1) $y = x^3 - 5x^2 + 7x - 5$

a) $y = x^3 - 5x^2 + 7x - 5$

$\frac{dy}{dx} = 3x^2 - 10x + 7$

find critical value $\frac{dy}{dx} = 0$

$3x^2 - 10x + 7 = 0$

$(3x-7)(x-1) = 0$

$x = \frac{7}{3}, 1$

$f'(0) = 3(0)^2 - 10(0) + 7$

$= 7$

$f'(2) = 3(2)^2 - 10(2) + 7$

$= -1$

$f'(3) = 3^3 - 10(3) + 7$

$= 4$

b) $y = x^3 - 5x^2 - 7x - 5$

$\frac{dy}{dx} = 3x^2 - 10x - 7$

$\frac{d^2y}{dx^2} = 6x - 10$

find x when $\frac{d^2y}{dx^2} = 0$

$6x - 10 = 0$

$x = \frac{5}{3}$

$f''(0) = 6(0) - 10$

$= -10$

$f''(2) = 6(2) - 10$

$= 2$

$\therefore$ y or f(x) is concave downward at $(-\infty, \frac{5}{3})$

and concave upward at $(\frac{5}{3}, \infty)$

2. Suppose that a firm's short-run production function is given by

$$Q(L) = 6L^2 - L^3$$

where $Q(L)$ is the output level, and L is the number of workers

a. Derive the average product of labor (AP_L) and the marginal product of labor (MP_L).
b. What size of the work force (L^{**}) maximizes the average output per labor, Q(L)/L?
c. Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point.
d. Given that the firm faces the demand function

$$Q = 100 - 2P$$

derive the marginal revenue product (*MRP*) function.

a) $Q(L) = 6L^2 - L^3$

$$APL = \frac{Q}{L} = \frac{6L^2 - L^3}{L} = 6L - L^2 \quad \#$$

$$MPL = \frac{dQ}{dL} = \frac{d(6L^2 - L^3)}{dL} = 12L - 3L^2 \quad \#$$

b) Max APL

$$\frac{d\,APL}{dL} = 6 - 2L = 0$$

$$6 = 2L$$

$$L = 3$$

$$\frac{d^2 APL}{d^2 L} = -2$$

$$-2 < 0$$

Hence, $L=3$ maximize the APL

c) MP_L cross AP_L

$12L - 3L^2 = 6L - L^2$

$-2L^2 + 6L = 0$

$-2L(L-3) = 0$

$L = 0, 3$

$L = 3$ #

∴ From the Answer 2.B we know that APL is maximize at $L = 3$. Moreover, MP_L and AP_L also cross at $L = 3$ so we can conclude that AP_L and MP_L cross at maximum point.

d) $MPR = \frac{dTR}{dL} = MPL \cdot MR$

$MP_L = 12L - 3L^2$

Find MR

$TR = P.Q$

$= (50 - 0.5Q)Q$

$= 50Q - 0.5Q^2$

$MR = \frac{dTR}{dQ}$

$= 50 - Q$

$MPR = MP_L \cdot MR$

$= (12L - 3L^2)(50 - Q)$

$= (12L - 3L^2)(50 - 6L^2 + L^3)$ #

3) a) $TR = PQ$

$$= \left(40 + \frac{105}{Q} - \frac{3}{2}Q^2\right)Q$$

$$TR = 40Q + 105 - \frac{3}{2}Q^3$$

max output : $\frac{dTR}{dQ} = 0$

$$40 - \frac{9}{2}Q^2 = 0$$

$$40 = \frac{9Q^2}{2}$$

$$Q^2 = \frac{80}{9}$$

$$Q = 2.9814$$

elasticity for Demand $= \frac{dQ}{dP} \cdot \frac{P}{Q}$

$Q = 2.9814$; $P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$

$$P = 40 + \frac{105}{\sqrt{\frac{80}{9}}} - \frac{\cancel{3}}{\cancel{2}}\left(\frac{\cancel{80}^{40}}{\cancel{9}_3}\right)$$

$$= 40 + \frac{105\sqrt{9}}{\sqrt{80}} - \frac{40}{3}$$

$$= 40 + \frac{315}{4\sqrt{5}} - \frac{40}{3}$$

$$P = 61.8847$$

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

$$\frac{dP}{dQ} = -\frac{105}{Q^2} - 3Q$$

$\therefore$ elasticity $= \frac{dQ}{dP} \cdot \frac{P}{Q}$

$$= \frac{1}{-\frac{105}{Q^2} - 3Q} \times \frac{P}{Q}$$

$$= \frac{1}{-\frac{105}{2.9814^2} - 3(2.9814)} \times \frac{61.8847}{2.9814}$$

$$= -1$$

b) $\pi = TR - TC$

$$\pi = (PQ) - TC$$

$$= \left(40 + \frac{105}{Q} - \frac{3}{2}Q^2\right)Q - (6Q^3 - 81Q^2 - 175Q - 10)$$

$$= 40Q + 105 - \frac{3}{2}Q^3 - 6Q^3 + 81Q^2 + 175Q + 10$$

$$= -\frac{15}{2}Q^3 + 81Q^2 + 215Q + 115$$

c) Profit max : $\frac{d\pi}{dQ} = 0$

$$\frac{d\pi}{dQ} = -\frac{15}{2}Q^3 + 81Q^2 + 215Q + 115 = 0$$

$$= -\frac{45}{2}Q^2 + 162Q + 215 = 0$$

$$22.5Q^2 - 162Q - 215 = 0$$

$$Q = 8.345, \cancel{-1.145}$$

Second order condition

$$\frac{d\pi'}{dQ} = -90Q + 16$$

$Q = 8.345$; $= -90(8.345) + 16$

$$= -735.05$$

d) before tax; $\pi = TR - TC$

$$\underbrace{\frac{d\pi}{dQ} = \frac{dTR}{dQ}}_{MR} - \underbrace{\frac{TC}{dQ}}_{MC} = 0$$

$$MR - MC = 0$$

$$\boxed{MR = MC}$$

After tax ; $\pi = TR - TC - tax$

$$\frac{d\pi}{dQ} = \frac{d}{dQ}(TR - TC - tax) = 0$$

lumpsum tax = constant

$$\underbrace{\frac{dTR}{dQ}}_{MR} - \underbrace{\frac{dTC}{dQ}}_{MC} - \frac{d\,tax}{dQ} = 0$$

$$MR - MC - 0 = 0$$

$$\boxed{MR = MC}$$

$\therefore$ there is no change in lumpsum tax's effect.

4. The swimming pool maintenance sector consists of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$, where q is the number of swimming pools serviced per day.

a. What is the short-run supply curve for each pool maintenance firm? What is the short-run supply curve for the market as a whole?

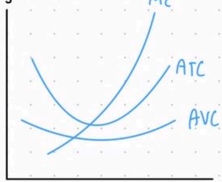
b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

c. Suppose the government imposed a €3 tax on chemicals per pool maintained. How would this tax change the market equilibrium?

d. How would the burden of this tax be shared between owners of swimming pools and the firms that offer pool maintenance services?

e. Calculate the total loss of producer surplus as a result of the new tax. Show that this loss equals the change in total short-run profits in this industry.

a) Supply Curve



1) find $MC = \frac{\Delta TC}{\Delta Q} = \frac{dTC}{dQ}$

$STC = 0.5q^2 + 10q + 5$

$MC = \frac{dTC}{dQ} = q + 10$

$MC = q + 10$

2) find avc

from $STC = 0.5q^2 + 10q + 5$

$VC = 0.5q^2 + 10q$

$AVC = \frac{VC}{Q} = \frac{0.5Q^2 + 10Q}{Q}$

$AVC = 0.5Q + 10$

3) supply

$P = MC$

$P = Q + 10$

$Q = P - 10$ #

4) Market supply

$Q_m = q_1 + q_2 + \ldots + q_n$

$n = 100$

$Q_m = 100(P - 10)$

$= 100P - 1000$

$Q^s_m = 100P - 1000$ #

b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

$$Q_m^D = Q_m^S$$

$$1100 - 50P = 100P - 1000$$

$$2100 = 150P$$

$$P = 14$$

$$Q_m = 400$$

Market price = 14 ฿

Quantity demand = 400

$\downarrow$ 100 Firm

So q (q of 1 firm) $= \frac{400}{100} = 4$

Short Run Profit

$$\pi = P \cdot Q - TC$$

$$= 14(4) - [(0.5(4)^2) + 10(4) + 5]$$

$$\pi = 3$$

Unit tax

c. Suppose the government imposed a €3 tax on chemicals per pool maintained. How would this tax change the market equilibrium?

tax make TC ↑

$STC = 0.5q^2 + 10q + 5 + 3q$

$STC = 0.5q^2 + 13q + 5$

find new supply

$MC = \frac{d\,STC}{dQ} = q + 13$

$MC = P$

$P = q + 13$

$q = -13 + P$ firm supply

find market supply

$Q^m = 100(-13 + P)$

$Q^m = -1300 + 100P$

find equilibrium

$Q^D = Q^S$

$1100 - 50P = -1300 + 100P$

$2400 = 150P$

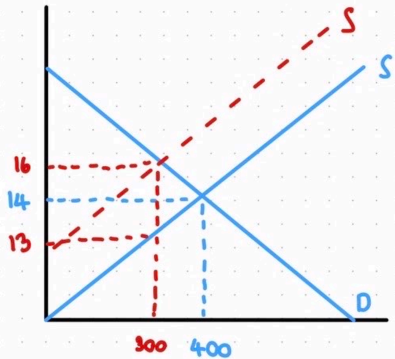
$P = 16$

$Q = -1300 + 100(16)$

$= -1300 + 1600$

$Q = 300$

d. How would the burden of this tax be shared between owners of swimming pools and the firms that offer pool maintenance services?



Consumer burden $= P^N - P^0 = 16 - 14$

$= 2$ baht / piece

$\frac{2}{3} \times 100 = 66.64\ \%$

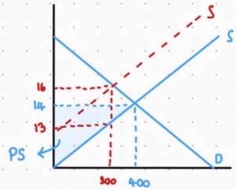
Producer burden = tax − what consumer pay

$= 3 - 2$

$= 1$ baht / piece

$\frac{1}{3} \times 100 = 33.34\ \%$

e. Calculate the total loss of producer surplus as a result of the new tax. Show that this loss equals the change in total short-run profits in this industry.

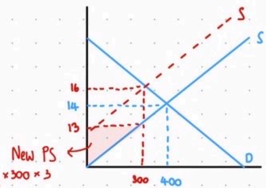


find intersect $(Q=0)$

$Q = -1000 + 100P$

$0 = -1000 + 100P$

$P = 10$



$\frac{1}{2} \times 300 \times 3$

$= 450$

change in PS $= 450 - 800$

$= -350$

find New Profit

New Profit $= P \cdot Q - (TC \text{ (with tax)})$

$= 13q - [0.5q^2 + 10q + 5]$

let Q

$q = 300/100 = 3$

$= 13(3) - (0.5(3) + 10(3) + 5)$

$= 2.5$

change in short run profit

$= 2.5 - 3$

$= -0.5$

$\therefore$ Profit $= -0.5(100)$

$= -50$