Decision Under Uncertainty

EE431/438

Summary and Examples

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- uncertain situation: more than one state of the world
- at an instant time, only one state can occur
- if we can assign a probability to each state, we are in a situation of risk
- $\sum_{i=1}^{n} \pi_i = 1$, $E(X) = \sum_{i=1}^{n} \pi_i X_i$
- a fair price : the price = expected value
- people may or may not be willing to pay the fair price; depending on their degree of risk aversion

- Rational individuals will maximise expected utility
- Risk averters: certainty equivalence (CE) .... expected value of a gamble (EG)
- the gap between CE and EG is called ....
- Risk averters are not willing to pay the fair price of a gamble.
- The maximum price an investor would pay for a gamble is equal to its CE.
- A risky asset must compensate the investors for the risk ; risk premium

Mr.Sharp has an initial wealth of £36 and has two options. One option is for him to do nothing and keep the money. The other option is to buy a common stock of ABC company. This would cost him the £36. His wealth will then be £64 if the price of ABC common stock rises and £16 if it falls. There is a 50% probability that the price will rise. Mr. Sharp's utility function is U = $W^{0.5}$, where W is his wealth.

(a) What is Mr. Sharp's expected wealth if he buys the common stock? Does he prefer to buy the common stock?

(b) If the probability that the price will rise increases to 0.9, what will be Mr. Sharp's expected wealth? Will he prefer to buy the common stock now? Calculate the risk premium.

Review from last time (3) : State Preference Model

- the states of the world and the outcome for each state
- an asset which pay exactly \$1 if a particular state occur = a pure security or an Arrow-Debrue security
- there exists a pure security for every state of the world : the market is complete
- any security can be thought of as a combination of pure securities
- the price of any security can be calculated by using the price of the pure security
- a fair price of a pure scurity associated with state *i* is equal to the probability for state *i*

Security	Payoff state 1	Payoff state 2	Security Price
A	10	20	8
B	30	10	9

(a) Determine the prices of the two pure secuities

(b) If security C pay $10 if state 1 occurs and $0 otherwise, what should be the price of security C?

(c) If you want to buy a completelyrisk-free portfolio (i.e. one that has the same payoff in both states of nature), how many shares of *A* and *B* you would buy ?

- A risk averter is willing to pay buy an insurance to avoid the risk
- two states: good time and bad time
- $V(W_g, W_b) = \pi U(W_g) + (1-\pi)U(W_b)$
- initial wealth, $\overline{W}$; price of pure security for good time, P_G ; price of pure security for bad time, P_B
- budget constraint : $\overline{W} = P_G W_g + P_B W_b$
- $MRS_{X,Y} = \dfrac{P_x}{P_y}$; $MRS_{W_b W_g} = \dfrac{P_B}{P_G}$
- $\dfrac{MU_{W_{........}}}{MU_{W_{.....}}} = \dfrac{P_B}{P_G}$;
- If the price of the pure securities are fair; $W_g = W_b$, regardless of the form of the utility function
- the individual's wealth does not vary across the states; certainty line

Review from last time (5) : Insurance

- Example: insurance
- initially, $W_g = \overline{W}$, $W_b = \overline{W} - L$; the probability for the good state is π, the probability for a bad state is $1 - \pi$
- an insurance againt the bad state: price = $(1 - \pi)$
- an individual will buy L units of insurance
- $W_g = \overline{W} - (1 - \pi)L$, $W_b = \overline{W} - L - (1 - \pi)L + L = W_g$
- fair price = fully insurance
- Notice that the individual is willing to pay $(1 - \pi)$L upfront to avoid the risk