B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 1/2015

Practice Problem 7

(Derivatives of More-Than-One Independent Variable Function)

Question 1: Determine $Z_x,\ and\ Z_y$

a. $Z = (x^2y+2)^2$

b. $Z = x^{1/2}y^{1/3}$

c. $Z = xy - ln(xy)$

Question 2:

Consider a function $Z = f(x,y) = x^3 + 3x^2y + 6xy^2 - y^3$

a. Derive the Hessian matrix.
b. Evaluate the value of the Hessian matrix where x = 2 and y =3

Question 3:

a. Given that $Z = \frac{x^3-y^3}{x^2y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

b. Given that $Z = \frac{x-y}{x+y}$, find $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = ?$

Question 4

Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2 .$$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

a. What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative
b. Is the product X considered an inferior product?
c. What is the level of quantity demanded if $P_x = 10,\ P_y = 25\ \ and\ I = 10$?
d. Calculate the own-price elasticity of demand, and evaluate the value when $P_x = 10,\ P_y = 25\ \ and\ I = 10$.
e. Calculate the cross-price elasticity of demand when $P_x = 10,\ P_y = 25\ \ and\ I = 10$.
f. Calculate income elasticity of demand when $P_x = 10,\ P_y = 25\ \ and\ I = 10$. Is the product a necessary or luxurious product?

Question 5: Consider a simple market model where

Demand: $q = -p + \sqrt{I}$
Supply: $q = 3p - w^2$

where q is quantity of output, p is price, I is the level of income, and w is the price of factor input

a) Derive the solution of all the endogenous variables.
b) Use the partial derivative to show that, under the equilibrium, an increase in W causes a decrease in quantity, but an increase in price.

c) Predict the impact of an increase in income on equilibrium quantity and price.

Question 6: Find the total differentials for the following functions:

a. $z = 4x^3 - 13xy - 6y^5$
b. $z = (2x^2 - y)(3x - 4y^3)$
c. $z = 8x^{\frac{1}{2}}y^{\frac{1}{4}}w^{\frac{1}{4}}$

Question 7: Suppose that Mr. A's utility depends on two commodities: x_1 and x_2. His utility function is given by

$$U = 25x_1 + 27x_2 - 3x_1^2 - 7x_1x_2 - 4x_2^2$$

(a) Determine the marginal utility function of each commodity.

(b) Find the marginal rate of substitution (MRS) between the two commodities when $x_1 = 2$ and $x_2 = 1$.

Question 8: Given the equation for the production isoquant

$$18[0.2K^{-0.4} + 0.8L^{0.4}]^{-2.5} = 1936$$

Use the implicit function rule to find the marginal rate of technical substitution (MRTS) of L for K.

Question 9: The demand for money, *M*, in the United States for the period 1929-1952 has been estimated as

$$M = 0.14Y + 76.03(r-2)^{-0.84} \quad , \qquad (r > 2)$$

where *Y* is the annual national income, and *r* is the interest rate measured in percent per year. Find $\frac{\partial M}{\partial Y}$ and $\frac{\partial M}{\partial r}$ and discuss their signs.

Question 10: Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using *L* units of labor. Assume that $f'(L) > 0$ and $f''(L) < 0$, so *f* is strictly increasing and strictly concave. If the firm gets *P* baht per unit produced and pays *w* baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$.

***Question 11**: Let the demand for a commodity be

$$Q_d = D(P, Y_0, T_0), \qquad \frac{\partial D}{\partial P} < 0; \frac{\partial D}{\partial Y_0} > 0; \frac{\partial D}{\partial t_0} < 0$$

$$Q_s = S(P, T_0), \qquad \frac{\partial S}{\partial P} > 0; \frac{\partial S}{\partial T_0} < 0$$

Where P is the price, Y_0 is the consumer's income, t_0 is the taste for the commodity and T_0 is the tax on the commodity. All derivatives are continuous. Use implicit function rule to find $\frac{\partial P^*}{\partial t_0}$, $\frac{\partial P^*}{\partial T_0}$, $\frac{\partial Q^*}{\partial Y_0}$, and $\frac{\partial Q^*}{\partial T_0}$. Discuss their economic implications.