# Assignment
# Simultaneous Equations Model

**Simultaneous Equations System**

$$y_{1t} = \beta_{10} + \beta_{11} y_{2t} + \beta_{12} x_{1t} + \beta_{13} x_{2t} + u_{1t} \quad (1)$$

$$y_{2t} = \beta_{20} + \beta_{21} y_{1t} + \beta_{22} x_{3t} + \beta_{23} x_{4t} + u_{2t} \quad (2)$$

Endogenous variables in this system include $y_{1t}$ and $y_{2t}$
Exogenous variables in this system include $x_{1t}$, $x_{2t}$, $x_{3t}$, and $x_{4t}$

1. State reduce form model of these system models.
2. Estimate reduce form model using OLS and prediction of the endogenous variables.
3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.
4. Estimate the structural models of these system equations using OLS, 2SLS, and 3SLS. Concerning on the asymptotic property, which estimated results are the most appropriated results? Why?

1. State reduce form model of these system models.

$$y_{1t} = \pi_{10} + \pi_{11}x_{1t} + \pi_{12}x_{2t} + \pi_{13}x_{3t} + \pi_{14}x_{4t} + w_{1t}$$
$$y_{2t} = \pi_{20} + \pi_{21}x_{1t} + \pi_{22}x_{2t} + \pi_{23}x_{3t} + \pi_{24}x_{4t} + w_{2t}$$

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

```
. reg y1 x1 x2 x3 x4

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(4, 495)       =    189.18
       Model |  7700218.45         4  1925054.61   Prob > F        =    0.0000
    Residual |  5037024.74       495  10175.8076   R-squared       =    0.6045
-------------+----------------------------------   Adj R-squared   =    0.6013
       Total |  12737243.2       499  25525.5375   Root MSE        =    100.88

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .7351842   .0458451    16.04   0.000     .6451092    .8252591
          x2 |  -1.370992   .1062249   -12.91   0.000    -1.579699   -1.162284
          x3 |  -.3716883   .1571014    -2.37   0.018    -.6803562   -.0630205
          x4 |   .9451346    .050141    18.85   0.000     .8466191     1.04365
       _cons |  -.9002973   5.618986    -0.16   0.873     -11.9403    10.13971
------------------------------------------------------------------------------

. predict y1hat, xb

. reg y2 x*

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(4, 495)       =    716.26
       Model |  36037999.6         4  9009499.91   Prob > F        =    0.0000
    Residual |  6226340.32       495  12578.4653   R-squared       =    0.8527
-------------+----------------------------------   Adj R-squared   =    0.8515
       Total |    42264340       499  84698.0761   Root MSE        =    112.15

------------------------------------------------------------------------------
          y2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |    .265324   .0509709     5.21   0.000      .165178      .36547
          x2 |  -.5918622   .1181016    -5.01   0.000    -.8239044    -.35982
          x3 |  -1.292387   .1746665    -7.40   0.000    -1.635566   -.9492077
          x4 |   2.946954   .0557471    52.86   0.000     2.837424    3.056484
       _cons |  -2.189755   6.247227    -0.35   0.726    -14.46411     10.0846
------------------------------------------------------------------------------

. predict y2hat, xb
```

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.

```
. reg y1 y2hat x1 x2

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(3, 496)       =    252.69
       Model |  7699473.59         3   2566491.2   Prob > F        =    0.0000
    Residual |   5037769.6       496  10156.7936   R-squared       =    0.6045
-------------+----------------------------------   Adj R-squared   =    0.6021
       Total |  12737243.2       499  25525.5375   Root MSE        =    100.78

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       y2hat |   .3201675   .0168776    18.97   0.000      .287007    .3533279
          x1 |   .6506053   .0459881    14.15   0.000     .5602498    .7409609
          x2 |   -1.1816   .1060642   -11.14   0.000    -1.389991   -.9732098
       _cons |  -.0825759   5.588965    -0.01   0.988    -11.06354    10.89839
------------------------------------------------------------------------------
```

```
. reg y2 y1hat x3 x4

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(3, 496)       =    956.02
       Model |  36032833.3         3  12010944.4   Prob > F        =    0.0000
    Residual |  6231506.64       496  12563.5215   R-squared       =    0.8526
-------------+----------------------------------   Adj R-squared   =    0.8517
       Total |    42264340       499  84698.0761   Root MSE        =    112.09

------------------------------------------------------------------------------
          y2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       y1hat |   .3887183   .0540226     7.20   0.000     .2825769    .4948596
          x3 |  -1.149046   .1751029    -6.56   0.000    -1.493081   -.8050109
          x4 |   2.578087   .0743161    34.69   0.000     2.432074    2.724101
       _cons |  -.4182315   5.814539    -0.07   0.943     -11.8424    11.00593
------------------------------------------------------------------------------

.
```

4. Estimate the structural models of these system equations using OLS, 2SLS, and 3SLS. Concerning on the asymptotic property, which estimated results are the most appropriated results? Why?

```
. reg y1 y2 x1 x2

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(3, 496)       =    881.99
       Model |  10726508.6         3  3575502.87   Prob > F        =    0.0000
    Residual |  2010734.57       496  4053.90034   R-squared       =    0.8421
-------------+----------------------------------   Adj R-squared   =    0.8412
       Total |  12737243.2       499  25525.5375   Root MSE        =     63.67

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y2 |    .399428   .0098383    40.60   0.000     .3800981    .4187579
          x1 |   .6301685   .0290345    21.70   0.000     .5731227    .6872144
          x2 |  -1.161171   .0669997   -17.33   0.000    -1.29281   -1.029533
       _cons |  -5.839926   3.518283    -1.66   0.098    -12.7525    1.072649
------------------------------------------------------------------------------

. est store m1

. reg y2 y1 x3 x4

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(3, 496)       =   4051.67
       Model |  40607309.2         3  13535769.7   Prob > F        =    0.0000
    Residual |  1657030.81       496  3340.78792   R-squared       =    0.9608
-------------+----------------------------------   Adj R-squared   =    0.9606
       Total |    42264340       499  84698.0761   Root MSE        =      57.8

------------------------------------------------------------------------------
          y2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y1 |   .7478655   .0189107    39.55   0.000     .7107106    .7850204
          x3 |  -1.050503   .0901201   -11.66   0.000    -1.227568   -.8734393
          x4 |   2.250549    .033475    67.23   0.000     2.184778    2.316319
       _cons |  -17.91048   2.827999    -6.33   0.000    -23.46682   -12.35415
------------------------------------------------------------------------------

. est store m2

. ivregress 2sls y1 x1 x2 (y2=x1 x2 x3 x4)

Instrumental variables (2SLS) regression          Number of obs   =        500
                                                  Wald chi2(3)    =    1693.05
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.8215
                                                  Root MSE        =     67.437

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y2 |   .3201675   .0112935    28.35   0.000     .2980326    .3423023
          x1 |   .6506053   .0307725    21.14   0.000     .5902922    .7109184
```

```
          x2 |    -1.1816   .0709719   -16.65   0.000    -1.320703   -1.042498
       _cons |   -.082576   3.739805    -0.02   0.982     -7.41246    7.247308
------------------------------------------------------------------------------
Instrumented:  y2
Instruments:   x1 x2 x3 x4

. est store m3

. ivregress 2sls y2 x3 x4 (y1=x1 x2 x3 x4)

Instrumental variables (2SLS) regression          Number of obs   =        500
                                                  Wald chi2(3)    =    6295.02
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.9323
                                                  Root MSE        =     75.657

------------------------------------------------------------------------------
          y2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y1 |   .3887183   .0364645    10.66   0.000     .3172491    .4601874
          x3 |  -1.149046   .1181921    -9.72   0.000    -1.380698   -.9173935
          x4 |   2.578087   .0501624    51.39   0.000     2.479771    2.676404
       _cons |  -.4182316   3.924735    -0.11   0.915    -8.110571    7.274108
------------------------------------------------------------------------------
Instrumented:  y1
Instruments:   x3 x4 x1 x2

. est store m4

. reg3 (y1 y2 x1 x2) (y2 y1 x3 x4), inst(x1 x2 x3 x4) 3sls

Three-stage least-squares regression
------------------------------------------------------------------------------
Equation             Obs   Parms        RMSE    "R-sq"       chi2         P
------------------------------------------------------------------------------
y1                   500       3    67.50756    0.8211    1694.59    0.0000
y2                   500       3    75.63899    0.9323    6295.31    0.0000
------------------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y1           |
          y2 |   .3198315   .0112879    28.33   0.000     .2977076    .3419554
          x1 |   .6650667   .0266978    24.91   0.000     .6127399    .7173935
          x2 |  -1.140358    .055968   -20.38   0.000    -1.250054   -1.030663
       _cons |   .9128191    3.58841     0.25   0.799    -6.120335    7.945973
-------------+----------------------------------------------------------------
y2           |
          y1 |   .3890072   .0364571    10.67   0.000     .3175526    .4604618
          x3 |  -1.185279   .0738911   -16.04   0.000    -1.330103   -1.040455
          x4 |   2.576427   .0499839    51.55   0.000      2.47846    2.674394
       _cons |  -.3246522   3.917497    -0.08   0.934    -8.002806    7.353501
------------------------------------------------------------------------------
Endogenous variables:  y1 y2
Exogenous variables:   x1 x2 x3 x4
------------------------------------------------------------------------------

. est store m5

. est table m1 m2 m3 m4 m5, se stat(N rss rss_1 rss_2 rmse rmse_1 rmse_2 F chi2
chi2_1 chi2_2 r2 r2_1 r2_2 r2_a)

-------------------------------------------------------------------------------
    Variable |     m1           m2           m3           m4           m5
-------------+-----------------------------------------------------------------
             |
_            |
          y2 |  .39942802                .32016746
             |  .00983831                 .0112935
          x1 |  .63016855                 .6506053
             |  .02903452                .03077254
          x2 | -1.1611715               -1.1816004
             |  .06699971                .07097193
          y1 |               .74786547                .38871825
             |               .01891067                .03646451
          x3 |              -1.0505035               -1.1490458
             |               .09012008                 .1181921
```

| | | | | |
|---|---|---|---|---|
| x4 | 2.2505488 | | 2.5780874 | |
| | .03347504 | | .05016238 | |
| _cons | -5.8399263 | -17.910485 | -.08257597 | -.41823164 |
| | 3.5182829 | 2.8279987 | 3.7398053 | 3.924735 |
| y1 | | | | |
| y2 | | | | .31983146 |
| | | | | .01128791 |
| x1 | | | | .6650667 |
| | | | | .02669784 |
| x2 | | | | -1.1403583 |
| | | | | .05596803 |
| _cons | | | | .91281906 |
| | | | | 3.58841 |
| y2 | | | | |
| y1 | | | | .38900721 |
| | | | | .03645709 |
| x3 | | | | -1.1852792 |
| | | | | .07389113 |
| x4 | | | | 2.5764269 |
| | | | | .0499839 |
| _cons | | | | -.32465224 |
| | | | | 3.9174972 |
| Statistics | | | | |
| N | 500 | 500 | 500 | 500 | 500 |
| rss | 2010734.6 | 1657030.8 | 2273850.3 | 2862011.9 | |
| rss_1 | | | | | 2278635.4 |
| rss_2 | | | | | 2860628.6 |
| rmse | 63.670247 | 57.799549 | 67.436641 | 75.657279 | |
| rmse_1 | | | | | 67.50756 |
| rmse_2 | | | | | 75.638993 |
| F | 881.99082 | 4051.6699 | | | |
| chi2 | | | 1693.0476 | 6295.0181 | |
| chi2_1 | | | | | 1694.5862 |
| chi2_2 | | | | | 6295.3051 |
| r2 | .84213738 | .96079364 | .82148019 | .93228306 | |
| r2_1 | | | | | .82110451 |
| r2_2 | | | | | .93231579 |
| r2_a | .84118257 | .96055651 | .82040043 | .93187348 | |

legend: b/se

| | y1 | | | y2 | | |
|---|---|---|---|---|---|---|
| | OLS | 2SLS | 3SLS | OLS | 2SLS | 3SLS |
| y1 | | | | 0.7479 | 0.3887 | 0.3890 |
| | | | | (0.0189) | (0.0365) | (0.0365) |
| y2 | 0.3994 | 0.3202 | 0.3198 | | | |
| | (0.0098) | (0.0113) | (0.0113) | | | |
| x1 | 0.6302 | 0.6506 | 0.6651 | | | |
| | (0.0290) | (0.0308) | (0.0267) | | | |
| x2 | -1.1612 | -1.1816 | -1.1404 | | | |
| | (0.0670) | (0.0710) | (0.0560) | | | |
| x3 | | | | -1.0505 | -1.1490 | -1.1853 |
| | | | | (0.0901) | (0.1182) | (0.0739) |
| x4 | | | | 2.2505 | 2.5781 | 2.5764 |
| | | | | (0.0335) | (0.0502) | (0.0500) |
| Constant | -5.8399 | -0.0826 | 0.9128 | -17.9105 | -0.4182 | -0.3247 |
| | (3.5183) | (3.7398) | (3.5884) | (2.8280) | (3.9247) | (3.9175) |
| Obs. | 500 | 500 | 500 | 500 | 500 | 500 |
| RSS | 2010735 | 2273850 | 2278635 | 1657031 | 2862012 | 2860629 |
| RMSE | 63.67 | 67.44 | 67.51 | 57.80 | 75.66 | 75.64 |
| F-test | 881.99 | | | 4051.67 | | |
| Chi2-test | | 1693.05 | 1694.59 | | 6295.02 | 6295.31 |
| R-squares | 0.8421 | 0.8215 | 0.8211 | 0.9608 | 0.9323 | 0.9323 |
| Adj. R2 | 0.8412 | 0.8204 | | 0.9606 | 0.9319 | |

Note: Value in parenthesis is Standard Error (S.E.)

Data Generating Process

```
set obs 500
g t=_n
tsset t
g x1=rnormal(10,100)
g x2=rnormal(-30,45)
g x3=rnormal(1,30)
g x4=rnormal(15,90)
matrix C = (1, .8 \ .8, 1)
corr2data u1 u2, n(500) means(0 0) sds(70 75) corr(C)
g y1=1 in 1
g y2=2 in 1
forvalue t=1(1)500 {
  replace y1=0.5+0.7*x1-1.2*x2+u1 in `t'
  replace y2=0.5-1.1*x3+2.5*x4+u2 in `t'
  replace y1=0.5+0.3*y2+0.7*x1-1.2*x2+u1 in `t'
  replace y2=0.5+0.4*y1-1.1*x3+2.5*x4+u2 in `t'
  replace y1=0.5+0.3*y2+0.7*x1-1.2*x2+u1 in `t'
  replace y2=0.5+0.4*y1-1.1*x3+2.5*x4+u2 in `t'
  replace y1=0.5+0.3*y2+0.7*x1-1.2*x2+u1 in `t'
  replace y2=0.5+0.4*y1-1.1*x3+2.5*x4+u2 in `t'
}
```

* Compare 2SLS vs GMM

```
. reg y1 y2hat x1 x2

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(3, 496)       =    252.69
       Model |  7699473.59         3   2566491.2   Prob > F        =    0.0000
    Residual |   5037769.6       496  10156.7936   R-squared       =    0.6045
-------------+----------------------------------   Adj R-squared   =    0.6021
       Total |  12737243.2       499  25525.5375   Root MSE        =    100.78

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       y2hat |   .3201675   .0168776    18.97   0.000      .287007    .3533279
          x1 |   .6506053   .0459881    14.15   0.000     .5602498    .7409609
          x2 |    -1.1816   .1060642   -11.14   0.000    -1.389991   -.9732098
       _cons |  -.0825759   5.588965    -0.01   0.988    -11.06354    10.89839
------------------------------------------------------------------------------

. ivregress 2sls y1 x1 x2 (y2=x1 x2 x3 x4)

Instrumental variables (2SLS) regression          Number of obs   =        500
                                                  Wald chi2(3)    =    1693.05
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.8215
                                                  Root MSE        =     67.437

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y2 |   .3201675   .0112935    28.35   0.000     .2980326    .3423023
          x1 |   .6506053   .0307725    21.14   0.000     .5902922    .7109184
          x2 |    -1.1816   .0709719   -16.65   0.000    -1.320703   -1.042498
       _cons |   -.082576   3.739805    -0.02   0.982     -7.41246    7.247308
------------------------------------------------------------------------------
Instrumented:  y2
Instruments:   x1 x2 x3 x4

. ivregress gmm y1 x1 x2 (y2=x1 x2 x3 x4)

Instrumental variables (GMM) regression           Number of obs   =        500
                                                  Wald chi2(3)    =    1596.25
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.8216
GMM weight matrix: Robust                         Root MSE        =     67.405

------------------------------------------------------------------------------
             |               Robust
          y1 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y2 |   .3204869   .0116394    27.53   0.000     .2976741    .3432998
          x1 |   .6507274   .0298274    21.82   0.000     .5922668     .709188
          x2 |   -1.18233   .0695007   -17.01   0.000    -1.318549   -1.046111
       _cons |  -.0439821   3.491438    -0.01   0.990    -6.887074    6.79911
------------------------------------------------------------------------------
Instrumented:  y2
Instruments:   x1 x2 x3 x4

.
```