

Logit/Probit using Panel Data

The model: $p_{it} = \Pr[y_{it}=1] = E(y_{it} | x_{it}) = F(x'_{it}\beta)$

and

$$y_{it} = 1 \text{ if } y_{it}^* > 0$$
$$= 0 \text{ if } y_{it}^* \leq 0$$

where: $y_{it}^* = x'_{it}\beta + u_{it}$

Then,

$$\Pr[y_{it} = 1] = \Pr[y_{it}^* > 0] = \Pr[u_{it} > -x'_{it}\beta] = F(x'_{it}\beta)$$

Fixed-Effects Logit Model

For panel data, it is more likely to have individual fixed effect. Then, the model can be

with
$$y_{it}^* = x'_{it}\beta + \alpha_i + \varepsilon_{it}$$

$$\Pr[y_{it} = 1] = \Pr[y_{it}^* > 0] = \Pr[\varepsilon_{it} > -x'_{it}\beta - \alpha_i] = F(x'_{it}\beta + \alpha_i)$$

Equality of the last term can be held if F is symmetric around zero.

However, with the fixed effect α_i and a fixed T , when $n \rightarrow \infty$, number of α_i will increase, then, α_i cannot be consistently estimated for fixed T .

This is called *incidental parameters problem*.

Max. Conditional Likelihood Function

To solve the problem, maximize conditional likelihood function:

$$L_c = \prod_{i=1}^N \Pr \left(y_{i1}, \dots, y_{iT} \mid \sum_{t=1}^T y_{it} \right)$$

Proof: Assume 2 periods

Unconditional Function: $L_c = \prod_{i=1}^N \Pr(y_{i1}) \Pr(y_{i2})$

The condition $\sum_{t=1}^2 y_{it} = y_{i1} + y_{i2}$ can be 0, 1, or 2

1. $y_{i1} + y_{i2} = 0$ then $\Pr[y_{i1} = 0, y_{i2} = 0 \mid y_{i1} + y_{i2} = 0] = 1$
2. $y_{i1} + y_{i2} = 2$ then $\Pr[y_{i1} = 1, y_{i2} = 1 \mid y_{i1} + y_{i2} = 2] = 1$

For the first two cases $\log 1 = 0$, log-likelihood will not change, thus, they are not matter.

Max. Conditional Likelihood Function

Proof: Assume 2 periods (Cont.)

For Logit model: $\Pr[y_{it} = 1] = \frac{e^{\alpha_i + x'_{it}\beta}}{1 + e^{\alpha_i + x'_{it}\beta}}$

3.1 $y_{i1} + y_{i2} = 1$ then

$$\Pr[y_{i1} = 1, y_{i2} = 0 \mid y_{i1} + y_{i2} = 1] = \frac{\Pr[y_{i1} = 1, y_{i2} = 0]}{\Pr[y_{i1} + y_{i2} = 1]}$$

$$= \frac{e^{\alpha_i + x'_{i1}\beta}}{e^{\alpha_i + x'_{i1}\beta} + e^{\alpha_i + x'_{i2}\beta}} = \left(\frac{e^{\alpha_i + x'_{i1}\beta}}{e^{\alpha_i + x'_{i1}\beta}} \right) \frac{e^{\alpha_i + x'_{i1}\beta}}{e^{\alpha_i + x'_{i1}\beta} + e^{\alpha_i + x'_{i2}\beta}} = \frac{1}{1 + e^{(x_{i2} - x_{i1})'\beta}}$$

3.2 $y_{i1} + y_{i2} = 1$ then

$$\Pr[y_{i1} = 0, y_{i2} = 1 \mid y_{i1} + y_{i2} = 1] = \frac{\Pr[y_{i1} = 0, y_{i2} = 1]}{\Pr[y_{i1} + y_{i2} = 1]}$$

$$= \frac{e^{(x_{i2} - x_{i1})'\beta}}{1 + e^{(x_{i2} - x_{i1})'\beta}}$$

Fixed-Effects Probit Model

By using maximize conditional log-likelihood function, fixed-effect logit can be consistently estimated.

Due to different functional form, maximize conditional log-likelihood function will not lead to consistent estimators of the fixed-effect probit model.

Test for Fixed-Effects

Hausman-type test based on the difference between conditional MLE and logit MLE can be used and is χ^2 with K degree of freedom.

Random-Effects Model

Random effects MLE assumes $\alpha_i \sim N[0, \sigma_\alpha^2]$

The model can be estimated by maximizing the following log-likelihood function:

where:

$$\sum_{t=1}^{n_i} \ln f(y_{it} | \mathbf{x}_{it}, \beta, \sigma_\alpha^2)$$

$$f(y_i | \mathbf{X}_i, \beta, \sigma_\alpha^2) = \int_{-\infty}^{\infty} f(y_i | \mathbf{X}_i, \beta, \sigma_\alpha^2) \frac{1}{\sqrt{2\pi\sigma_\alpha^2}} \exp\left(\frac{-\alpha_i}{2\sigma_\alpha^2}\right)^2 d\alpha_i$$

Since there is no closed-form solution for this log-likelihood function, the estimation is usually employed numerically using *Guass-Hermite quadrature methods*.

Test Pooled vs Panel Logit Estimators

To test whether the pooled estimator or the panel estimator should be employed, the test whether $\rho = 0$ should be performed.

where:
$$\rho = \frac{\sigma_{\alpha}^2}{\sigma_{\alpha}^2 + \sigma_{\varepsilon}^2}$$

If $\rho = 0$, it means that panel-level variance component σ_{α}^2 is unimportant.

Then, panel logit estimator is no different from the pooled logit estimator.

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where:
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Then, panel probit estimator is no different from the pooled probit estimator.