Solution key Lecture#6

CHAPTER 8: INDEX MODELS

PROBLEM SETS

1. The advantage of the index model, compared to the Markowitz procedure, is the vastly reduced number of estimates required. In addition, the large number of estimates required for the Markowitz procedure can result in large aggregate estimation errors when implementing the procedure. The disadvantage of the index model arises from the model's assumption that return residuals are uncorrelated. This assumption will be incorrect if the index used omits a significant risk factor.

7. a. The two figures depict the stocks' security characteristic lines (SCL). Stock A has higher firm-specific risk because the deviations of the observations from the SCL are larger for Stock A than for Stock B. Deviations are measured by the vertical distance of each observation from the SCL.

   b. Beta is the slope of the SCL, which is the measure of systematic risk. The SCL for Stock B is steeper; hence Stock B's systematic risk is greater.

   c. The R^2 (or squared correlation coefficient) of the SCL is the ratio of the explained variance of the stock's return to total variance, and the total variance is the sum of the explained variance plus the unexplained variance (the stock's residual variance):

   $$R^2 = \frac{\beta_i^2 \sigma_M^2}{\beta_i^2 \sigma_M^2 + \sigma^2(e_i)}$$

   Since the explained variance for Stock B is greater than for Stock A (the explained variance is $\beta_B^2 \sigma_M^2$, which is greater since its beta is higher), *and* its residual variance $\sigma^2(e_B)$ is smaller, its R^2 is higher than Stock A's.

   d. Alpha is the intercept of the SCL with the expected return axis. Stock A has a small positive alpha whereas Stock B has a negative alpha; hence, Stock A's alpha is larger.

   e. The correlation coefficient is simply the square root of R^2, so Stock B's correlation with the market is higher.

9. The standard deviation of each stock can be derived from the following equation for R^2:

$$R_i^2 = \frac{\beta_i^2 \sigma_M^2}{\sigma_i^2} = \frac{\text{Explained variance}}{\text{Total variance}}$$

Therefore:

$$\sigma_A^2 = \frac{\beta_A^2 \sigma_M^2}{R_A^2} = \frac{0.7^2 \times 20^2}{0.20} = 980$$

$$\sigma_A = 31.30\%$$

For stock B:

$$\sigma_B^2 = \frac{1.2^2 \times 20^2}{0.12} = 4{,}800$$

$$\sigma_B = 69.28\%$$

10. The systematic risk for A is:

$$\beta_A^2 \times \sigma_M^2 = 0.70^2 \times 20^2 = 196$$

The firm-specific risk of A (the residual variance) is the difference between A's total risk and its systematic risk:

980 – 196 = 784

The systematic risk for B is:

$$\beta_B^2 \times \sigma_M^2 = 1.20^2 \times 20^2 = 576$$

B's firm-specific risk (residual variance) is:

4800 – 576 = 4224

11. The covariance between the returns of A and B is (since the residuals are assumed to be uncorrelated):

$$\mathrm{Cov}(r_A, r_B) = \beta_A \beta_B \sigma_M^2 = 0.70 \times 1.20 \times 400 = 336$$

The correlation coefficient between the returns of A and B is:

$$\rho_{AB} = \frac{\mathrm{Cov}(r_A, r_B)}{\sigma_A \sigma_B} = \frac{336}{31.30 \times 69.28} = 0.155$$

12. Note that the correlation is the square root of R^2: $\rho = \sqrt{R^2}$

$$Cov(r_{A,} r_M) = \rho \sigma_A \sigma_M = 0.20^{1/2} \times 31.30 \times 20 = 280$$

$$Cov(r_{B,} r_M) = \rho \sigma_B \sigma_M = 0.12^{1/2} \times 69.28 \times 20 = 480$$

CHAPTER 9: THE CAPITAL ASSET PRICING MODEL

PROBLEM SETS

3. a. False. $\beta = 0$ implies $E(r) = r_f$, not zero.

   b. False. Investors require a risk premium only for bearing systematic (undiversifiable or market) risk. Total volatility includes diversifiable risk.

   c. False. Your portfolio should be invested 75% in the market portfolio and 25% in T-bills. Then: $\beta_P = (0.75 \times 1) + (0.25 \times 0) = 0.75$

8. The appropriate discount rate for the project is:

$$r_f + \beta \times [E(r_M) - r_f] = .08 + [1.8 \times (.16 - .08)] = .224 = 22.4\%$$

Using this discount rate:

$$\text{NPV} = -\$40 + \sum_{t=1}^{10} \frac{\$15}{1.224^t} = -\$40 + [\$15 \times \text{Annuity factor (22.4\%, 10 years)}] = \$18.09$$

The internal rate of return (IRR) for the project is 35.73%. Recall from your introductory finance class that NPV is positive if IRR > discount rate (or, equivalently, hurdle rate). The highest value that beta can take before the hurdle rate exceeds the IRR is determined by:

$$.3573 = .08 + \beta \times (.16 - .08) \Rightarrow \beta = .2773/.08 = 3.47$$

20. $r_1 = 19\%$; $r_2 = 16\%$; $\beta_1 = 1.5$; $\beta_2 = 1$

a. To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b. If $r_f = 6\%$ and $r_M = 14\%$, then (using the notation alpha for the abnormal return):

$$\alpha_1 = .19 - [.06 + 1.5 \times (.14 - .06)] = .19 - .18 = 1\%$$

$$\alpha_2 = .16 - [.06 + 1 \times (.14 - .06)] = .16 - .14 = 2\%$$

Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If $r_f = 3\%$ and $r_M = 15\%$, then:

$$\alpha_1 = .19 - [.03 + 1.5 \times (.15 - .03)] = .19 - .21 = -2\%$$

$$\alpha_2 = .16 - [.03 + 1 \times (.15 \text{ x} - .03)] = .16 - .15 = 1\%$$

Here, not only does the second investor appear to be the superior stock selector, but the first investor's predictions appear valueless (or worse).