Quiz 2: Solution

1. Find $\dfrac{dy}{dx}$ for $\quad (1+y)^k = kxa^y \quad$ where a and k are some fixed positive real number.

Solution:

Since y is defined in terms of x implicitly, we will use **implicit differentiation** to find $\dfrac{dy}{dx}$.

$$\begin{aligned}
\frac{d}{dx}\left((1+y)^k\right) &= \frac{d}{dx}\left(kxa^y\right) \\
k(1+y)^{k-1}\frac{d}{dx}(1+y) &= k\left[xa^y\ln(a)\frac{dy}{dx} + a^y\frac{dx}{dx}\right] \\
k(1+y)^{k-1}\frac{dy}{dx} &= k\left[xa^y\ln(a)\frac{dy}{dx} + a^y\right] \\
\left[k(1+y)^{k-1}\right]\frac{dy}{dx} &= kxa^y\ln(a)\frac{dy}{dx} + ka^y \\
\left[k(x+y)^{k-1} - kxa^y\ln(a)\right]\frac{dy}{dx} &= ka^y \\
\frac{dy}{dx} &= \frac{ka^y}{k(x+y)^{k-1} - kxa^y\ln(a)} \\
&= \frac{a^y}{(x+y)^{k-1} - xa^y\ln(a)}
\end{aligned}$$

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