Problem Sets

Basic

1. What is the relationship between forward rates and the market's expectation of future short rates? Explain in the context of both the expectations and liquidity preference theories of the term structure of interest rates.
2. Under the expectations hypothesis, if the yield curve is upward-sloping, the market must expect an increase in short-term interest rates. True/false/uncertain? Why?
3. Under the liquidity preference theory, if inflation is expected to be falling over the next few years, long-term interest rates will be higher than short-term rates. True/false/uncertain? Why?

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4. If the liquidity preference hypothesis is true, what shape should the term structure curve have in a period where interest rates are expected to be constant?
   a. Upward sloping.
   b. Downward sloping.
   c. Flat.

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5. Which of the following is **true** according to the pure expectations theory? Forward rates:
   a. Exclusively represent expected future short rates.
   b. Are biased estimates of market expectations.
   c. Always overestimate future short rates.

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6. Assuming the pure expectations theory is correct, an upward-sloping yield curve implies:
   a. Interest rates are expected to increase in the future.
   b. Longer-term bonds are riskier than short-term bonds.
   c. Interest rates are expected to decline in the future.

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Intermediate

7. The following is a list of prices for zero-coupon bonds of various maturities. Calculate the yields to maturity of each bond and the implied sequence of forward rates.

Maturity (Years)	Price of Bond
1	$943.40
2	898.47
3	847.62
4	792.16

8. Assuming that the expectations hypothesis is valid, compute the expected price path of the 4-year bond in the previous problem as time passes. What is the rate of return of the bond in each year? Show that the expected return equals the forward rate for each year.
9. Consider the following $1,000 par value zero-coupon bonds:

Bond	Years to Maturity	YTM(%)
A	1	5%
B	2	6
C	3	6.5
D	4	7

According to the expectations hypothesis, what is the expected 1-year interest rate 3 years from now?

10. The term structure for zero-coupon bonds is currently:

Maturity (Years)	YTM (%)
1	4%
2	5
3	6

Next year at this time, *you* expect it to be:

Maturity (Years)	YTM (%)
1	5%
2	6
3	7

a. What do *you* expect the rate of return to be over the coming year on a 3-year zero-coupon bond?
b. Under the expectations theory, what yields to maturity does *the market* expect to observe on 1- and 2-year zeros at the end of the year? Is the market's expectation of the return on the 3-year bond greater or less than yours?

11. The yield to maturity on 1-year zero-coupon bonds is currently 7%; the YTM on 2-year zeros is 8%. The Treasury plans to issue a 2-year maturity *coupon* bond, paying coupons once per year with a coupon rate of 9%. The face value of the bond is $100.

a. At what price will the bond sell?
b. What will the yield to maturity on the bond be?
c. If the expectations theory of the yield curve is correct, what is the market expectation of the price that the bond will sell for next year?
d. Recalculate your answer to (*c*) if you believe in the liquidity preference theory and you believe that the liquidity premium is 1%.

12. Below is a list of prices for zero-coupon bonds of various maturities.

Maturity (Years)	Price of $1,000 Par Bond (Zero-Coupon)
1	$943.40
2	873.52
3	816.37

a. An 8.5% coupon $1,000 par bond pays an annual coupon and will mature in 3 years. What should the yield to maturity on the bond be?
b. If at the end of the first year the yield curve flattens out at 8%, what will be the 1-year holding-period return on the coupon bond?

13. Prices of zero-coupon bonds reveal the following pattern of forward rates:

Year	Forward Rate
1	5%
2	7
3	8

In addition to the zero-coupon bond, investors also may purchase a 3-year bond making annual payments of $60 with par value $1,000.

a. What is the price of the coupon bond?
b. What is the yield to maturity of the coupon bond?
c. Under the expectations hypothesis, what is the expected realized compound yield of the coupon bond?
d. If you forecast that the yield curve in 1 year will be flat at 7%, what is your forecast for the expected rate of return on the coupon bond for the 1-year holding period?

14. You observe the following term structure:

	Effective Annual YTM
1-year zero-coupon bond	6.1%
2-year zero-coupon bond	6.2
3-year zero-coupon bond	6.3
4-year zero-coupon bond	6.4

a. If you believe that the term structure next year will be the same as today's, will the 1-year or the 4-year zeros provide a greater expected 1-year return?
b. What if you believe in the expectations hypothesis?

15. The yield to maturity (YTM) on 1-year zero-coupon bonds is 5% and the YTM on 2-year zeros is 6%. The yield to maturity on 2-year-maturity coupon bonds with coupon rates of 12% (paid annually) is 5.8%. What arbitrage opportunity is available for an investment banking firm? What is the profit on the activity?

16. Suppose that a 1-year zero-coupon bond with face value $100 currently sells at $94.34, while a 2-year zero sells at $84.99. You are considering the purchase of a 2-year-maturity bond making *annual* coupon payments. The face value of the bond is $100, and the coupon rate is 12% per year.
a. What is the yield to maturity of the 2-year zero? The 2-year coupon bond?
b. What is the forward rate for the second year?
c. If the expectations hypothesis is accepted, what are (1) the expected price of the coupon bond at the end of the first year and (2) the expected holding-period return on the coupon bond over the first year?
d. Will the expected rate of return be higher or lower if you accept the liquidity preference hypothesis?

Challenge

17. The current yield curve for default-free zero-coupon bonds is as follows:

Maturity (Years)	YTM (%)
1	10%
2	11
3	12

a. What are the implied 1-year forward rates?
b. Assume that the pure expectations hypothesis of the term structure is correct. If market expectations are accurate, what will be the pure yield curve (that is, the yields to maturity on 1- and 2-year zero coupon bonds) next year?
c. If you purchase a 2-year zero-coupon bond now, what is the expected total rate of return over the next year? What if you purchase a 3-year zero-coupon bond? (*Hint:* Compute the current and expected future prices.) Ignore taxes.
d. What should be the current price of a 3-year maturity bond with a 12% coupon rate paid annually? If you purchased it at that price, what would your total expected rate of return be over the next year (coupon plus price change)? Ignore taxes.

18. Suppose that the prices of zero-coupon bonds with various maturities are given in the following table. The face value of each bond is $1,000.

Maturity (Years)	Price
1	$925.93
2	853.39
3	782.92
4	715.00
5	650.00

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a. Calculate the forward rate of interest for each year.
b. How could you construct a 1-year forward loan beginning in year 3? Confirm that the rate on that loan equals the forward rate.
c. Repeat (*b*) for a 1-year forward loan beginning in year 4.

19. Continue to use the data in the preceding problem. Suppose that you want to construct a *2-year* maturity forward loan commencing in 3 years.
a. Suppose that you buy *today* one 3-year maturity zero-coupon bond. How many 5-year maturity zeros would you have to sell to make your initial cash flow equal to zero?
b. What are the cash flows on this strategy in each year?
c. What is the effective 2-year interest rate on the effective 3-year-ahead forward loan?
d. Confirm that the effective 2-year interest rate equals $(1 + f_4) \times (1 + f_5) - 1$. You can therefore interpret the 2-year loan rate as a 2-year forward rate for the last 2 years. Alternatively, show that the effective 2-year forward rate equals

$$\frac{(1 + y_5)^5}{(1 + y_3)^3} - 1$$

CFA® PROBLEMS

1. Briefly explain why bonds of different maturities have different yields in terms of the expectations and liquidity preference hypotheses. Briefly describe the implications of each hypothesis when the yield curve is (1) upward-sloping and (2) downward-sloping.

2. Which one of the following statements about the term structure of interest rates is true?
a. The expectations hypothesis indicates a flat yield curve if anticipated future short-term rates exceed current short-term rates.
b. The expectations hypothesis contends that the long-term rate is equal to the anticipated short-term rate.
c. The liquidity premium theory indicates that, all else being equal, longer maturities will have lower yields.
d. The liquidity preference theory contends that lenders prefer to buy securities at the short end of the yield curve.

3. The following table shows yields to maturity of zero-coupon Treasury securities.

Term to Maturity (Years)	Yield to Maturity (%)
1	3.50%
2	4.50
3	5.00
4	5.50
5	6.00
10	6.60

a. Calculate the forward 1-year rate of interest for year 3.
b. Describe the conditions under which the calculated forward rate would be an unbiased estimate of the 1-year spot rate of interest for that year.
c. Assume that a few months earlier, the forward 1-year rate of interest for that year had been significantly higher than it is now. What factors could account for the decline in the forward rate?

4. The 6-month Treasury bill spot rate is 4%, and the 1-year Treasury bill spot rate is 5%. What is the implied 6-month forward rate for 6 months from now?

5. The tables below show, respectively, the characteristics of two annual-pay bonds from the same issuer with the same priority in the event of default, and spot interest rates. Neither bond's price is consistent with the spot rates. Using the information in these tables, recommend either bond *A* or bond *B* for purchase.

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