EE312 Macroeconomic Theory

Chapter 7

A Two-Period Model:

The Consumption-Savings Decision

Agenda

- **Two-period model set-up and optimality conditions**

- Implications for behavioral responses of consumption

- Credit market equilibrium

Two-Period Model

- The consumer makes **intertemporal choice** between current consumption and future consumption.
  - Savings (lending) and dissaving (borrowing).
  - **The real interest rate** is the relative price of future consumption in terms of current consumption.
- Decisions to be affected by changes in the real interest rate and in current and future incomes.

The Consumer

- Assume the consumer receives **exogenous income (endowment economy)**.
- The consumer's budget constraint:
  - c = current consumption goods;
  - s = current **savings;**
  - y = current real income;
  - t = current lump-sum taxes.
  - The current disposable income (y - t) equals current consumption plus savings (c + s).

The consumer's budget constraint

$$c + s = y - t$$

- Assuming bonds directly traded in the credit market.
- If s > 0, the consumer is **a lender**.
- If s < 0, the consumer is **a borrower**.

- Assume one single real interest rate (r) for borrowers and lenders.

- One unit of **current consumption** can be exchanged for (1+r) units of **future consumption goods** in the credit market.
  - The relative price of future consumption in terms of current consumption is 1/(1+r).

Future budget constraint

- The consumer's future budget constraint:
  - c' = future consumption;
  - y' = future income;
  - t' = future taxes.
- If s < 0, the consumer pays the interest and principal on loan.

$$c' = y' - t' + (1+r)s$$

Lifetime (consolidated) budget constraint

$$c' = y' - t' + (1+r)s$$

$$s = \frac{c' - y' + t'}{(1+r)}$$

but $c + s = y - t$

$$c + \frac{c' - y' + t'}{(1+r)} = y - t$$

$$c + \frac{c'}{(1+r)} = y + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)}$$

- The PV of **lifetime consumption** equals PV of **lifetime income** minus PV of **lifetime taxes**.
- The lifetime disposable income is the same as **lifetime wealth (*we*)**

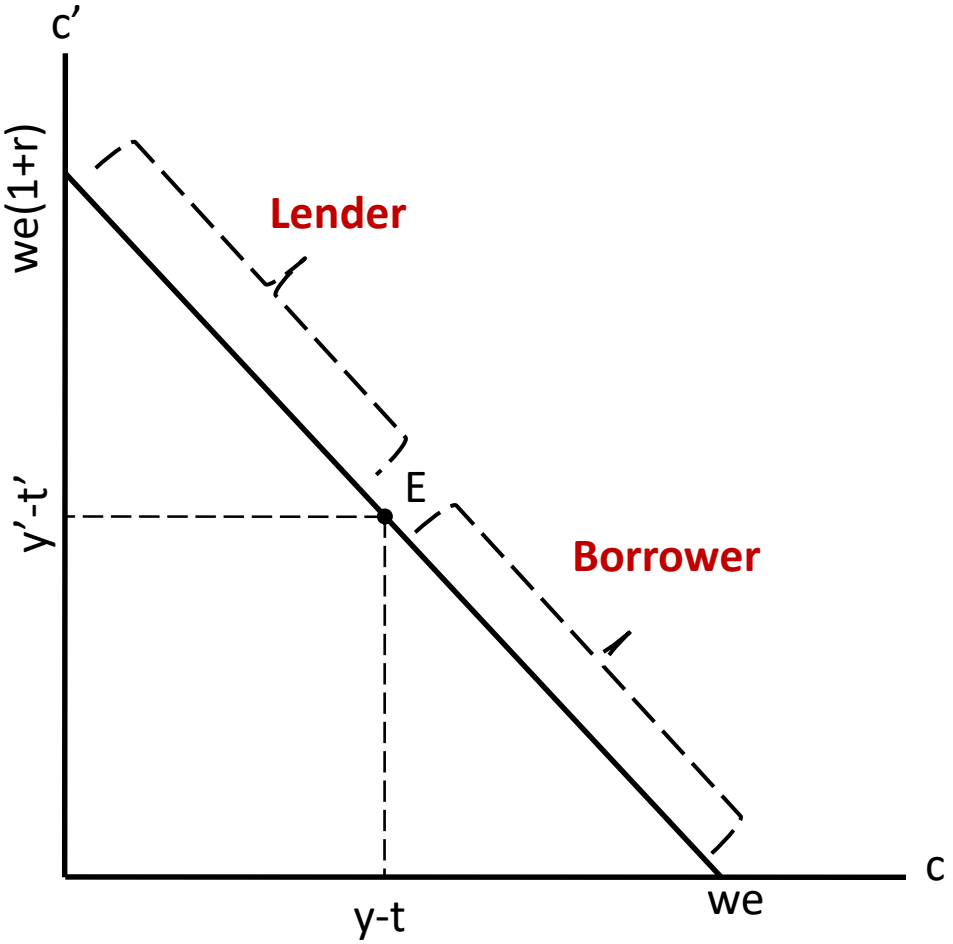
$$we = y + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)}$$

$$c + \frac{c'}{(1+r)} = we$$

$$c' = -(1+r)c + we(1+r)$$

Consumer's lifetime budget constraint

- E = endowment point.
  - If c = y – t and c' = y' - t', then s = 0.
  - Above E, the consumer is **a lender**.
  - Below E, **a borrower**.

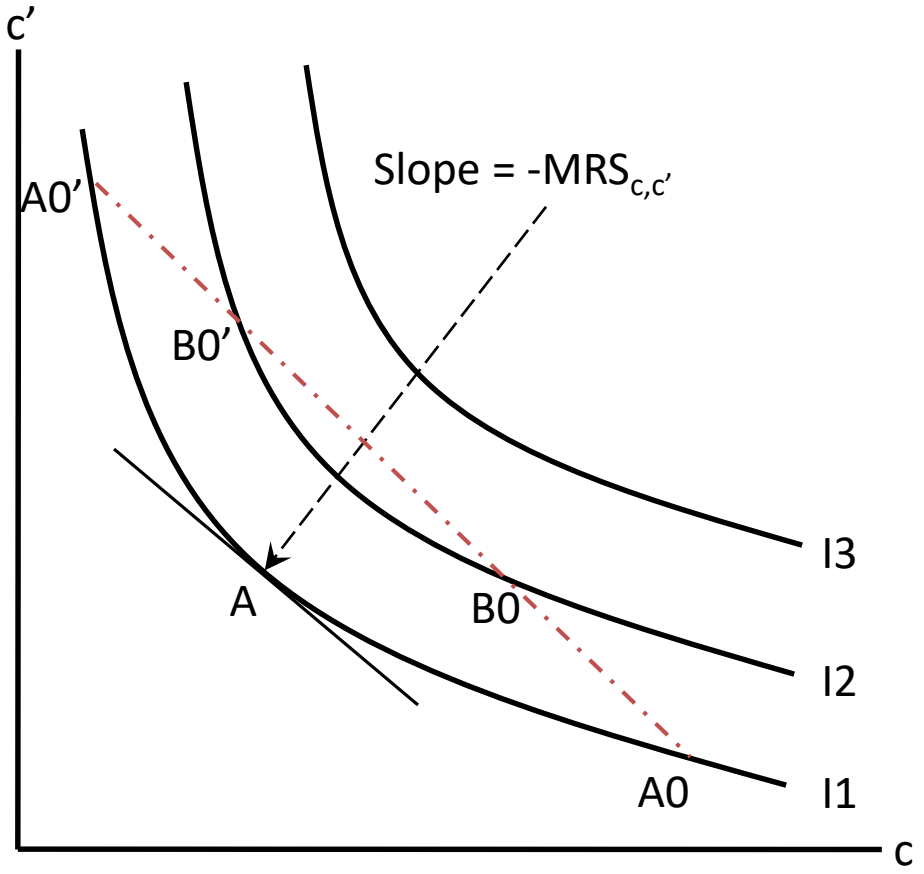


The Consumer's Preferences

- **A consumption bundle** is a combination of current and future consumption goods.
- Properties of consumer preferences:
  - More is preferred to less.
  - **Diversity** in the consumption bundle is preferred (*consumption smoothing*).
  - Current and future consumption goods are **normal goods**.

The consumer's indifference curves

- The ICs are downward-sloped and **convex**.
- Slope = -$MRS_{c,c'}$ = **the marginal rate of substitution** of c for c' is falling.



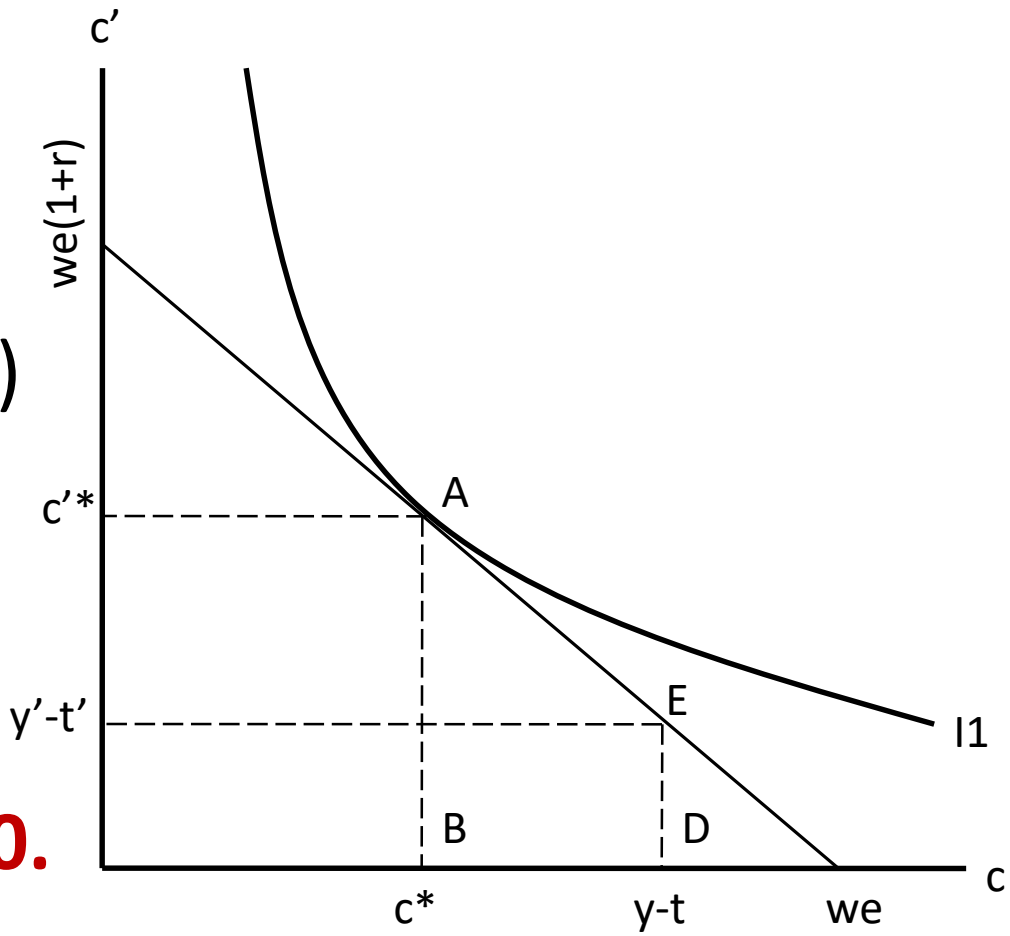
Consumer optimization

$$MRS_{c,c'} = 1 + r$$

- The consumer is optimizing where the marginal rate of substitution of c for c' equals the relative price of c in terms of c'.
  - **The willingness to trade off c for c' equals the market rate of trading c for c'.**
- The optimal consumption bundle is (c, c') = (c*, c'*).

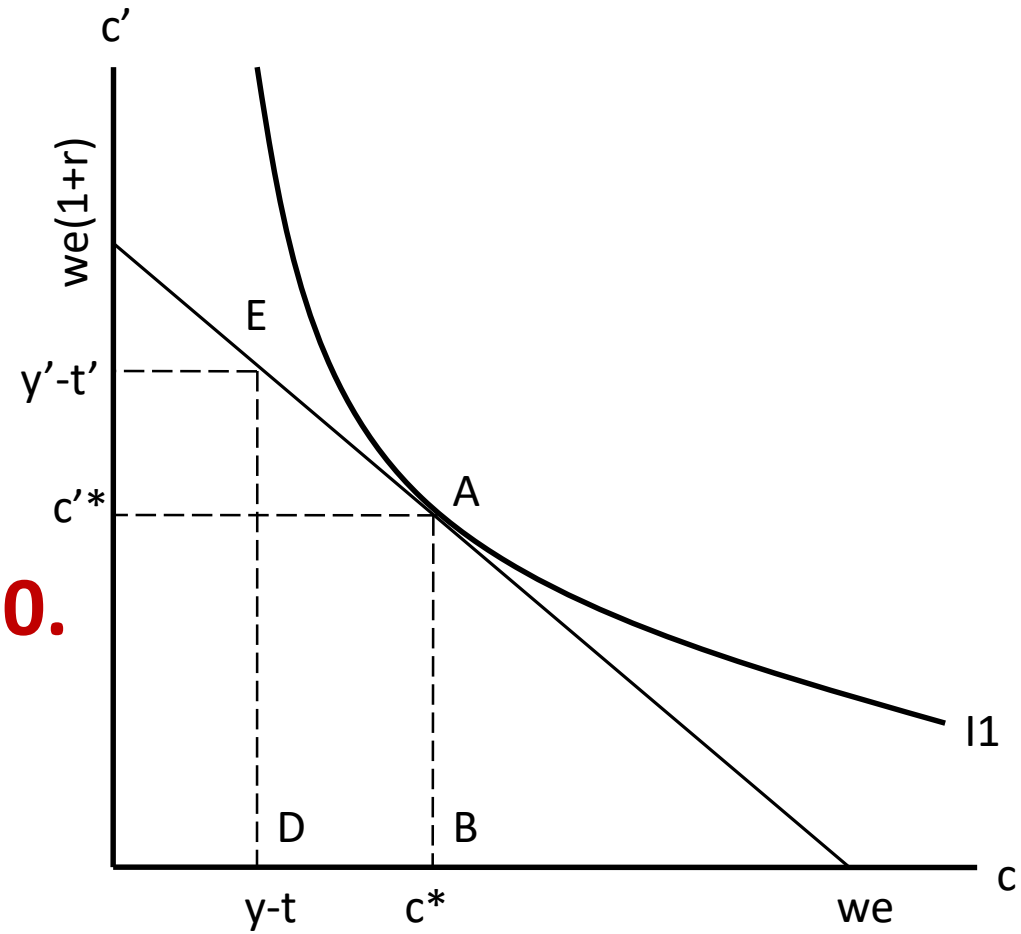
The consumer is **a lender**.

- At A, (c, c') = (c*, c'*) **and c*< (y - t).**
- s = y – t – c* = **BD > 0.**



The consumer is **a borrower**.

- At A, **c* > (y - t).**
- -s = c*- y + t = **BD < 0.**



Agenda

- ~~Two-period model set-up and optimality condition~~

- **Implications for behavioral responses of consumption**

- Credit market equilibrium

An increase in current income

- An increase in **current income** results in an increase in **lifetime wealth**.
  - A pure positive income effect.
  - The budget line shifts *horizontally* to the right.

$$we_1 = y_1 + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$we_2 = y_2 + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$\Delta we = we_2 - we_1 = y_2 - y_1$$

- Both current and future consumptions increase (normal goods).
  - The increase in c is **smaller** than the increase in y.
  - savings increase; hence, c' increases.
  - The consumer prefers **diversity** in the consumption bundle --- **consumption smoothing**.

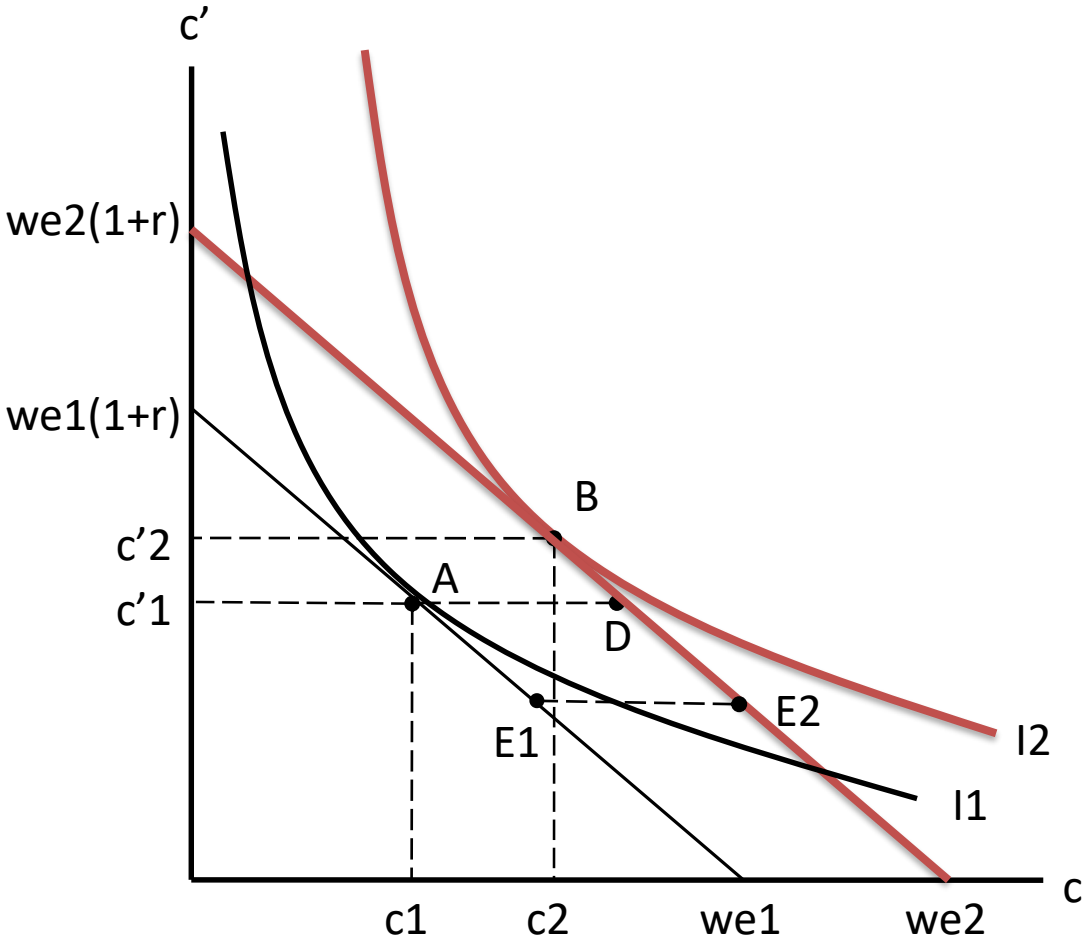
$$\Delta s = \Delta y - \Delta t - \Delta c$$

$$\Delta t = 0; \Delta y > \Delta c$$

$$\Delta s > 0$$

Increase in current y for **a lender**.

- Both c and c’ increase (A to B).
- $\Delta c = c_1c_2 < \Delta Y = AD$; $\Delta s > 0$.
- So c’ increases = $c'_1c'_2$, given r and t.



An increase in future income

- An increase in **future income** results in an increase in **lifetime wealth**.
  - The budget line shifts *vertically* to the top.

$$we_1 = y + \frac{y_1'}{1+r} - t - \frac{t'}{1+r}$$

$$we_2 = y + \frac{y_2'}{1+r} - t - \frac{t'}{1+r}$$

$$\Delta we = we_2 - we_1 = \frac{y_2' - y_1'}{1+r}$$

- Future consumption increases, but by a lesser amount than the increase in future income.
  - **savings** decrease; **current consumption** increases.
  - The increase in future income is smoothened into increases in **both** future and current consumption.

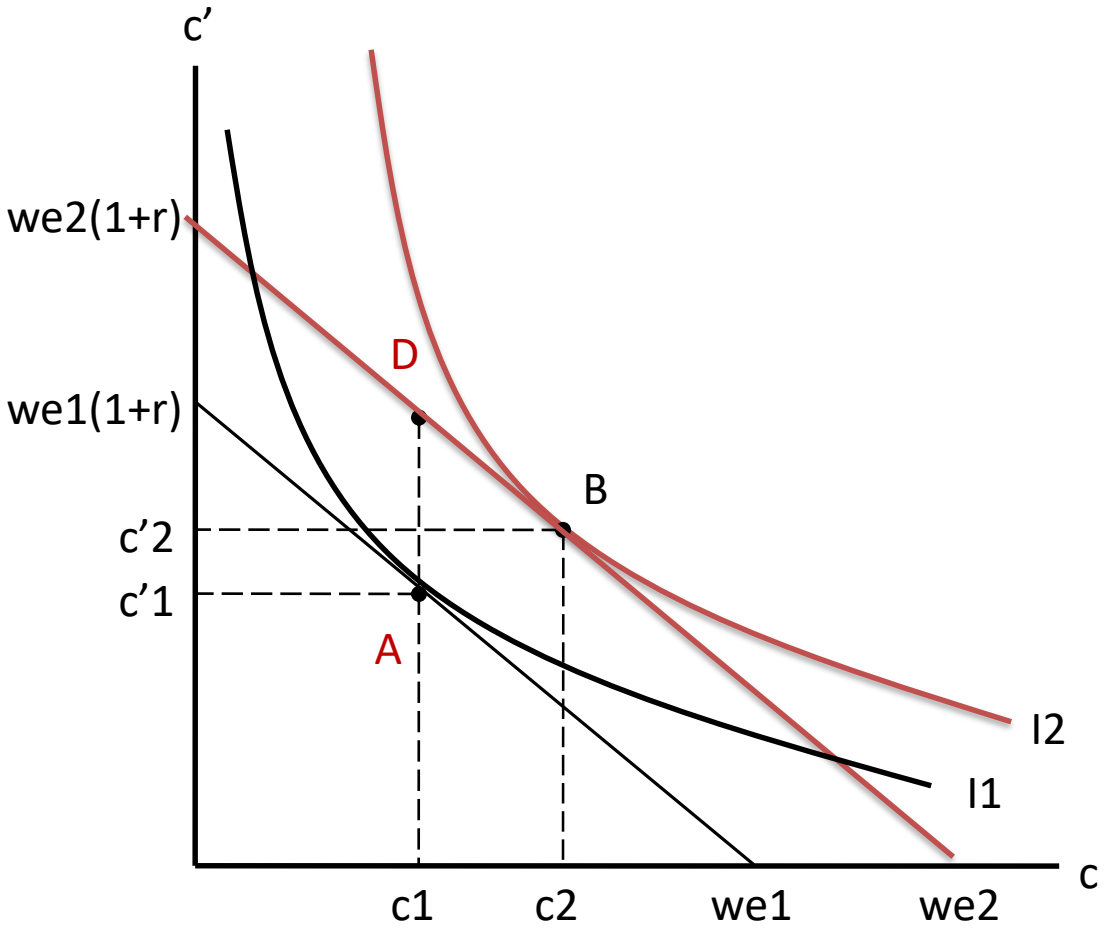
$$\Delta s = \Delta y - \Delta t - \Delta c$$

$$\Delta t = \Delta y = 0;$$

$$\text{So } \Delta s < 0; \Delta c > 0$$

Increase in future income

- Both c and c’ increase (A to B).
- $\Delta c' = c'_1 c'_2 <$ (**$\Delta y' =$ AD**);
- $\Delta s < 0$.

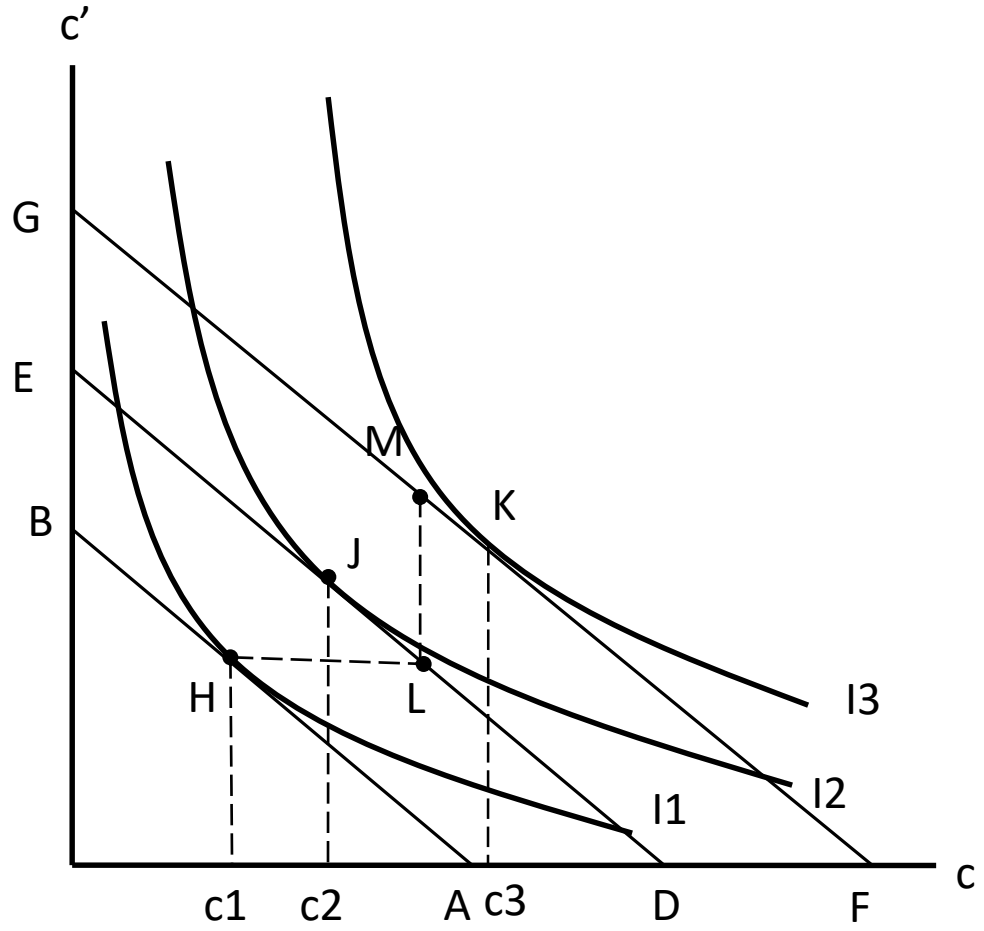


Temporary and permanent increases in income

- **A permanent increase** in income has a larger positive effect on lifetime wealth and current income than **a temporary increase**.
  - **Milton Friedman's 'permanent income hypothesis'**: the level of current consumption depends on the level of permanent income'.
- **The consumer will tend to save most of the temporary increase in income.**

Temporary versus permanent Δy

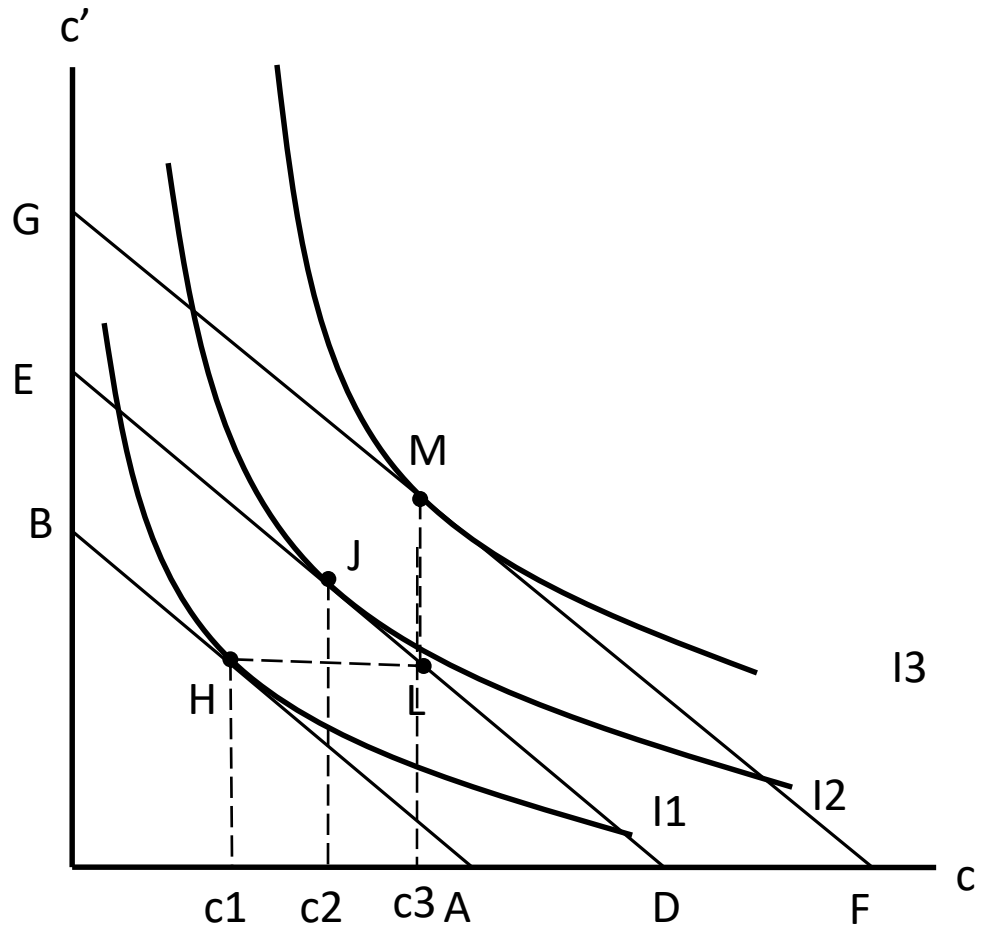
- HJ = effect of **temporary (one-period) rise** in y.
- HK = effect of **permanent rise** in y.



- **A temporary increase** in y = HL: the budget line shifts from AB to ED.
  - The consumption bundle rises from H to J.
  - Current consumption rises less than current income; savings increase --- **consumption smoothing**.
- **A permanent increase** in $y = y_2 - y_1 = y'_2 - y'_1$: the budget line shifts from AB to GF.
  - $y_2 - y_1 =$ HL $= y'_2 - y'_1 =$ LM.
  - The consumption bundle rises from H to K.
  - Larger effect on current consumption: c_1c_3

Bigger response under permanent Δy than temporary

- HJ = effect of **temporary (one-period) rise** in y.
- HM = effect of **permanent rise** in y.
  - So, saving does not change!
- **Effect on saving in then ambiguous**



Effect of a tax cut?

- The effect of the government's tax cut on consumption depends on whether the cut is temporary or permanent.
  - **If temporary**, the increase in consumption will be small; ***most of the increased income is saved***.
  - **If permanent**, the increase in consumption will be large.

An increase in the real interest rate

- 1/(1+r) is the relative price of future consumption in terms of current consumption.
  - The slope of the budget line is -(1+r).
  - A change in the real interest rate causes the budget line to rotate around the endowment point.
  - Changes in **intertemporal decision** between current and future consumption goods.
  - Analysis of **the substitution effect** and **income effect**.

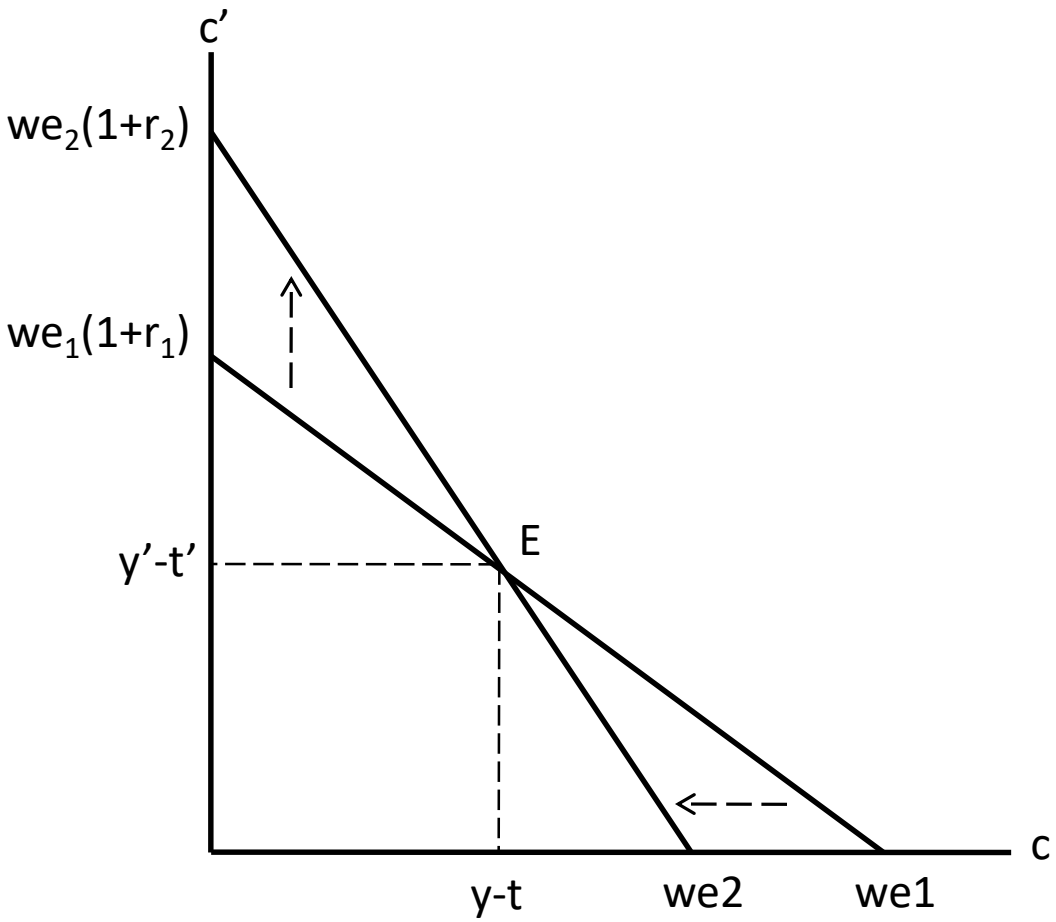
$$we = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$we(1+r) = (y-t)(1+r) + y' - t'$$

- The budget line rotates upwards as the real interest rate increases.
  - ‘we’ decreases (horizontal intercept).
  - we(1+r) increases if r increases (vertical intercept).
  - The endowment point (E) remains the same (no change in the initial endowment of y - t, y’- t’).

The real interest rate rises.

- The increase in the real interest rate rotates the budget line upwards with the same E.



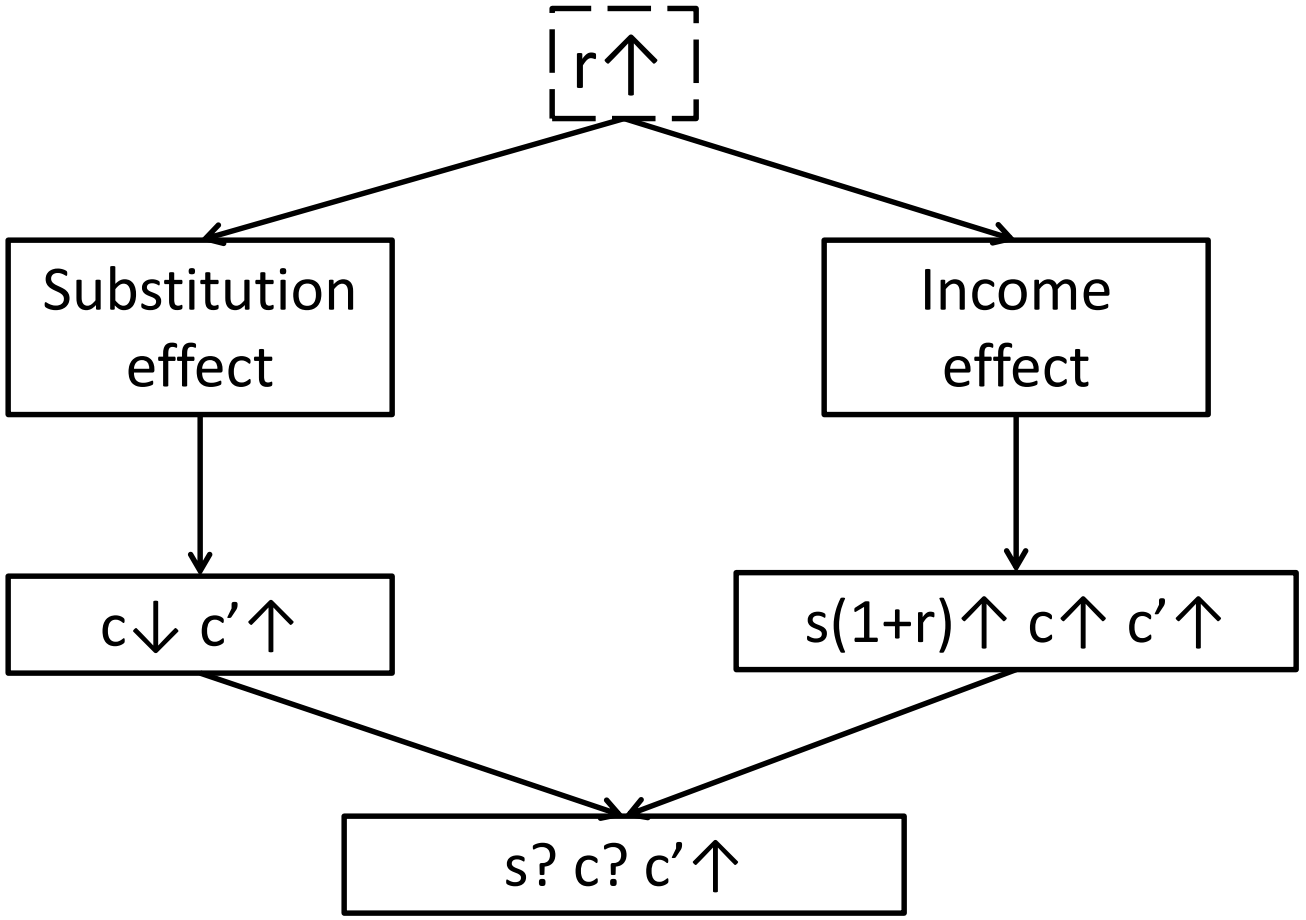
The dual effects of a higher r

- An increase in the real interest rate (r) causes a change in the relative price of current and future consumptions.
  - **Future consumption** becomes cheaper; **current consumption** more expensive.
  - Higher returns on savings for a lender; higher borrowing cost for a borrower.
  - **Lender**: cheaper future consumption.
  - **Borrower**: more expensive current consumption.

Effect of higher r on **the lender**

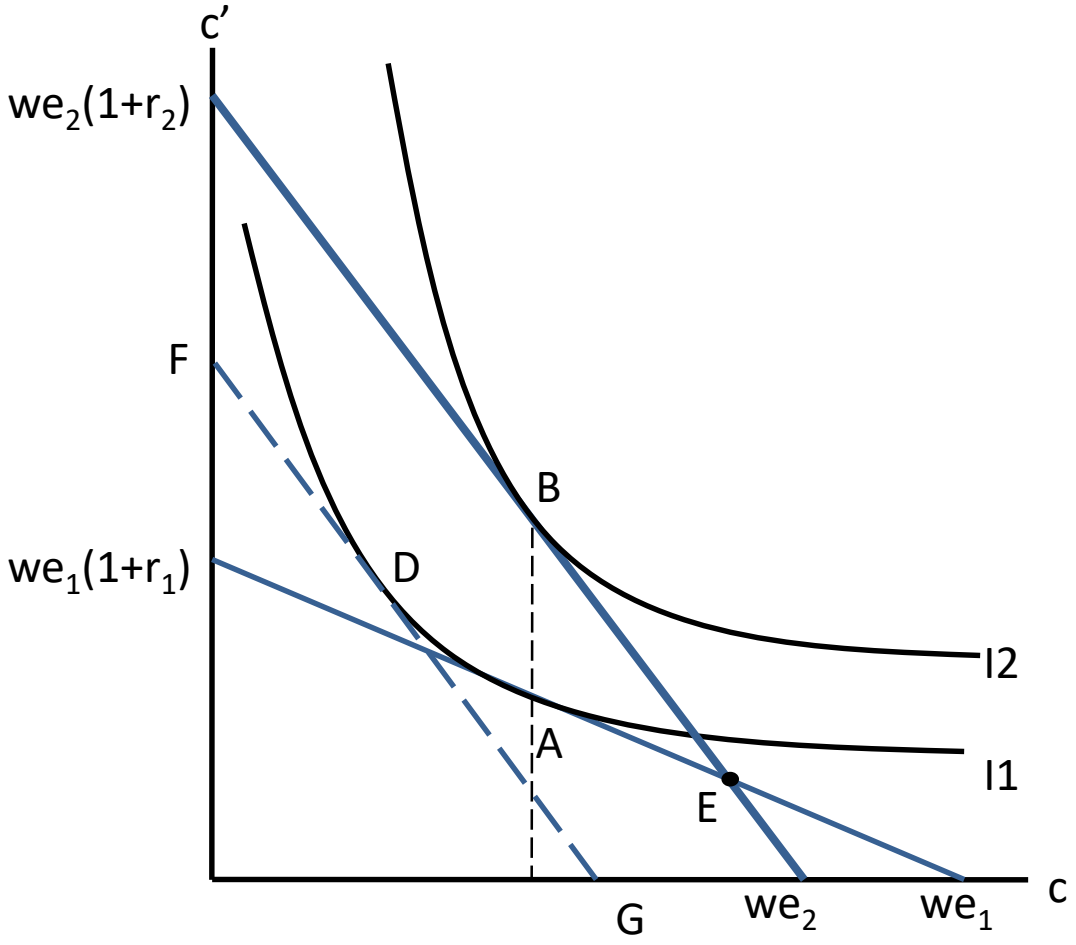
- Future consumption becomes cheaper in terms of current consumption.
  - **The substitution effect**: more future consumption for less current consumption.
  - **The income effect**: given savings yield more future income --- higher current and future consumptions.
- Higher future consumption; unclear current consumption and savings, given y and y'.

The consumer is a lender



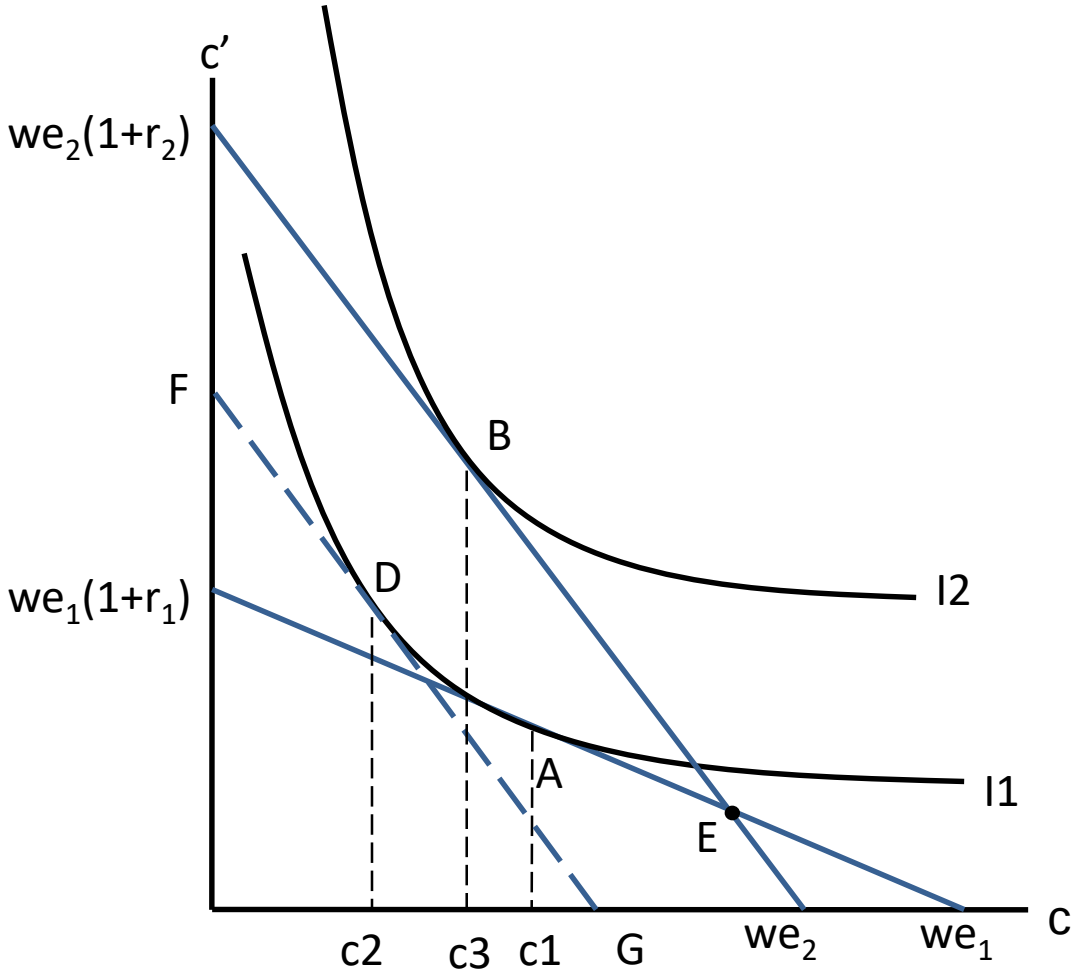
An increase in r for a lender.

- AD = **substitution effect**; lower c for higher c'.
- DB = **income effect**; higher c and c'.
- Net effect: higher c'; unclear c.



Stronger substitution effect

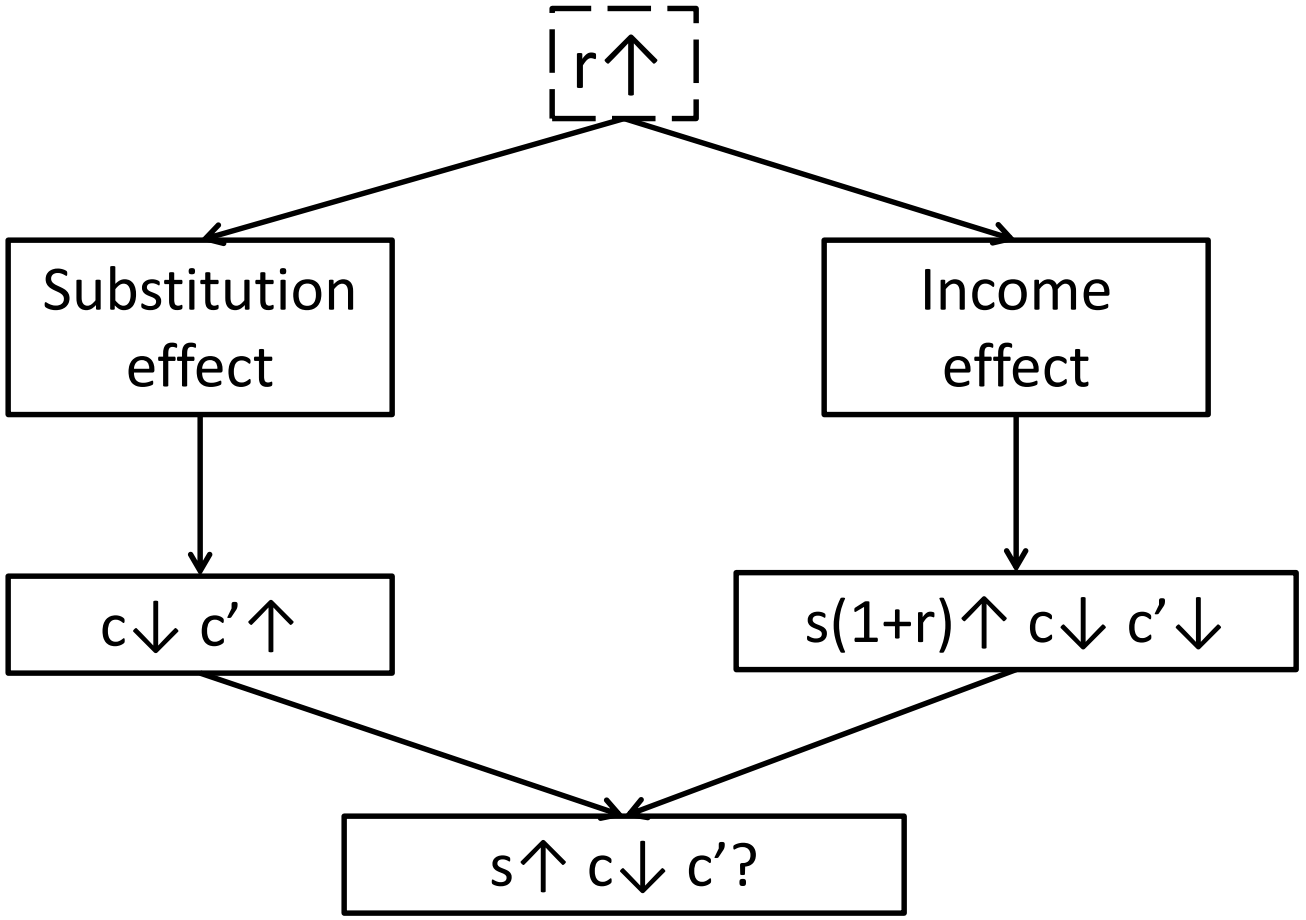
- AD = substitution effect; lower c.
- DB = income effect; higher c.
- AD > DB; net lower c at c3, **assuming a lender**.



Effect of higher r on **the borrower**

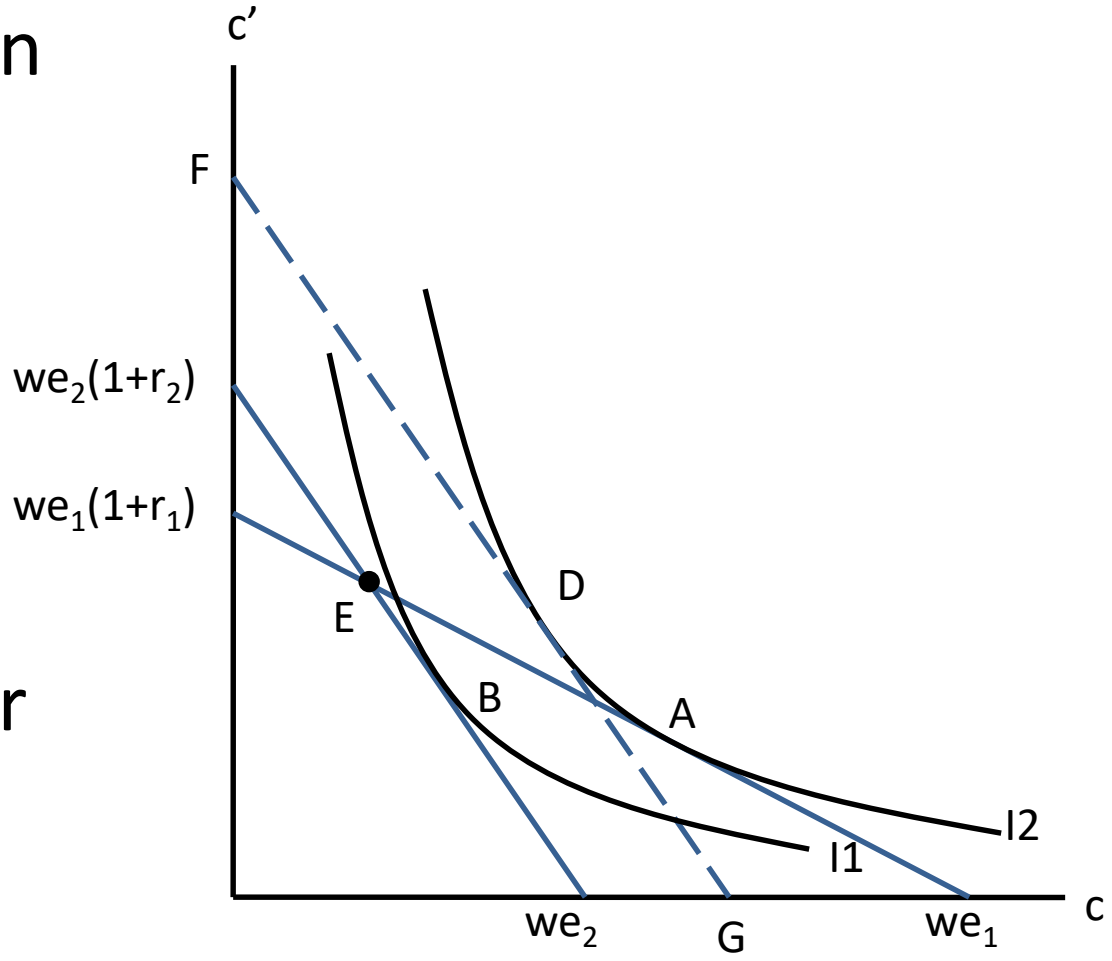
- Current consumption becomes more expensive in terms of future consumption.
  - **The substitution effect**: more future consumption and less current consumption.
  - **The income effect**: loans for current consumption become more expensive --- reduced current and future consumptions.
- Current consumption decreases while savings increase; unclear future consumption.

The consumer is a borrower



An increase in r for a borrower

- AD = substitution effect; lower c and higher c'.
- DB = income effect; lower c and c'.
- Net effect: lower c; unclear c'.



Conclusions on effects of Δr

- A higher real interest rate (r) has an **intertemporal substitution effect**.
  - Future consumption is substituted for current consumption --- savings increase.
- Positive income effect for lenders but negative income effect for borrowers.
- No theoretical certainty that current consumption will fall if the real interest rate rises; an empirical matter then.

Summing up: Behavioral responses

Variable	Current consumption	Saving
Income (increase)		
• Temporary		
Current	Increase/Small	Increase
Future	***(consumption smoothing)***	Decrease
• Permanent	Increase/Larger	Depends
Tax cut (more cut)		
• Temporary	Increase/Small	Increase
• Permanent	Increase/Large	Depends
Interest rate (increase)		
• Borrower	Decrease	Increases
• Saver	Depends	Depends

Agenda

- Two-period model set-up and optimality condition

- Implications for behavioral responses of consumption

- **Credit market equilibrium**

Credit market equilibrium

- Two key players
  - Private sector
    - Consumption / saving / borrowing decision
  - Government sector
    - Run fiscal policy (G and T)
    - Debt-financing policy by bond issuance

The government sector

- G = current government purchase of goods.
- G'= future government purchase of goods.
- T = current taxes collected by the government.
  - N = number of consumers, each paying the current tax of t; so T = Nt.
- T'= future taxes; and T' = Nt'.
- **Government borrows by issuing bonds (B) at the real interest rate of r.**

Government's current budget constraint

- Government spending is financed by taxes and bond issue in each period.
- Government's current budget constraint:
  - $G = T + B$ or $G - T = B$.
    - $B > 0$ government is a borrower;
    - $B < 0$ government is a lender.
- Government's future budget constraint:
  - $G' + (1+r)B = T'$

Government present-value budget constraint

- **The present value of government purchases must equal the present value of taxes.**

$$G = T + B$$

$$G' + (1+r)B = T'$$

$$B = \frac{(T' - G')}{1+r}$$

$$\textit{then} \quad G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

Competitive equilibrium

- Consumers and government interact in the credit market.
  - Trading of future consumption goods for current consumption goods through the **real interest rate**.
- **Equilibrium conditions:**
  - Each consumer optimizes current and future consumptions and savings, given r.
  - Government budget constraint holds.
  - The credit market clears.

The credit market clears.

- Aggregate private savings (S^p) equals government borrowing (B) or $S^p = B$.
  - National savings is the sum of aggregate private savings and government savings; $S = S^p + S^g$.

$$S = S^p + S^g$$

$$S^p = B; S^g = -B$$

$$S = B - B = 0$$

The income-expenditure identity

- The credit-market clearing implies that the income-expenditure identity holds.

$$S^p = Y - C - T$$

$$B = G - T$$

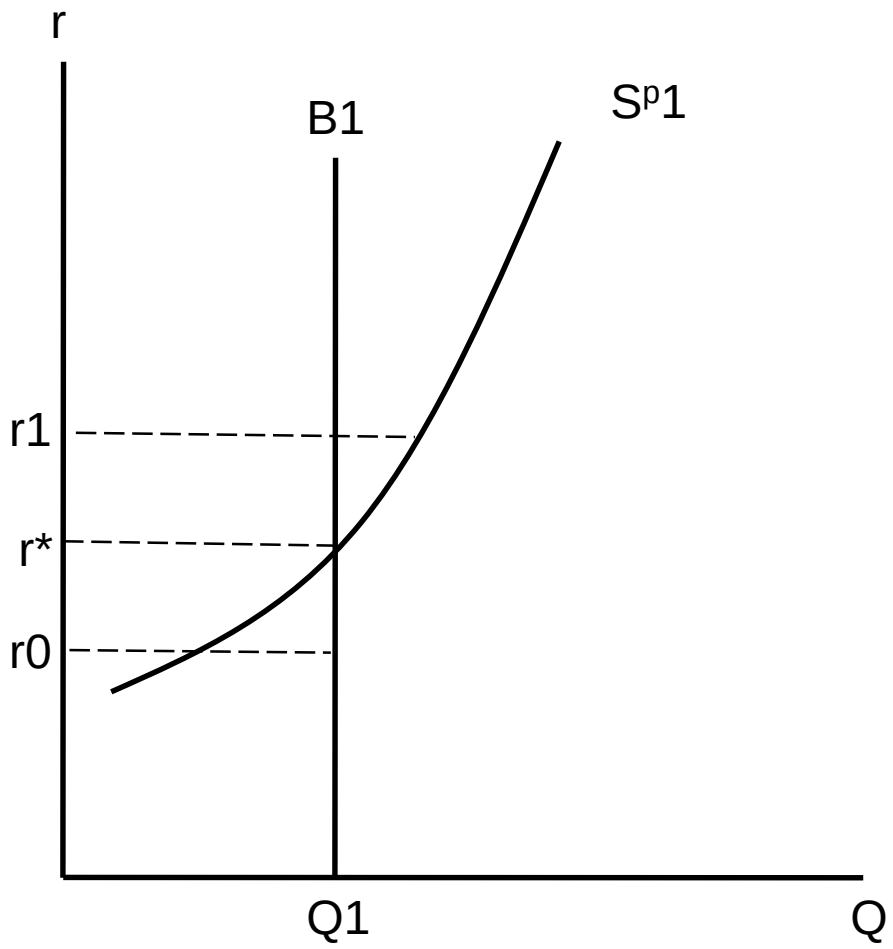
$$S^p = B$$

$$Y - C - T = G - T$$

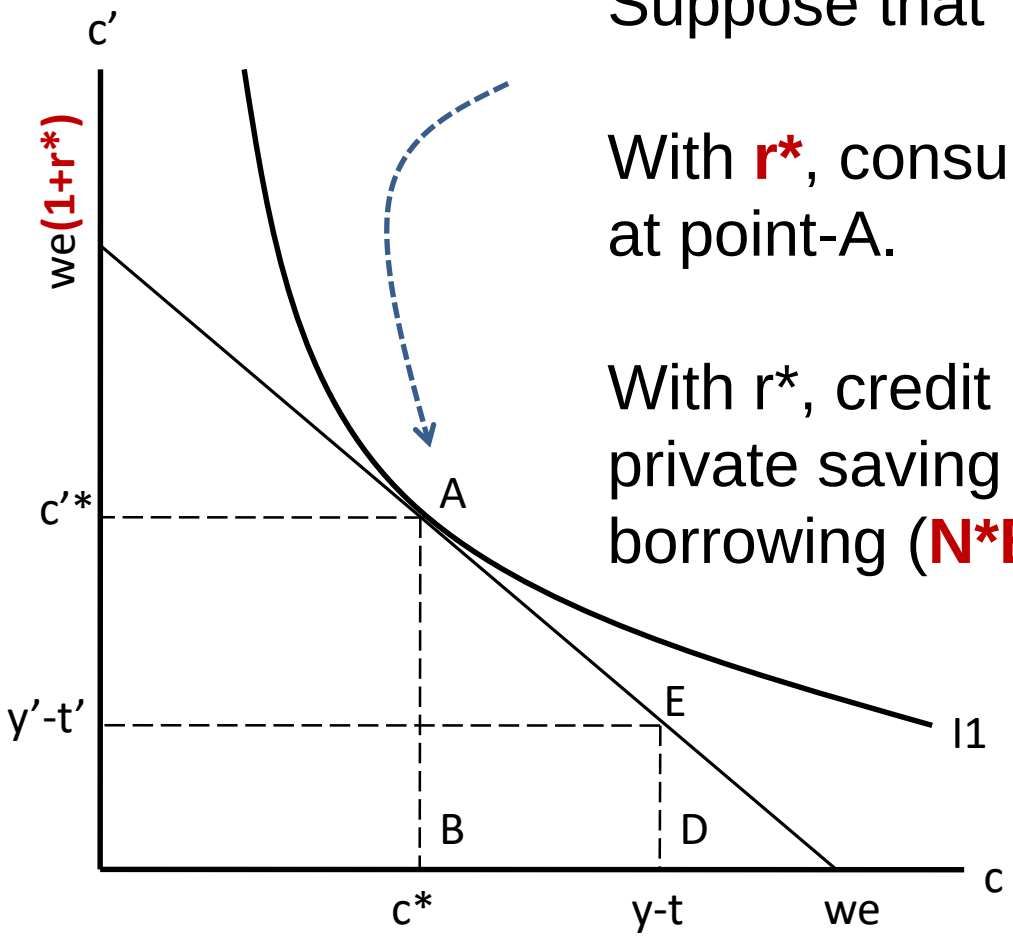
$$Y = C + G$$

Equilibrium representation

- Suppose a given budget deficit equal to G – T = B1
  - B1 is demand for credit
- r* is the equilibrium interest rate
  - r1: excess supply
  - r0: excess demand



Equilibrium representation



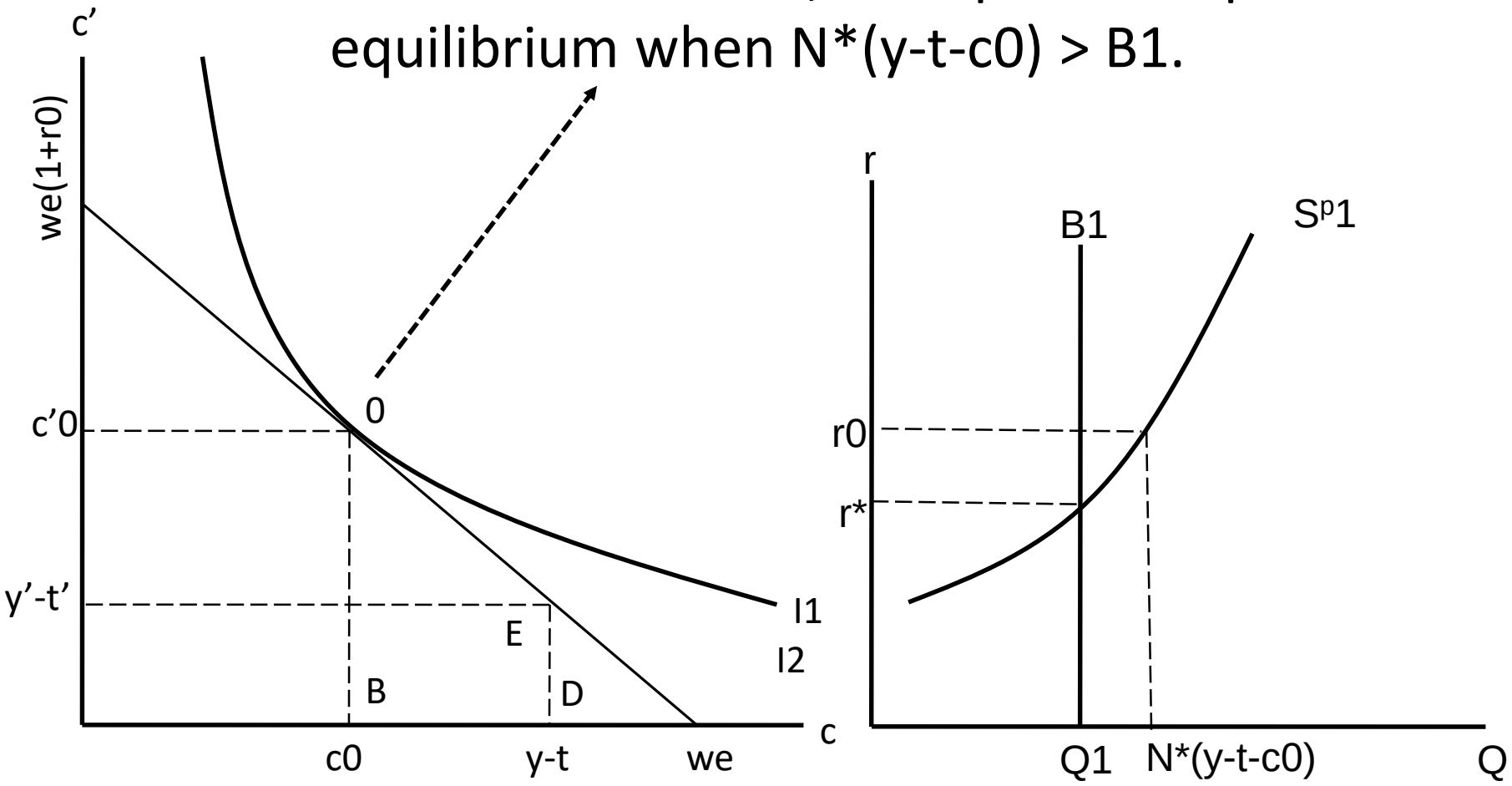
Suppose that N(y-t-c*) = G - T

With **r***, consumers are maximizing utility at point-A.

With r*, credit market is cleared; net private saving is equal to net government borrowing (**N*BD = G – T = B1**)

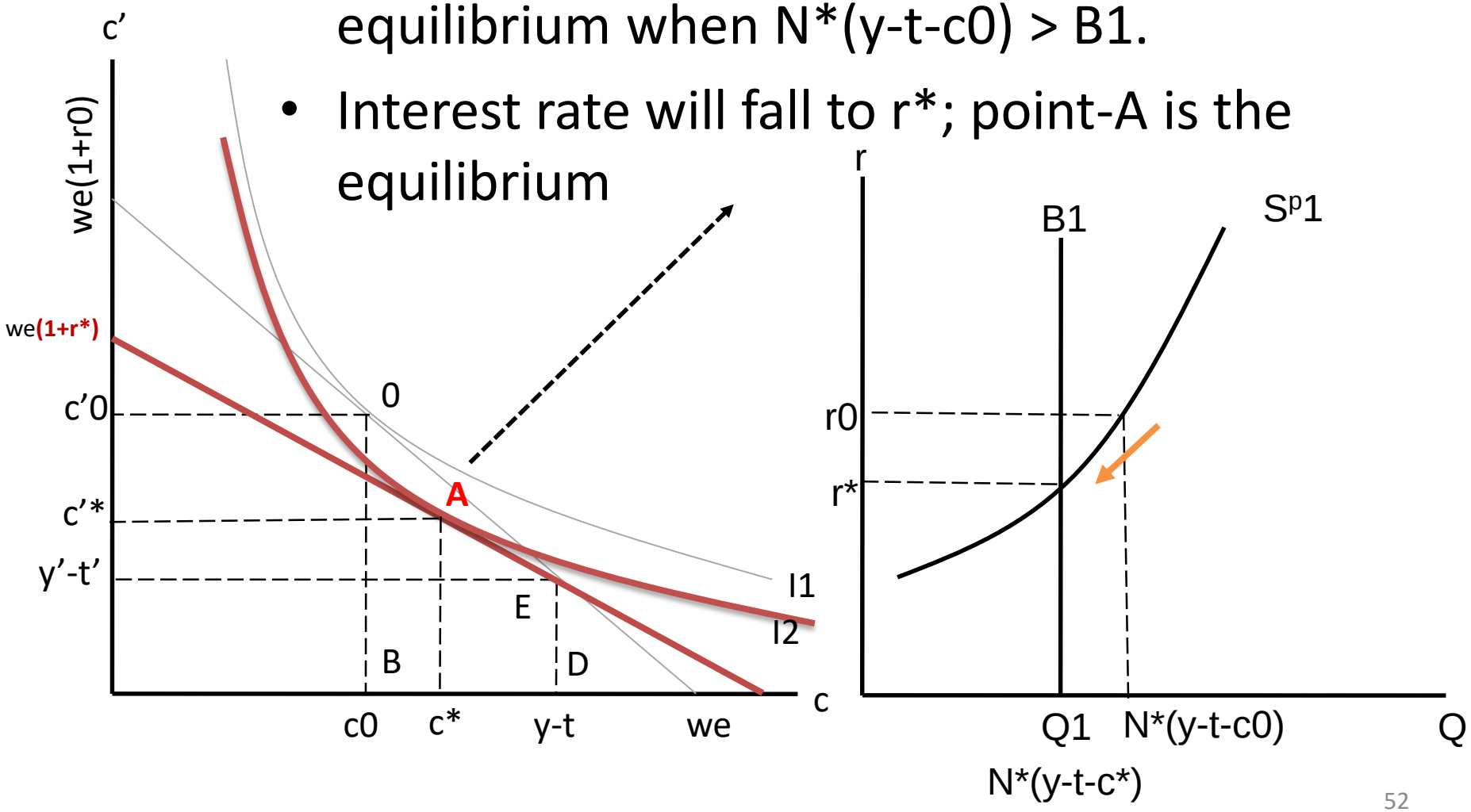
Disequilibrium and adjustment

- **Question**: Given r0, does point-0 represent the equilibrium when N*(y-t-c0) > B1.



Disequilibrium and adjustment

- **Question**: Given r0, does point-0 represent the equilibrium when N*(y-t-c0) > B1.
- Interest rate will fall to r*; point-A is the equilibrium



Impact of exogenous variables on equilibrium variables

- Taking variables {Y, Y', T, T', G, G'} as given, we can determine an equilibrium allocation and price, i.e. C* (current consumption), C*' (Future consumption), r* (real interest rate)

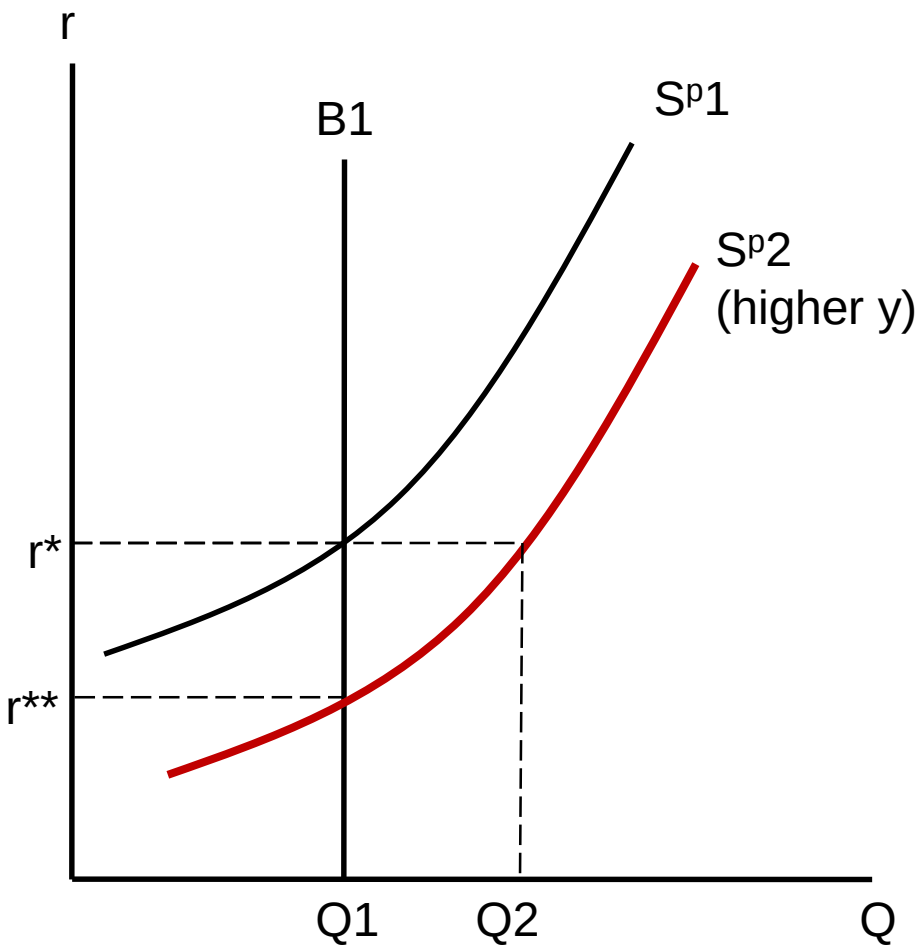
- **Question:** what happens to the equilibrium if the given exogenous variables are varied!

Questions

- How does an increase in current income affect (equilibrium) real interest rate etc.?
- How does an increase in future income affect (equilibrium) real interest rate etc.?
- How does an increase in permanent income affect (equilibrium) real interest rate etc.?
- **How does a tax cut affect (equilibrium) real interest rate etc.?**

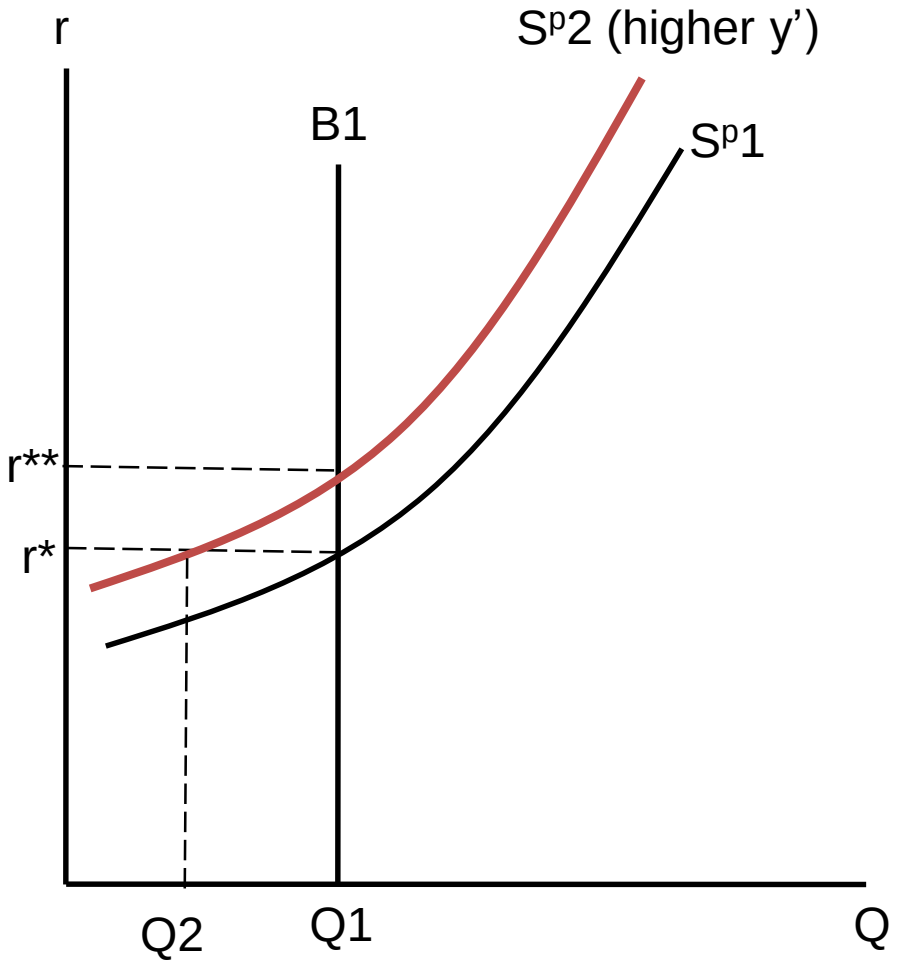
How does an increase in current income affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “r”.



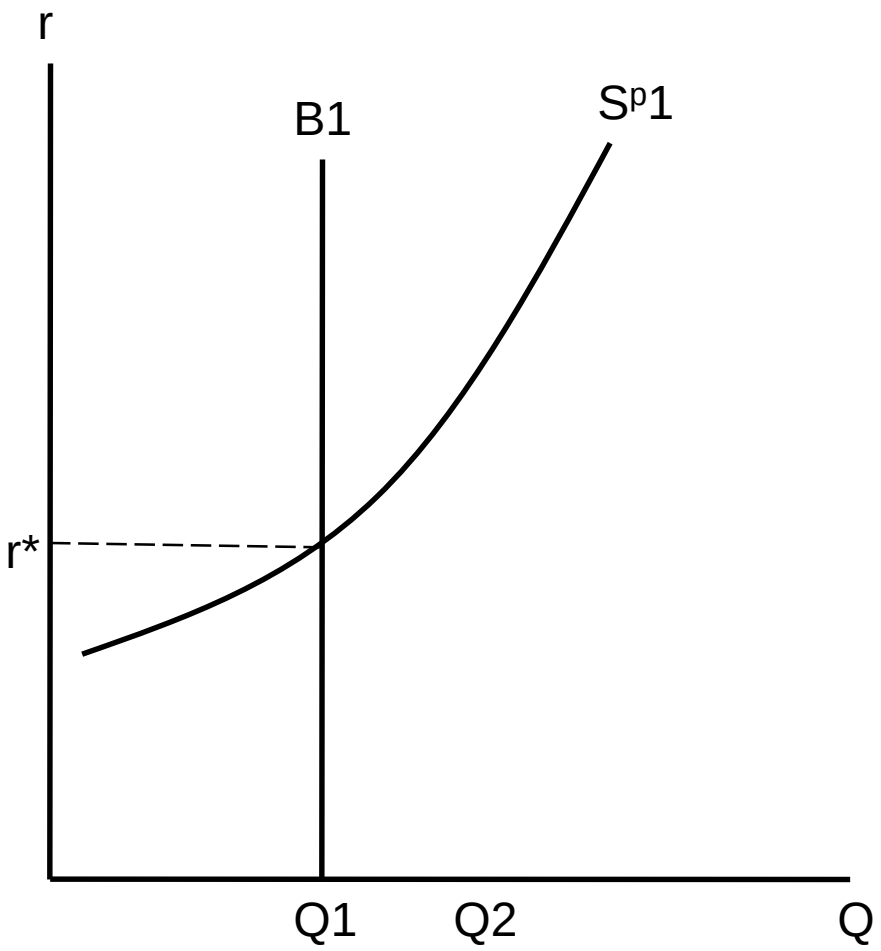
How does an increase in future income affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “r”.



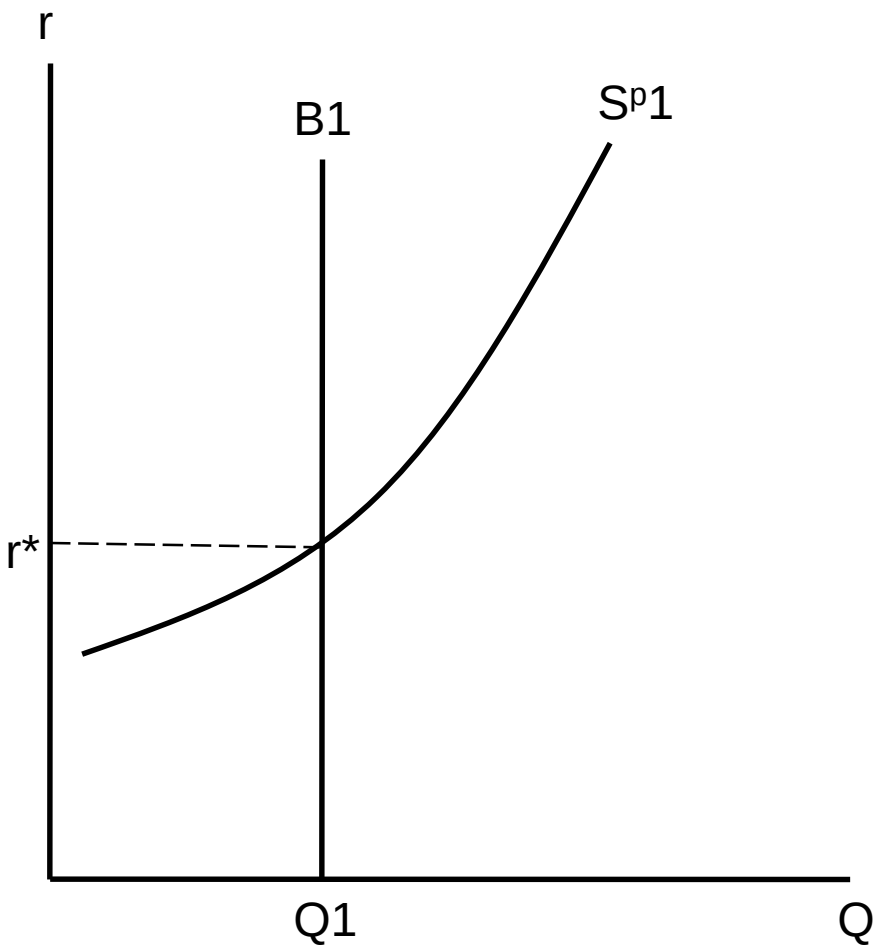
How does an increase in permanent income affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “r”.
- **Exercise!**



How does a tax cut affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “r”.



The Ricardian Equivalence

- A change in current taxes with *an equal and opposite change* in the present value of future taxes has **no effect** on the *real interest rate* and the *consumption of individual consumers*.
  - Assume equilibrium in the credit market, given r.
  - Current and future government spending are held constant.
  - Consumers' life-time budget constraint and government's present-value budget constraint hold.

Algebraic formulation

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

$$G + \frac{G'}{1+r} = Nt + \frac{Nt'}{1+r}$$

$$t + \frac{t'}{1+r} = \frac{1}{N}(G + \frac{G'}{1+r}) \quad (eq.8.24)$$

$$but\ c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N}(G + \frac{G'}{1+r}) \quad (eq.8.26)$$

$$t + \frac{t'}{1+r} = \frac{1}{N}(G + \frac{G'}{1+r}) \quad (eq.8.24)$$

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N}(G + \frac{G'}{1+r}) \quad (eq.8.26)$$

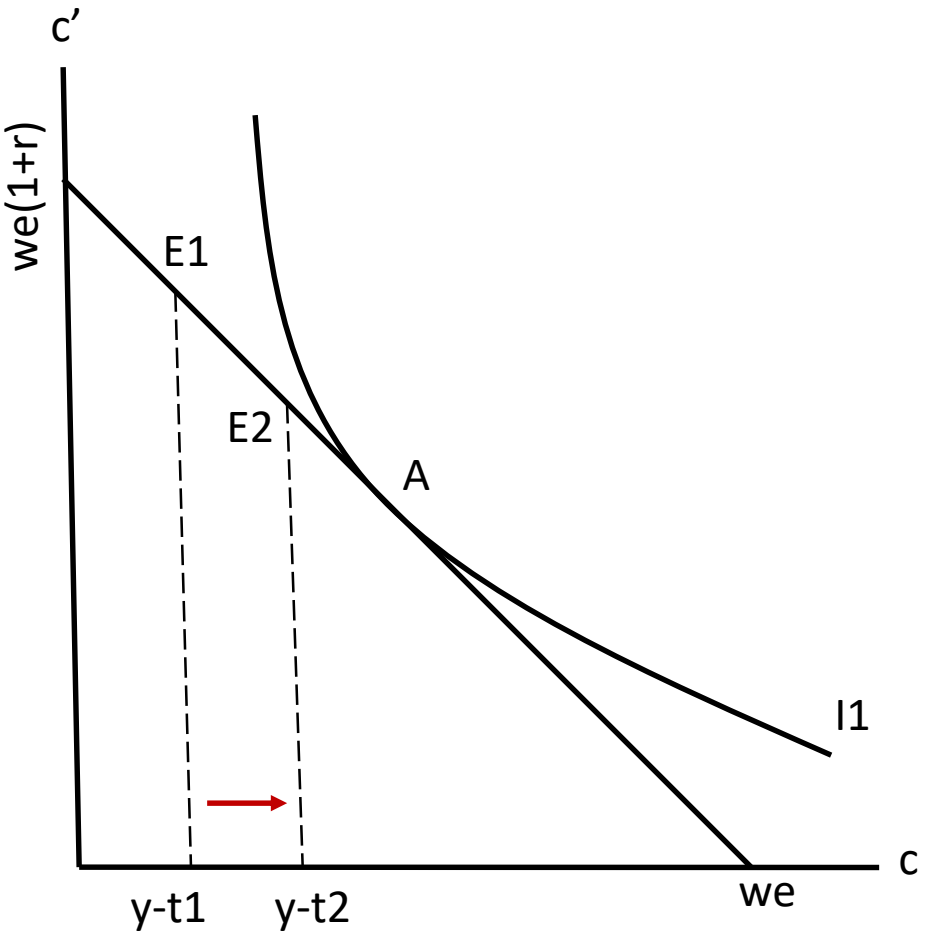
- The change in current taxes (Δt) must be matched by ***Δt' = –Δt*(1+r)*** so that equation 8.24 holds.
  - Equation 8.26 remains unchanged, given r (as y, y', G, G' and N are the same).
  - And Y = C + G; the credit market clears.
  - **No welfare change for consumers.**

- But private and government savings do change due to the **different timings of taxes**.
  - A decrease in current taxes increases private savings and reduces government savings by the same amount ($S^p = Y - C - T$ and $S^g = T - G$).

- **Consumers respond to a tax cut by *increasing private savings by the same amount*.**
  - Private savings increase to pay for higher future taxes.
  - The consumption bundle remains the same.
  - $\Delta S^p = \Delta B = \Delta T$ so the credit market equilibrium remains.

A current tax cut for a borrower

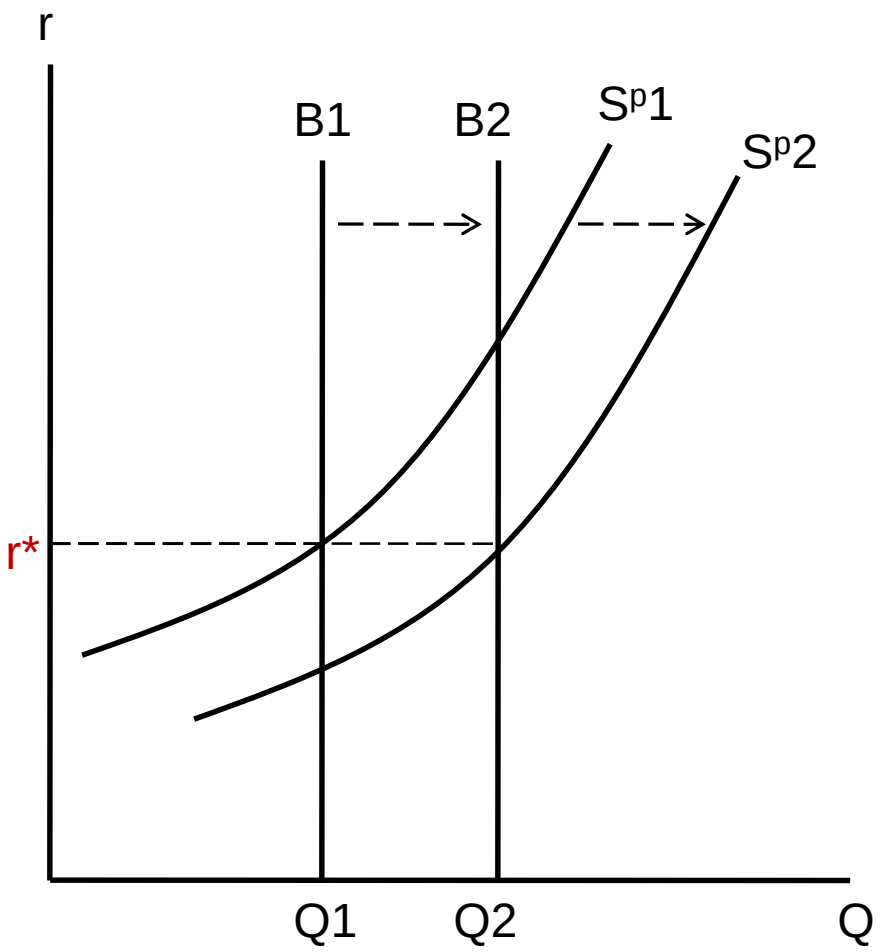
- **A current tax cut equals a future tax increase.**
- Lifetime wealth and consumption bundle (A) are the same.
- Only the endowment point changes from E1 to E2.



Unchanged credit market

- **Private savings and government borrowing increase by the same amount.**

- Equilibrium r is the same.



A tax cut is not a free lunch!

- A current tax cut gives all consumers higher current disposable income.
- But consumers must bear higher future taxes by the same amount.
- **No welfare gain for consumers!**

Ricardian equivalence assumptions: When it does not hold?

- A tax change affects every consumer by the same amount so that the present-value tax burden is unchanged.
  - If some consumers receive higher tax cuts, then their lifetime wealth and consumption choices (and the real interest rate) change.
  - Future tax burdens may be shared unequally.

- Government can redistribute wealth among income classes through tax policy!

- Government debt is paid off during the lifetimes of current consumers.
  - But the government can postpone debt payment (and future taxes) to next generations.
  - The old receive tax cuts and higher disposable incomes; the young pay higher future taxes.

- The government can redistribute wealth between generations!
  - The effect of the social security programs.

- The tax is lump-sum.
  - All taxes cause distortions in the relative prices and consumption choices.
  - Welfare loss is greater than tax revenues.

- **Perfect credit markets**: consumers can borrow and lend as much at the same interest rate.
  - But consumers have limits on borrowing.
  - The borrowing rate is higher than the lending rate.
  - Government borrows at a lower rate.
  - *Credit-constrained consumers* benefit from a tax cut.