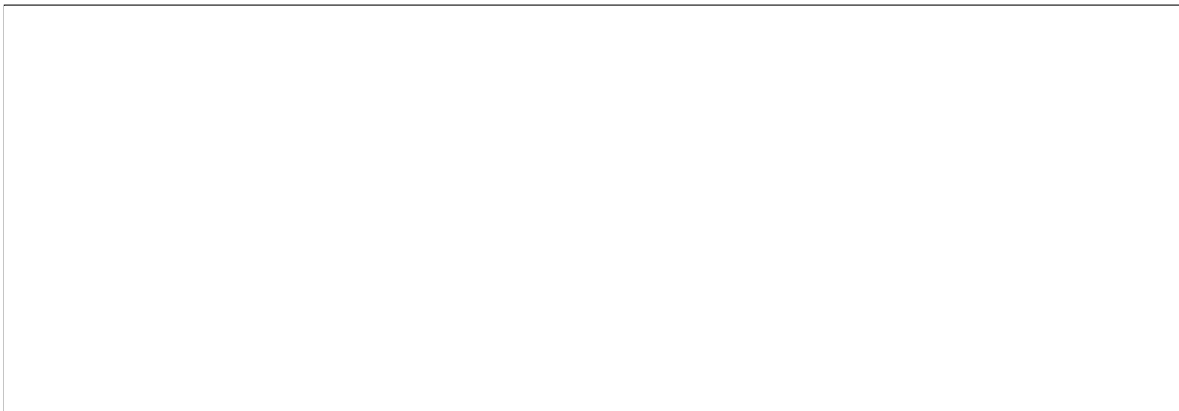
the necessary condition for AR model is that series  $r_t$  have to be weak stationarity.



Unconditional mean



Unconditional variance



Unconditional autocorrelations

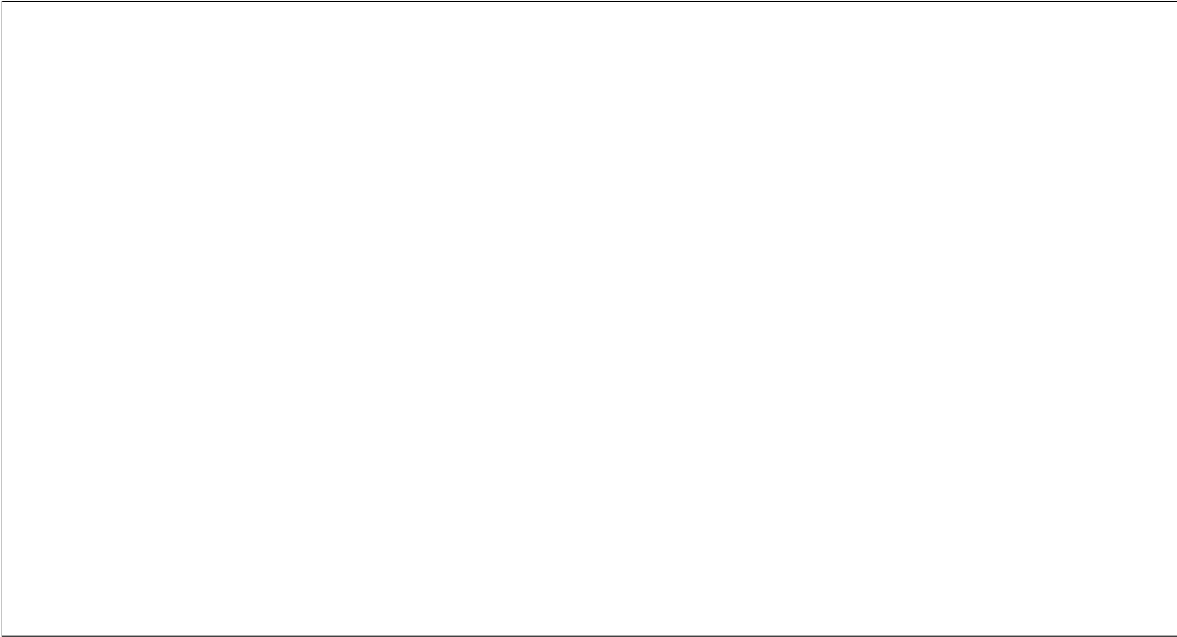
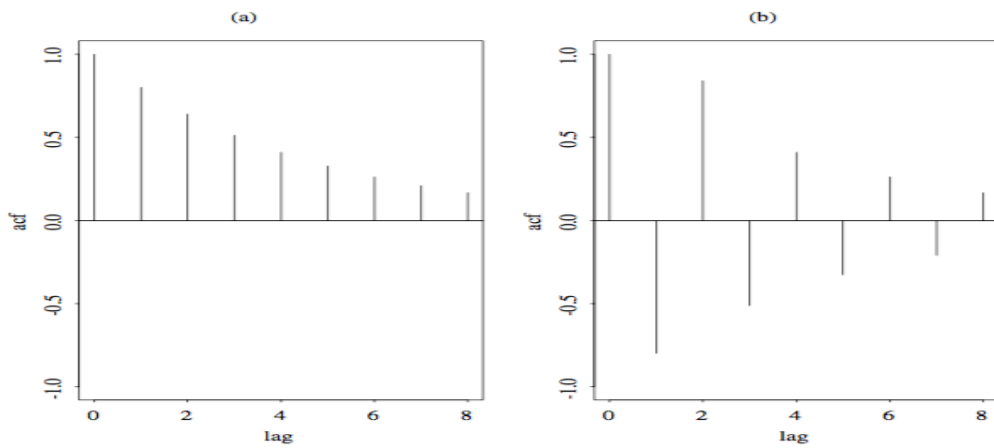


Figure The autocorrelation function of an AR(1) model: (a) for  $\phi_1 = 0.8$ , and (b) for  $\phi_1 = -0.8$ .

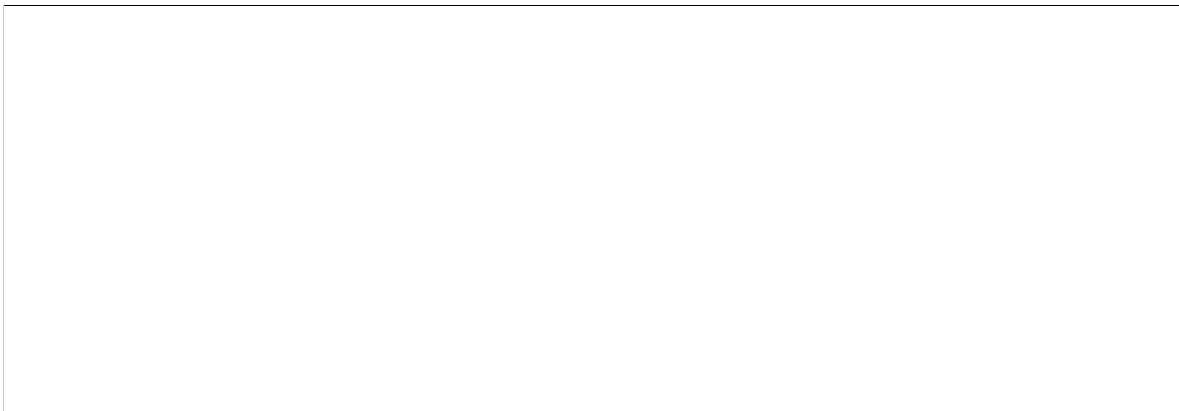


AR (2) model

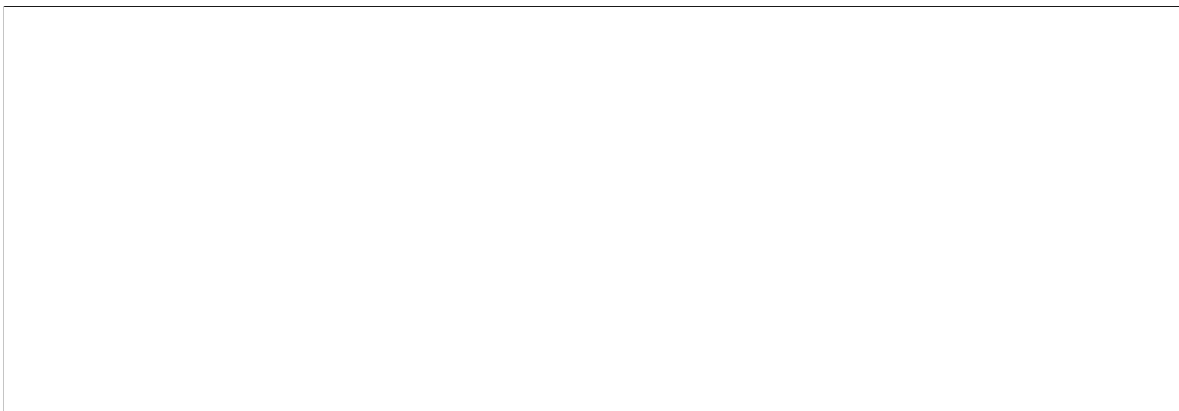
$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + a_t$$



Unconditional mean



Unconditional variance



Unconditional autocorrelations

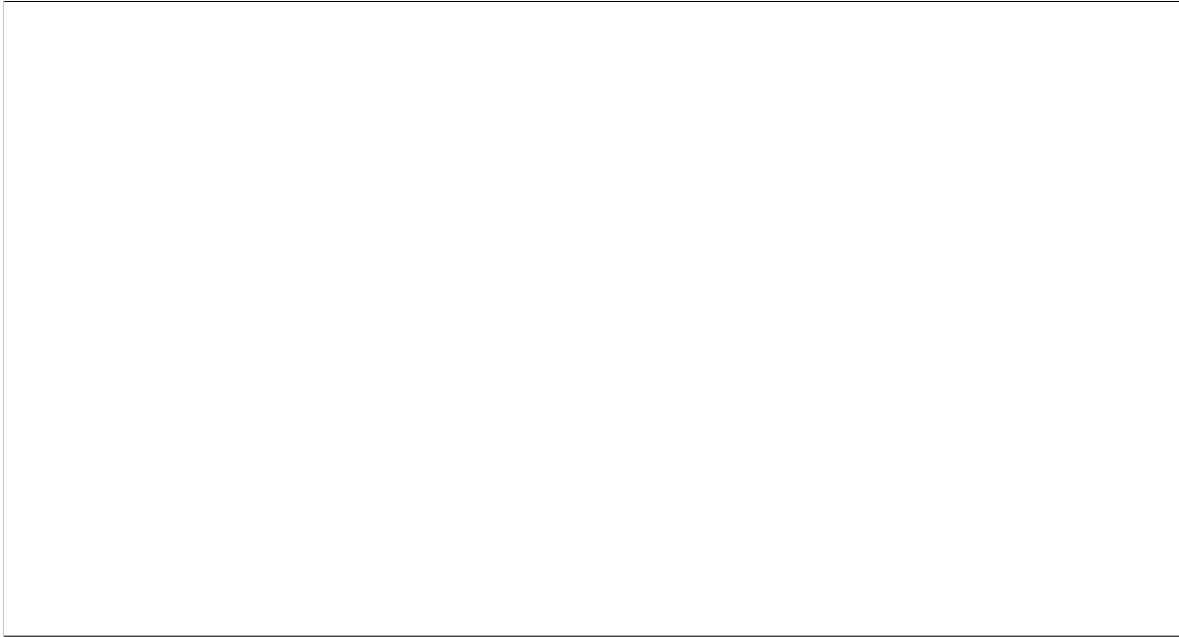
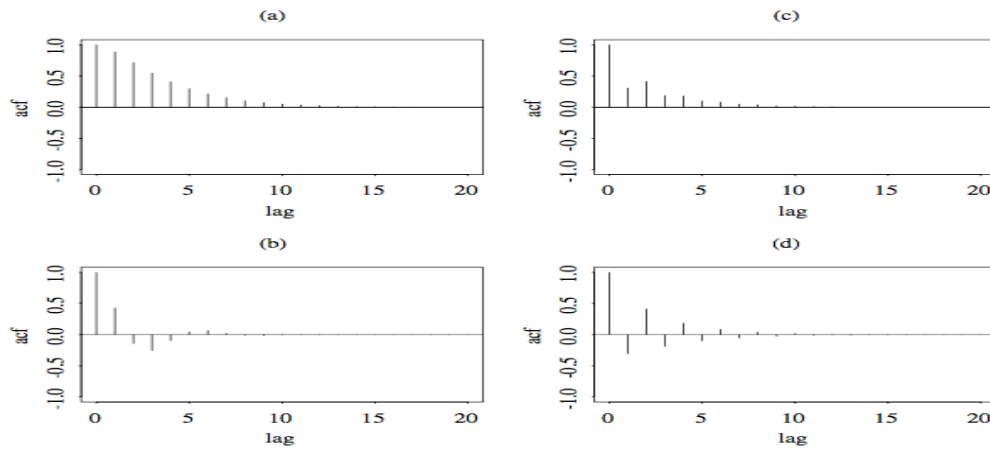


Figure The autocorrelation function of an AR(2) model: (a)  $\phi_1 = 1.2$  and  $\phi_2 = -0.35$ , (b)  $\phi_1 = 0.6$  and  $\phi_2 = -0.4$ , (c)  $\phi_1 = 0.2$  and  $\phi_2 = 0.35$ , (d)  $\phi_1 = -0.2$  and  $\phi_2 = 0.35$ .



### 2.4.2 Stationarity of AR(p) Model

In case of AR(p)

$$r_t = \phi_0 + \phi_1 r_{t-1} + \dots + \phi_p r_{t-p} + a_t$$

We can write down the Unconditional Mean as :

$$E(r_t) = \frac{\phi_0}{1 - \phi_1 - \dots - \phi_p},$$

which the Polynomial equation can be expressed as

$$x^p - \phi_1 x^{p-1} - \phi_2 x^{p-2} - \dots - \phi_p = 0$$

The above equation is also known as Equation in which if Characteristic roots has the value less than 1 in modulus, we can say that the model is stationary.

Moreover, AR(p) , the ACF can be written as difference equation

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) \rho_l = 0$$

where  $l > 0$

The graph of ACF of AR(p) has the pattern as the graph of sine and cosine.