Risk Preferences

PROSPECT THEORY, EE416 SEM2/2019

Expected Value Theory

What would the perfectly rational person do if faced with an uncertain world?

The initial attempt to define rational behavior actually looked at Expected Value (EV).

If you are faced with a choice of accepting a coin toss which **pays you** $10,000 if you win (heads with p=.5 odds) or **you pay** $10,000 if you lose (tails with 1-p=.5 odds), expected value theory says you should look at the probability weighted money outcomes to see if you should accept the bet.

Expected Value Theory

Let's say that you now have 30k in income this year to spend.

The expected value of the outcome is:

EV = .5*(30k + 10k) + .5*(30k – 10k) = 30k

Thus, if you compare the expected value of the bet to your certain income of 30k, you should be indifferent between accepting the bet and rejecting the bet!

Expected Value Theory

- Suppose you have a lottery, $X = (x_1, p_1\,;\, x_2, p_2\,;\, ...\,;\, x_n, p_n)$
  - payoffs: $(x_1, x_2, ..., x_n)$
  - probabilities: $(p_1, p_2, ..., p_n)$, where: $p_1 + p_2 + ... + p_n = 1$
- (Very) old models used expected value (EV) to make choices:

$$EV(X) = \sum_{i=1}^{n} p_i x_i$$

Expected Value Theory

- Example: Consider a choice between two options (or, lotteries)
  - Option #1: Coin toss that pays \$100 if heads, and \$0 if tails
  - Option #2: \$48 for sure

Expected Value Theory

- What is the expected value of a coin toss that pays \$100 if heads, and \$0 if tails?

$$EV = (.5)(\$100) + (.5)(\$0) = \$50$$

- Expected value theory suggests that we should choose this risky lottery over \$48 for sure
  - ...but most people prefer the sure thing!

St. Petersburg Paradox

The Paradox challenges the old idea that people value random ventures according to its expected return/expected value.

St. Petersburg Paradox

- Consider a gamble where you continue to flip a coin until it lands on heads – once heads comes up the game is over. Payoffs are as follows:
  - If heads comes up the 1st flip, you get $2
  - If heads comes up on the 2nd flip, you get $4
  - 3rd flip, you get $8
  - 4th flip, you get $16...
- How much would you pay for this bet?

St. Petersburg Paradox

- Most people willing to pay between \$5 and \$20. But consider the following probabilities:
  - 1^{st} heads on first flip: $Pr = \frac{1}{2} \implies$ get \$2
  - 1^{st} heads on second flip: $Pr = \frac{1}{4} \implies$ get \$4
  - 1^{st} heads on third flip: $Pr = \frac{1}{8} \implies$ get \$8, and so on...
- Expected value of this bet:

$$EV = \frac{1}{2}(\$2) + \frac{1}{4}(\$4) + \frac{1}{8}(\$8) + \frac{1}{16}(\$16) + ...$$

- This simplifies to:

$$EV = \$1 + \$1 + \$1 + \$1 + ... = \infty$$

Bernoulli's Solution

In 1738, Daniel Bernoulli noted that there was a difference between the actual value of money and the psychological value of money

We now call that psychological value: utility

Bernoulli noticed that people dislike risk, and want to avoid the worst outcomes at great cost, that is, they are risk-averse.

This can be explained by diminishing marginal utility of wealth

Bernoulli's Solution

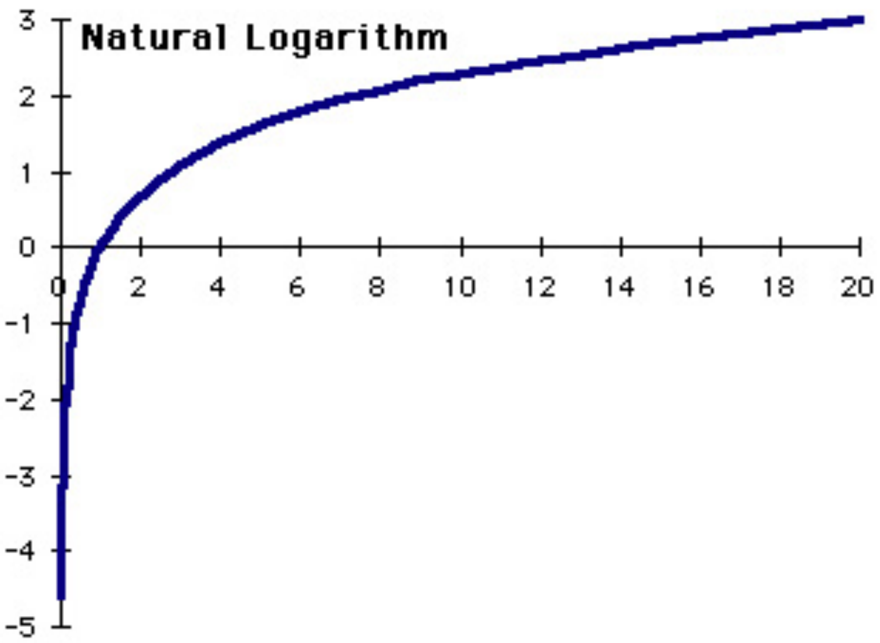
Bernoulli argued that the expected value measure was not correct because each successive increase in the amount of money you have increases your happiness less than the previous increase. This is called the principle of diminishing marginal utility where utility is a numerical measure of happiness.

Based on observed behavior (risk aversion) a reasonable utility function is : $U = \ln W$ where

U=utility, W= wealth, and ln denotes the natural logarithm.

Bernoulli's Solution

The natural log is a reasonable utility function for a risk averse individual.



Expected Utility theory

- (Very) old models used expected value (EV) to make choices:

$$EV(X) = \sum_{i=1}^{n} p_i x_i$$

- Updated models use expected utility (EU) instead:

$$EU(X) = \sum_{i=1}^{n} p_i u(x_i)$$

Expected Utility theory

If we go back to the "fair bet" example, we can show that most people will turn down even odds of 10k as follows.

Expected utility is the probability-weighted sum of utilities:

EU = .5*ln(30k + 10k) + .5*ln(30k – 10k) = .5*(10.60) + .5*(9.90) = 10.25 expected utils.

Which compares to a utility of refusing the bet of:

EU = 1.0*ln(30k) = 10.31 certain utils.

Clearly, a person who would like to be happier would refuse

the bet since 10.31 utils is better than 10.25 utils.

Bernoulli's Errors

- Expected Utility Theory has been widely accepted (and used) for many years, but it has its own issues.

- Consider this example:

Today Jack and Jill each have a wealth of $5 million.

Yesterday, Jack had $1 million and Jill had $9 million.

Are they equally happy?

Bernoulli's Errors

The expected utility model only takes into account final states of wealth, not changes in wealth.

- Jack and Jill both have final wealth of $5 mil (same final state).
- But they had different initial wealth (different changes).
- Kahneman & Tversky want to develop a model that allows changes in wealth to affect decision making.

Thought experiment A:

- Problem 1: Choose between…

Sure thing: win $900

Gamble: 90% win $1000, 10% win $0

- Problem 2: Choose between…

Sure thing: lose $900

Gamble: 90% lose $1000, 10% lose $0

Problem 1 vs Problem 2

- In Problem 1, most people...

Choose sure thing

Explained by risk aversion

- In Problem 2, most people...

Choose the gamble

Explained if people are risk-loving

Thought experiment B:

- Problem 3: In addition to whatever you own, you have been given $1000. Choose between...

Sure thing: win $500

Gamble: 50% win $1000, 50% win $0

- Problem 4: In addition to whatever you own, you have been given $2000. Choose between...

Sure thing: lose $500

Gamble: 50% lose $1000, 50% lose $0

Problem 3 vs. Problem 4

In this example, the final payoffs are exactly the same in the two problems!

- Sure thing: $1500
- Gamble: 50% win $1000 and 50% win $2000

Findings suggest that people are risk-averse over potential gains, and risk-loving over potential losses.

HOW CAN YOU BE BOTH RISK AVERSE/NEUTRAL IN GAINS AND RISK SEEKING IN LOSSES?

Thought experiment C:

- Would you accept the following bet? A coin toss where:

If head, win $150

If tail, lose $100

- Possible explanation?

Loss aversion: people feel the pain of a loss more acutely than the benefit of an equally-sized gain

Prospect theory

Kahneman & Tversky (1979) develop a new theory, Prospect Theory, to incorporate these observed behaviors.

Some key features they emphasize in their model:

- Evaluation of choices are made relative to a reference point.
- Diminishing sensitivity (risk-averse in gains, risk-loving in losses)
- Loss Aversion
- Probability weighting

Prospect theory

A person evaluates a prospect $(x, p; y, q)$ according to the functional

$$V(x, p; y, q) \quad = \quad \pi(p)\, v(x) \quad + \quad \pi(q)\, v(y).$$

Reminder: EU theory says use

$$U(x, p; y, q) = pu(w + x) + qu(w + y) + (1 - p - q)\, u(w)$$

What's new?

- $\pi(\cdot)$ is the probability-weighting function.
- $v(\cdot)$ is the value function.

Prospect Theory: Value function

Prospect theory attempts to explain risk aversion in gains and risk loving in losses by positing that the utility function is fundamentally wrong from biological principles as follows:

Prospect Theory: Value function

- The nervous system is set up to primarily detect differences, not absolute levels.
- A gain is perceived as a pleasurable change from the status quo (reference point) and the nervous system shows a decreasing response both to the intensity and duration of pleasurable stimuli.
- A loss is perceived as a painful change from the status quo (reference point) and the nervous system shows a decreasing response both to the intensity and duration of painful stimuli.

Prospect Theory: Value function

- The reference point is usually where you are (the status quo), but can be where you would like to be, or think you should be.
- We do not yet fully understand how reference points are determined, but they should be subject to phenomena such as adaptation and social pressure (social norms), etc.

Utility function vs. Value function

- Utility function:
  - Evaluation of choices are made over final states of wealth
  - x-axis = wealth, y-axis = utility
- Value function:
  - Evaluation of choices are made relative to a reference point
  - x-axis = gain/loss, y-axis = value

Utility function vs. Value function