EE431: Economics of financial market and institutions

Semester 1/2017

Assignment# 1

Instructions:

1. Attempt all.
2. Your homework is group-work; please write down the group member in your homework submitted. Please attach a cover letter to your homework. In your cover letter, please write down the name list of members (also ID) in your group.
3. Due date: Sept. 12th 2017 (Thursday) at the BE office (before 15.00).
4. Late homework will not be accepted.

Question 1: *Mean-Variance and project selection*

Consider two projects with the required amount of initial investment of $100 million. Both projects are assumed to be *indivisible*. (You can divide the scale of investment, i.e. no option for the fractional investment.) Financial cash flows of each project are summarized in the two tables below.

Project A

Probability	Receivable cash flow (million dollars)	Rate of return (%)
¼	80	
¼	100	
¼	120	
¼	140	

Project B

Probability	Receivable cash flow (million dollars)	Rate of return (%)
½	90	
½	130	

1.1) Define the rate of return as the net operating profit over the investment. Calculate the rate of under all possible outcomes.
1.2) Calculate the expected rate of return on each project.
1.3) Calculate the standard deviation of the rate of return on each project. (Hint: you must convert the figures for rate of return into decimal units.)
1.4) Which one of the project would you select? What kind of decision criteria do you use? Explain.

__Question 2:__ *Risk aversion coefficient in greater detail.*

In class, we discussed about the two types of risk aversion coefficient, i.e. CARA and CRRA. We argue that the two coefficients serve their role as the proxy for the ranking of the degree of risk aversion. This is because the two coefficients are used as the input for *approximating* the level of risk premium required by the individual. For the sake of an illustration, suppose that an individual, with his initial wealth set equal to "w", is facing with a risky option that generates the final outcome equal to $w + \varepsilon$, where ε is a random variable with $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma_\varepsilon^2$.

Arrow-Pratt showed that the level of risk premium can be approximate by the following formula;

$$R_P(w) = \frac{1}{2}\sigma_\varepsilon^2 R_A(w) = \frac{1}{2}\sigma^2\left[-\frac{u''(w)}{u'(w)}\right]$$

As one can see from the formula above, the level of risk premium ($R_P(w)$) is an increasing function in the coefficient of absolute risk aversion ($R_A(w)$), i.e. $\left[-\frac{u''(w)}{u'(w)}\right]$. Given the expression, the agent with bigger value of the CARA would require greater amount of risk premium, and hence associating with the greater degree of risk aversion.

Assume that you have a logarithm utility function for wealth U(W) = $\sqrt{W}$ and that you are faced with a 50/50 chance of winning or losing $1,000. Consider the following questions.

2.1) Calculate the mean and variance of the random pay-off. (Hint: ε can take two possible values, and hence represents the discrete random variable.)

2.2) How much will you pay to avoid this risk if your current level of wealth is $10,000?

2.3) Use the approximate formula given above to calculate the level of risk premium. Compare your solution with the exact solution obtained from the question 2.2

2.4) How much would you pay to avoid the risk if you level of wealth were instead $1,000,000?

<u>Question 3:</u> *Diversification and Risks*

Consider the following investment strategy of two investors, namely John and Robert.

John: He has $100 million in cash, and invested in a real estate in London. The property has its current value of $80 million. Future value of the estate depends on the possibility of fire incident occurred in London. Under the unfortunate event with the occurrence of a fire accident, his property will lose its full value. If not, his property remains at the current value of $80 million.

Robert: He also has $100 million in cash. However, he owns two properties located in London and Paris. The current value of both properties is $40 million. The value of his property could be changed depending upon whether there is any fire accident any of these two places. If a fire accident occurs at any location, the value of the property in that location will be all gone, i.e. losing its full value

A study has found that the chance of having a fire incident in each location is 10% equally, and the occurrence of the incidence is statistically independent. Suppose that there is no insurance market that can provide the coverage to this fire risk, consider the following problem.

3.1) Calculate the level of wealth for John when fire occurs in London, and when everything goes as normal without any fire incidents.

3.2) Given the *independence* assumption, what is the probability that both London and Paris would have a fire incident at the same time?

3.3) What about the chance that neither places would have a fire incident?

3.4) How about the chance that fire accident would occur at most one particular location, i.e. either London or Paris.

3.5) Calculate the level of contingent wealth of Robert under the four possible scenarios described in (3.2) – (3.4)

3.6) Calculate the expected value of wealth and the standard deviation of wealth for John and Robert.

3.7) Given the investment decision chosen by John and Robert, do you think which one of them would likely be bearing upon more risk? What do we learn about the risk mitigation method adopted by John and Robert?

<u>Question 4:</u>

Suppose that Mao is an expected utility maximizer, with the VNM utility function $u(w) = w^2$

4.1) What type of attitude toward risk does Mao's preference exhibit? Why?

4.2) What is Mao's certainty equivalent of the following lottery:

Probability	Money
.4	30
.5	100
.1	500

4.3) Calculate the amount of risk premium. Interpret your result.

4.4) What is the sign of the coefficient of absolute risk aversion?

<u>Question 5:</u> Certainty equivalence and risk premium

An investor with his initial wealth of $10 is weighing the two investment options between risky and riskless asset. Risky asset offers the net return of 200% if the economy enters into the boom state. But if the economy gets into the stagnate condition, net return of the risky asset would be -100% (negative 100%). For the riskless asset, the net return is always equal to 5%. We suppose that the chance that economy would be entering into a booming and stagnating stage are equal.

Suppose that the investment has VNM expected utility given by,

$$u(W) = \frac{(W^{1-\gamma}) - 1}{1 - \gamma}$$

where W is the level of wealth at the end of the investment horizon.

Consider the following problems

5.1) Calculate the expected utility that can be derived under the full investment on risky asset. Show you calculate under two cases, i.e. $\gamma = 0$ and $\gamma = \frac{1}{2}$.

5.2) Using the information in (5.1), calculate the certainty equivalence and the risk premium under both cases of coefficient values.

5.3) Following (5.2), how can we relate the size of risk premium to the value of γ. What is the value of risk premium when γ is equal to 0. Explain the reason why you obtain the number from your calculation.

5.4) Calculate the coefficient of risk aversion, both Absolute and Relative. Describe the relationship between the two coefficients of risk aversion and γ.

<u>Question 6:</u> Risk and insurance

Assume that the preference of Ken is determined by the amount of net outstanding value of his asset, and can be represented by the following utility function,

$$U = 24w - 2w^2$$

where w is the net asset value, measuring in terms of million dollar.

Suppose that Ken currently owns a house whose current value is $8 million. Housing value would change over the course of event. Under the normal situation, his house will remain at $8 million. Under the unfortunate event when flooding occurs, his house would be partially damaged by $6 million, i.e. the savage value of his house would be $2 million.

6.1) Suppose that p is the probability that flooding will not occur. Calculate the expected utility of holding his house.

6.2) What is the attitude of Ken toward risk?

6.3) Calculate the certainty equivalence if we assume that p = ½.

Now, an insurance company is offering Ken an insurance contract that covers the loss of flooding. The contract says the followings. For every single dollar of the insured amount, Ken would need to pay the company $y. That is, if the insured amount under the coverage is equal to $x, Ken would need to pay for the premium of $xy. This will ensure that he can receive $y if the flooding event occurs. (We call $y as premium per each dollar of insured amount.)

6.4) Suppose that "p" is the probability of flooding. What is the maximum amount of $y that Ken is willing to pay? How does the amount vary with respect to "p"? Explain your answer.

Question 7: Optimal portfolio selection

Suppose that an investor has an initial wealth (W_0) of $100. The investor faces with an investment decision problem with two choices of assets to be chosen. One is the risky asset while the other is risk-free asset. Assume that in the good state, risky asset offer 30% of net return to the investor. The return earned under the bad state for the risky asset is 5%. Suppose that the return on risk-free asset is equal to 10%. Consider the following problem with the utility function of the said investor given by,

$$u(W_1) = k_1 * ln(W_1) + k_2 W_1$$

where W_1 is the terminal-period wealth after the investment. k_1 and k_2 are two positive constants.

7.1) Derive CARA and CRRA.

7.2) Suppose that "a" is the amount of investment on risky asset (in dollar). Derive the condition that characterizes the optimal mixture of investment on risky asset and risk free asset. (Hint: At this stage, you only need to provide the optimality conditions. No need to solve out the explicit solution.)

7.3) Calculate the value of "a" when $k_1 = 1$ and $k_2 = 0$. Does the investor use the leverage in his investment strategy?

Question 8:

Suppose that an investor has the VNM preference given by $= -\frac{e^{-aW}}{a}, a > 0$. Consider the following problems.

8.1) What is the attitude toward risk of this investor? Explain your answer.

8.2) Does the preference exhibit an increasing/decreasing/constant relative risk aversion?

8.2) Following the investment theory, do you think that the investor would invest in the risky asset if the expected return of the risky asset is lower than that of risk-free asset. Justify your answer with economic reasoning.

8.3) Suppose that expected return of risky-asset is higher than that of risk-free asset. Do you think the investor would change the fraction of investment on risky asset if the level of his initial wealth increases?

<u>Question 9</u> (Two risky assets and one riskless asset)

In class, we discussed about two assets world, i.e one risky and one riskless. This questions go over an alternative setting in which we have two risky assets, and continue that have one risk-free asset.

Suppose that the utility function of an investor is given by

$U(W) = \frac{1}{2}W^k$ when W is the terminal period wealth.

Consider two risky assets with the return described by the below table.

Asset	*Good (1/3)*	*Normal (1/3)*	*Recession (1/3)*
A	15%	8%	-8%
B	9%	4%	-4%
C	2%	2%	2%

Suppose that the investor has an initial endowment of $10. Consider the following problems

9.1) which one of the three assets as shown in the table can one classify as the riskless asset? Why?

Now, let's further define that Wa represents the amount of dollar value invested on asset A, and that Wb represents the amount of dollar value invested on asset b.

9.2) Describe all possible contingent values of terminal period wealth, i.e. W1|G, W1|N and W1|R. (Hint: your expression should be the function defined on Wa and Wb.)

9.3) Using the expected utility approach, derive for the optimal Wa and Wb when k = ½. Does the investor adopt the leverage strategy in the investment? Explain.

9.4) Redo the analysis in (9.3) above when k=1. (Hint: you can simply answer this question without doing any algebra if you have a clear economics intuition.)